

Basics of Gauge-Higgs Unification



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2019/06/07 Lecture@E-ken

Requested topics

- (1) DM candidates in GHU
(arXiv:1704.04621, 1801.00686, 1803.01274)
- (2) Calculations of Higgs mass
(arXiv:hep-ph/0603237 etc)
- (3) GHU review
(Hosotani mechanism, finiteness of Higgs mass, SU(3) model etc)
- (4) Higgs phenomenology@LHC in GHU
(or future collider)

Requested topics

- (1) DM candidates in GHU $\Rightarrow$ seminar
(arXiv:1704.04621, 1801.00686, 1803.01274)
- (2) Calculations of Higgs mass $\Rightarrow$ lecture
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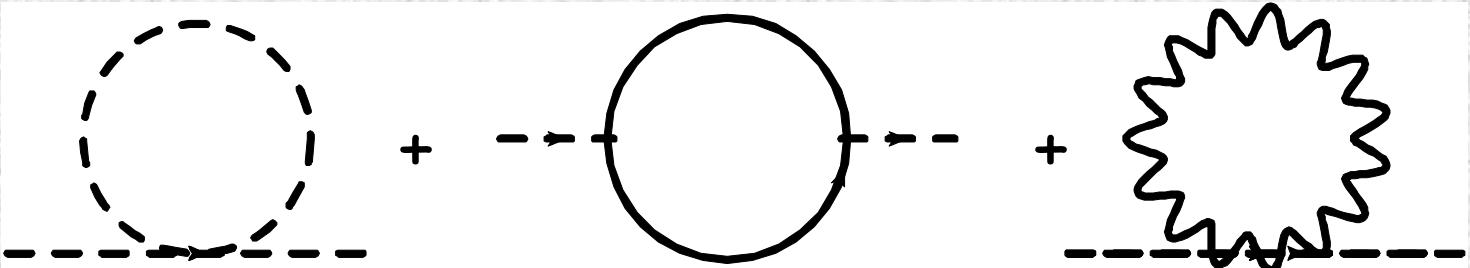
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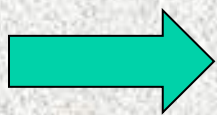
- ◆ Introduction
- ◆ Higgs mass calculation
- ◆ Gauge-Higgs sector
- ◆ Matter content &
Yukawa coupling
- ◆ EW symmetry breaking
- ◆ Summary

Introduction

One of the problems in the Standard Model:
Hierarchy Problem

Quantum corrections to the Higgs mass
is sensitive to the cutoff scale of the theory

$$\delta m_H^2 = \text{[diagram 1]} + \text{[diagram 2]} + \text{[diagram 3]} + \dots$$
The diagram shows three Feynman diagrams representing quantum corrections to the Higgs mass. Each diagram consists of a horizontal dashed line representing an external Higgs boson, with a loop attached to it. The first diagram has a dashed loop. The second diagram has a solid loop. The third diagram has a loop with a wavy, sun-like pattern. The diagrams are separated by plus signs, and the sequence ends with an ellipsis.



$$\delta m_H^2 \approx \frac{\Lambda^2}{16\pi^2}$$

Too large!!

(Natural cutoff scale is
Planck scale or GUT scale)

To get Higgs mass 125GeV ,
an **unnatural fine tuning of parameters** are required

$$m_H^2 = m_0^2 + \delta m^2 \approx \mathcal{O}\left((100 GeV)^2\right)$$

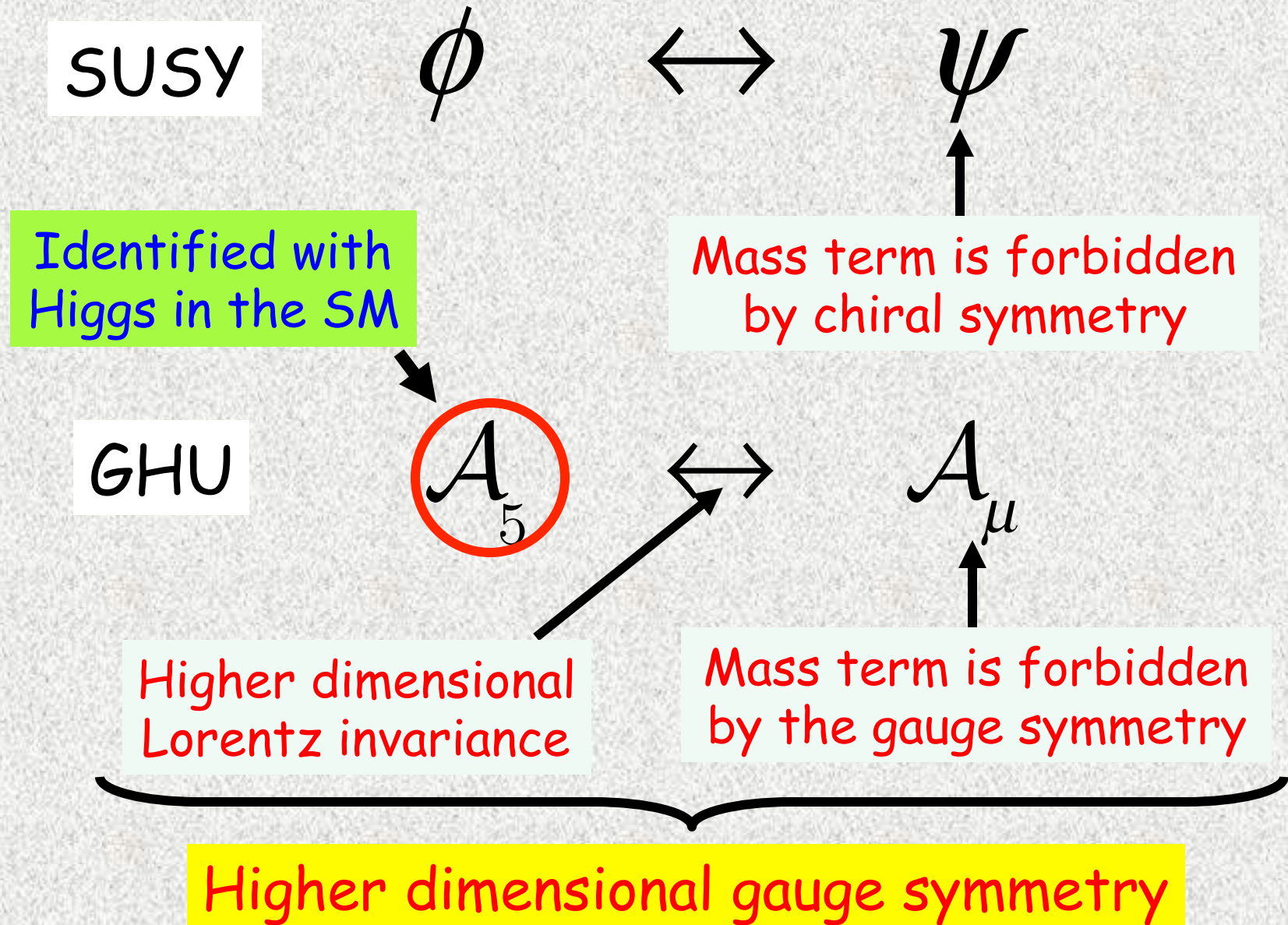
classical Quantum
corrections

Naively, we have $m_0^2, \delta m^2 \approx \mathcal{O}\left(\left(10^{18} GeV\right)^2\right)$

32 digits of fine tuning

[illegible]

Problem: We have **NO symmetry** forbidding the scalar mass



Indeed, the (local) mass term A_5^2 can be forbidden by the gauge symmetry for 5th component of the gauge field

$$\because A_5 \rightarrow A_5 + \partial_5 \mathcal{E}(x, y) + i \left[\mathcal{E}(x, y), A_5 \right]$$

In other words, no local counter term is allowed
 $\Rightarrow$ **No quadratic divergence, finite**

This symmetry is very useful in the orbifold model since it is operative even on the branes $G \rightarrow H$

Gersdorff, Irges & Quiros (2002)

$$\because A_5 \rightarrow A_5 + \partial_5 \underbrace{\mathcal{E}_{G/H}(x, y)}_{\mathbb{Z}_2 \text{ odd}} + i \left[\underbrace{\mathcal{E}_H(x, y)}_{\mathbb{Z}_2 \text{ even}}, A_5 \right]$$

No quadratic divergence from brane localized Higgs mass

Explicit calculations of Higgs mass

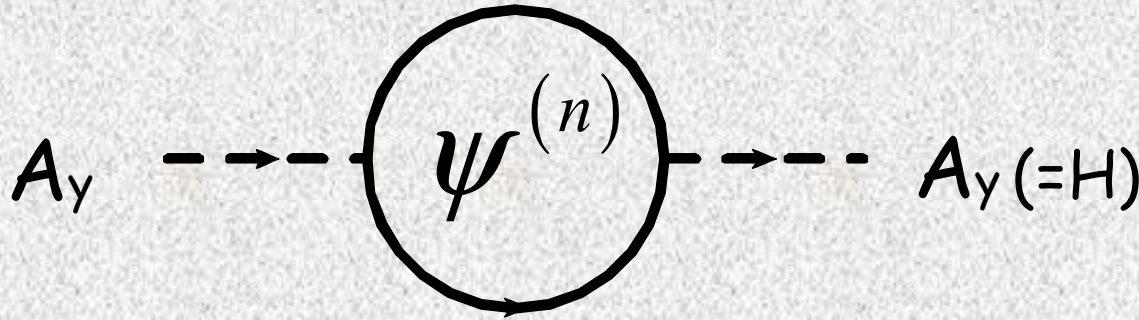
- D-dim QED on S^1 @1-loop Hatanaka, Inami & Lim (1998)
- 5D Non-Abelian gauge theory on S^1/Z_2 @1-loop Gersdorff, Irges & Quiros (2002)
- 6D Non-Abelian gauge theory on T^2 @1-loop Antoniadis, Benakli & Quiros (2001)
- 6D Scalar QED on S^2 @1-loop Lim, NM & Hasegawa (2006)
- 5D QED on S^1 @2-loop NM & Yamashita (2006); Hosotani, NM, Takenaga & Yamashita (2007)
- 5D Gravity on S^1 (GGH) Hasegawa, Lim & NM (2004)

...

Higgs mass calculation

Consider (D+1)-dim QED on S^1

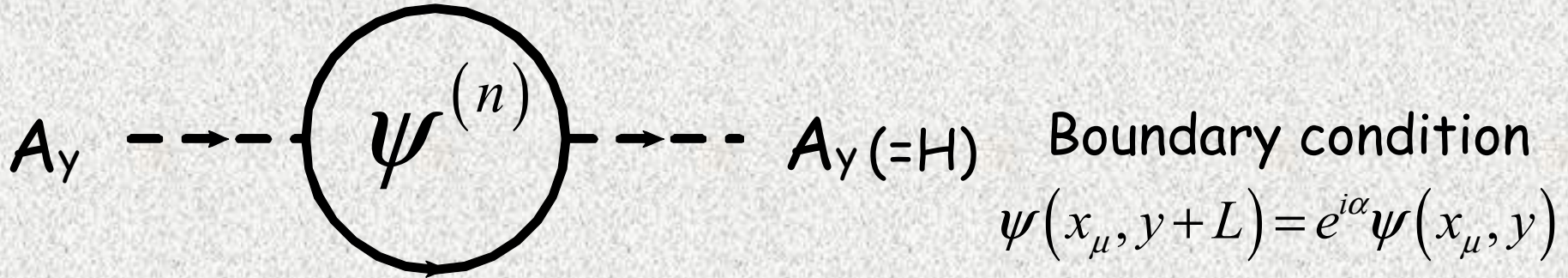
Hatanaka, Inami & Lim (1998)



$$\begin{aligned}
 m_H^2 &= ie_D^2 \int \frac{d^D k}{(2\pi)^D} \sum_{n=-\infty}^{\infty} \text{Tr} \left[\gamma_y \frac{1}{\not{k} - m} \gamma^y \frac{1}{\not{k} - m} \right] \quad (\text{No sum}) \quad L=2\pi R \\
 &\xrightarrow{L \rightarrow \infty} \frac{i}{D+1} e_{D+1}^2 \int \frac{d^{D+1} k}{(2\pi)^{D+1}} \text{Tr} \left[\gamma_M \frac{1}{\not{k} - m} \gamma^M \frac{1}{\not{k} - m} \right] (M=0,1,\dots,D) \\
 &= \frac{i}{D+1} e_{D+1}^2 2^{[(D+1)/2]} \int \frac{d^{D+1} k}{(2\pi)^{D+1}} \left[\frac{1-D}{k^2 - m^2} - \frac{2m^2}{(k^2 - m^2)^2} \right] \\
 &= \frac{i}{D+1} e_{D+1}^2 2^{[(D+1)/2]} \left(1-D + 2m^2 \frac{\partial}{\partial m^2} \right) \int \frac{d^{D+1} k}{(2\pi)^{D+1}} \frac{1}{k^2 - m^2} \\
 &= \frac{i}{D+1} e_{D+1}^2 2^{[(D+1)/2]} \frac{-i}{(4\pi)^{(D+1)/2}} \Gamma\left(\frac{1-D}{2}\right) \left(1-D + 2m^2 \frac{\partial}{\partial m^2} \right) (m^2)^{(D-1)/2} = 0
 \end{aligned}$$

Consider (D+1)-dim QED on S^1

Hatanaka, Inami & Lim (1998)



$$m_H^2 = ie_D^2 2^{[(D+1)/2]} \int \frac{d^D k}{(2\pi)^D} \sum_{n=-\infty}^{\infty} \left[-\frac{1}{\left(\left((2\pi n + \alpha)/L \right)^2 + \rho^2 \right)} + \frac{2\rho^2}{\left[\left((2\pi n + \alpha)/L \right)^2 + \rho^2 \right]^2} \right]$$

$$= -ie_D^2 2^{[(D+1)/2]} \int \frac{d^D k}{(2\pi)^D} \left(1 + \rho \frac{\partial}{\partial \rho} \right) \left(\frac{L}{2\rho} \right) \frac{\sinh(\rho L)}{\cosh(\rho L) - \cos \alpha}$$

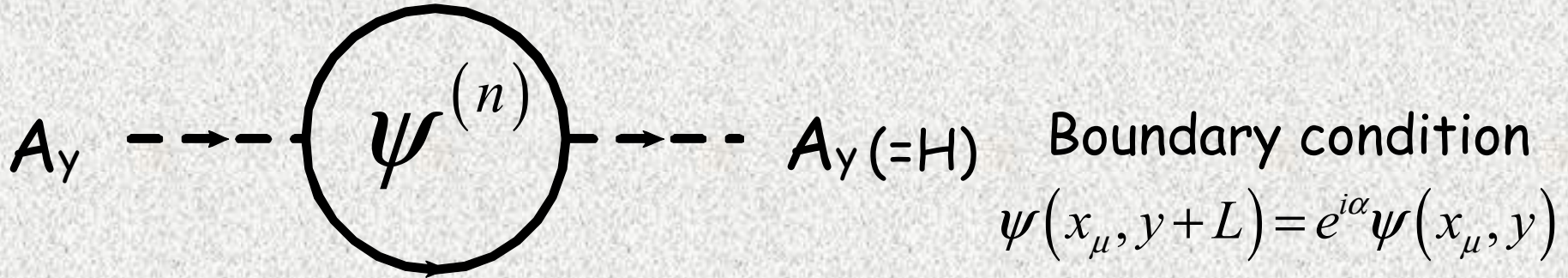
$L = 2\pi R$
 $\rho^2 = -k^2 + m^2$

$$\sum_n \frac{1}{\left(\frac{2\pi n + \alpha}{L} \right)^2 + \rho^2} - L \int \frac{dk_y}{2\pi} \frac{1}{k_y^2 + \rho^2} = \left(\frac{L}{2\rho} \right) \left[\frac{\sinh(\rho L)}{\cosh(\rho L) - \cos \alpha} - 1 \right]$$

Subtracting 0 in $L \rightarrow \infty$

Consider (D+1)-dim QED on S^1

Hatanaka, Inami & Lim (1998)



$$\begin{aligned}
 m_H^2 &= ie_D^2 2^{[(D+1)/2]} \int \frac{d^D k}{(2\pi)^D} \sum_{n=-\infty}^{\infty} \left[-\frac{1}{\left((2\pi n + \alpha)/L \right)^2 + \rho^2} + \frac{2\rho^2}{\left[\left((2\pi n + \alpha)/L \right)^2 + \rho^2 \right]^2} \right] \\
 &= -ie_D^2 2^{[(D+1)/2]} \int \frac{d^D k}{(2\pi)^D} \left(1 + \rho \frac{\partial}{\partial \rho} \right) \left(\frac{L}{2\rho} \right) \frac{\sinh(\rho L)}{\cosh(\rho L) - \cos \alpha} \quad \begin{array}{l} L=2\pi R \\ \rho^2 = -k^2 + m^2 \end{array} \\
 &= \frac{e_D^2 L^2}{2^{D-[(D+1)/2]} \pi^{D/2} \Gamma(D/2)} \int_0^\infty dk k_E^{D-1} \frac{1 - \cosh\left(\sqrt{k_E^2 + m^2} L\right) \cos \alpha}{\left[\cosh\left(\sqrt{k_E^2 + m^2} L\right) - \cos \alpha \right]^2} < \infty
 \end{aligned}$$

Superconvergent!!

Ex. take $D=4$ (5 dimension case) & $m=0, \alpha=\pi$

$$\begin{aligned}
 m_H^2 &= \frac{e_D^2 L^2}{2^{D-[(D+1)/2]} \pi^{D/2} \Gamma(D/2)} \int_0^\infty dk k_E^{D-1} \frac{1 - \cosh\left(\sqrt{k_E^2 + m^2} L\right) \cos \alpha}{\left[\cosh\left(\sqrt{k_E^2 + m^2} L\right) - \cos \alpha\right]^2} \\
 &= \frac{e_4^2}{4\pi^2} \frac{1}{(2\pi R)^2} \int_0^\infty ds s^3 \frac{1 - \cosh s \cos \alpha}{[\cosh s - \cos \alpha]^2} \Bigg|_{\alpha=\pi} \\
 &= \frac{9e_4^2}{16\pi^4 R^2} \zeta(3) = \frac{9e_4^2}{16\pi^4} \underbrace{\zeta(3)}_{1.2} m_W^2
 \end{aligned}$$

$m_W = \pi/R$

Higgs mass is too small
 → generic prediction of GHU

Way out to get 125 GeV Higgs mass

- 1: Realizing **small Higgs VEV** $a \ll 1$
by choosing appropriate matter content

$$m_H \sim m_W / (4\pi a) \quad (m_W = a/R)$$

Haba, Hosotani, Kawamura & Yamashita (2004)

Adachi, NM (2018)

- 2: $D > 5$ dimensions

F_{ij}^2 contains the Higgs quartic coupling $g^2[A_i, A_j]^2$
Higgs mass is generated at leading order

$m_H = 2m_W$ is predicted in 6D on T^2/Z_3 model

Scrucca, Serone, Silvestrini & Wulzer (2003)

- 3: Warped dimension (ex. Randall-Sundrum model)

Higgs mass is enhanced by curvature scale $k\pi R \sim 30$

Contino, Nomura & Pomarol (2003)

Gauge-Higgs sector

Model building of the gauge-Higgs unification

A_5 is an $SU(2)$ **adjoint** originally, not $SU(2)$ doublet
 $\Rightarrow$ need to enlarge the gauge group

$G \rightarrow SU(2)_L \times U(1)_Y$
 adj $\rightarrow$ doublet + other reps



Simplest G
 $SU(3)$

Consider 5D $SU(3)$ model on S^1/Z_2 with Parity: $P = \text{diag}(-, -, +)$

$$PA_\mu(x, y_i - y)P^\dagger = A_\mu(x, y_i + y), \quad PA_5(x, y_i - y)P^\dagger = -A_5(x, y_i + y)$$



$$A_\mu = \begin{pmatrix} (+,+) & (+,+) & (-,-) \\ (+,+) & (+,+) & (-,-) \\ (-,-) & (-,-) & (+,+) \end{pmatrix}, \quad A_5 = \begin{pmatrix} (-,-) & (-,-) & (+,+) \\ (-,-) & (-,-) & (+,+) \\ (+,+) & (+,+) & (-,-) \end{pmatrix}$$

Only (+,+) mode has massless mode (“0 mode”)

$$A_{\mu}^{(0)} = \frac{1}{2} \begin{pmatrix} W_{\mu}^3 + B_{\mu}^3 / \sqrt{3} & \sqrt{2} W_{\mu}^+ & 0 \\ \sqrt{2} W_{\mu}^- & -W_{\mu}^3 + B_{\mu}^3 / \sqrt{3} & 0 \\ 0 & 0 & -2B_{\mu} / \sqrt{3} \end{pmatrix}, A_5^{(0)} = \frac{1}{2} \begin{pmatrix} 0 & 0 & H^+ \\ 0 & 0 & H^0 \\ H^- & H^{0*} & 0 \end{pmatrix}$$

SU(2) × U(1) gauge fields

Higgs doublets

mode expansions

$$A_M^{(+,+)}(x, y) = \frac{1}{\sqrt{2\pi R}} \left[A_M^{(0)}(x) + \sqrt{2} \sum_{n=1}^{\infty} A_M^{(n)}(x) \cos\left(\frac{n}{R} y\right) \right]$$

$$A_M^{(-,-)}(x, y) = \frac{1}{\sqrt{\pi R}} \sum_{n=1}^{\infty} A_M^{(n)}(x) \sin\left(\frac{n}{R} y\right)$$

Gauge boson spectrum

$$M_{W_n} = \frac{n+a}{R}, M_{Z_n} = \frac{n+2a}{R}, M_{\gamma_n} = \frac{n}{R}, \left\langle A_5^{(0)} \right\rangle = \frac{a}{g_5 R}$$

$$M_{W_n} = \frac{n+a}{R}, M_{Z_n} = \frac{n+2a}{R}, M_{\gamma_n} = \frac{n}{R}, \langle A_5^{(0)} \rangle = \frac{a}{g_5 R}$$

- W, Z, γ are identified with zero modes:

$$M_W = a/R, \quad M_Z = 2a/R, \quad M_\gamma = 0$$

$SU(2) \times U(1) \rightarrow U(1)$ realized

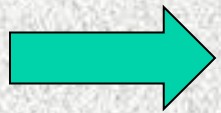
- $M_Z = 2M_W \rightarrow \cos\theta_W = \frac{1}{2}$
 $(\sin^2\theta_W = \frac{3}{4} \gg 0.23)$

- Non-zero KK modes of A_5 are eaten
 by non-zero KK modes of A_μ
 ("Higgs mechanism")

Hypercharge of the doublet

Check the hypercharge of Higgs doublet

$$\begin{aligned}\delta_{U(1)} A_5^{(0)} &= g [T^8, A_5^{(0)}] = \frac{g}{2\sqrt{3}} \left[\begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & -2 \end{pmatrix}, \begin{pmatrix} 0 & 0 & H^+ \\ 0 & 0 & H^0 \\ H^- & H^{0*} & 0 \end{pmatrix} \right] \\ &= \frac{g}{2\sqrt{3}} \begin{pmatrix} 0 & 0 & H^+ \\ 0 & 0 & H^0 \\ -2H^- & -2H^{0*} & 0 \end{pmatrix} - \frac{g}{2\sqrt{3}} \begin{pmatrix} 0 & 0 & -2H^+ \\ 0 & 0 & -2H^0 \\ H^- & H^{0*} & 0 \end{pmatrix} = \frac{g\sqrt{3}}{2} \begin{pmatrix} 0 & 0 & H^+ \\ 0 & 0 & H^0 \\ -H^- & -H^{0*} & 0 \end{pmatrix}\end{aligned}$$



$$\sin^2 \theta_W = \frac{g_Y^2}{g^2 + g_Y^2} = \frac{(\sqrt{3}g)^2}{g^2 + (\sqrt{3}g)^2} = \frac{3}{4} \neq 0.23(\text{Exp})$$

Too Big!!

Well-known by Fairlie, Manton (6D on S^2 w/ monopole bkgd)

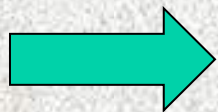
	G_2	$SO(5)$	$SU(3)$
$\sin^2 \Theta_W$	$1/4$	$1/2$	$3/4$

Way out to get a correct Θ_W

1: Additional U(1) $SU(3) \times U(1)' \rightarrow SU(2)_L \times U(1)_Y \times U(1)_X$
Scrucca, Serone & Silvestrini (2003)

$$A_Y = \frac{g'A_8 + \sqrt{3}gA'}{\sqrt{3g^2 + g'^2}}, A_X = \frac{\sqrt{3}gA_8 - g'A'}{\sqrt{3g^2 + g'^2}} \Rightarrow g_Y = \frac{\sqrt{3}gg'}{\sqrt{3g^2 + g'^2}}$$

$$\therefore A_8 = \frac{g'A_Y + \sqrt{3}gA_X}{\sqrt{3g^2 + g'^2}} \Rightarrow gA_8 \supset \frac{g'}{\sqrt{3g^2 + g'^2}} gA_Y$$



$$\sin^2 \theta_W = \frac{g_Y^2}{g^2 + g_Y^2} = \frac{3}{4 + 3g^2/g'^2}$$

Way out to get a correct Θ_W

2: Localized gauge kinetic terms

$$\mathcal{L} = -\frac{1}{2g_5^2} \text{Tr} F_{MN} F^{MN} - \underbrace{\left[\frac{1}{2g_4^2} \delta(y) + \frac{1}{2g_4'^2} \delta(y - \pi R) \right] \text{Tr} F_{\mu\nu} F^{\mu\nu}}_{\text{SU(2) x U(1) invariant}}$$

SU(3) invariant

SU(2) x U(1) invariant

4D effective
Gauge coupling

$$\frac{1}{g_{eff}^2} = \frac{1}{g_5^2} \int_0^{\pi R} dy \left(f_{A_\mu}^{(0)}(y) \right)^2 + \frac{1}{g_4^2} + \frac{1}{g_4'^2}$$

By tuning $g_4, g_4', \sin\Theta_W$ is adjustable

Matter Content
\$
Yukawa Coupling

Quark & Lepton embedding

Consider a fundamental rep of $SU(3)$

$$\mathbf{3} = (q, q-1, 1-2q)^T \text{ (} q: \text{ electric charge)}$$

Putting $q=2/3$, we get

$$\mathbf{3} = \mathbf{2}_{1/6} + \mathbf{1}_{-1/3} = (2/3, -1/3, -1/3)^T = (u_L, d_L, d_R)^T$$

This can be obtained by Z_2 parity as

$$\psi(-y) = P\gamma_5\psi(y), \quad P = \text{diag}(-, -, +)$$

Only fundamental reps cannot incorporate right-handed up-type quarks as well as leptons

Quark & Lepton embedding

Only fundamental reps cannot incorporate right-handed up-type quarks as well as leptons

As one of the embeddings, tensor product is useful

$$\psi(-y) = (P \otimes P) \gamma_5 \psi(y), \quad \psi(-y) = (P \otimes P \otimes P) \gamma_5 \psi(y)$$

$$\begin{aligned} \text{2-rank sym: } 6^* &= \left\{ \begin{array}{l} 3_{L-1/3} + 2_{L1/6} (Q) + 1_{L2/3} \\ 3_{R-1/3} + 2_{R1/6} + 1_{R2/3} (u_R) \end{array} \right. \\ \text{3-rank sym: } 10 &= \left\{ \begin{array}{l} 4_{L1/2} + 3_{L0} + 2_{L-1/2} (L) + 1_{L-1} \\ 4_{R1/2} + 3_{R0} + 2_{R-1/2} + 1_{R-1} (e_R) \end{array} \right. \end{aligned}$$

Many massless exotics $\Rightarrow$ brane localized mass term

Big Hurdle

In the gauge-Higgs unification,
Yukawa coupling = gauge coupling

How can we get fermion mass hierarchy???

As will be shown below,
fermion masses except for top quark are relatively easy

1: Localizing fermions@different point in 5th direction

Yukawa $\sim$ exponentially suppressed overlap integral
Arkani-Hamed & Schmaltz (1999)

2: Bulk fermions mixed with localized fermions
@the fixed points

Non-local Yukawa coupling Csaki, Grojean & Murayama (2002)

1: Yukawa coupling from localizing fermions @different points

1: To localize fermions at different points along the 5th direction, bulk masses are introduced

2: To be consistent with Z_2 orbifold, Z_2 parity of bulk mass must be odd $\Rightarrow$ kink mass

Consider a 5D fermion satisfying the following Dirac equation

$$0 = \left[i\Gamma^M D_M - M\epsilon(y) \right] \psi(x, y)$$

$$D_M = \partial_M - igA_M, \Gamma^M = (\gamma^\mu, i\gamma^5), (M = 0, 1, 2, 3, 5), \epsilon(y) = \begin{cases} 1 & (y > 0) \\ -1 & (y < 0) \end{cases}$$

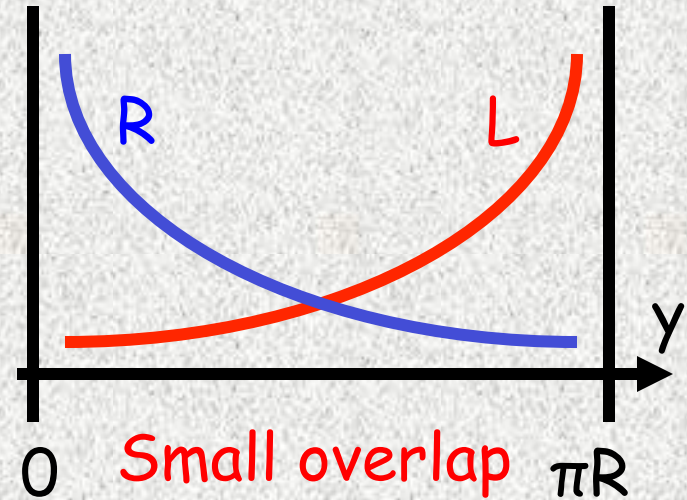
Focusing zero modes

$$\psi(x, y) : \psi_{L(R)}^{(0)}(x) f_{L(R)}^{(0)}(y), \gamma^5 \psi_{L(R)} = (-) \psi_{L(R)}$$

Zero mode wave functions

$$0 = [\partial_y - M\epsilon(y)] f_L^{(0)}(y) \rightarrow f_L^{(0)}(y) = \sqrt{\frac{M}{e^{2\pi MR} - 1}} e^{M|y|}$$

$$0 = [\partial_y + M\epsilon(y)] f_R^{(0)}(y) \rightarrow f_R^{(0)}(y) = \sqrt{\frac{M}{1 - e^{-2\pi MR}}} e^{-M|y|}$$



4D effective Yukawa coupling

$$Y = g_4 \int_0^{\pi R} dy f_L^{(0)}(y) f_R^{(0)}(y) = g_4 \int_0^{\pi R} dy \sqrt{\frac{M^2}{(1 - e^{-2\pi MR})(e^{2\pi MR} - 1)}}$$

$$\approx \pi MR g_4 e^{-\pi MR} \leq g_4 \Leftrightarrow m_f \leq m_W$$

$$\swarrow \pi MR \gg 1$$

Fermion masses **except top** is easy, but top is hard
 No need of unnatural fine-tuning for 5D parameters M, R

2: Mixing between bulk and boundary localized fermions

Csaki, Grojean & Murayama (2002)

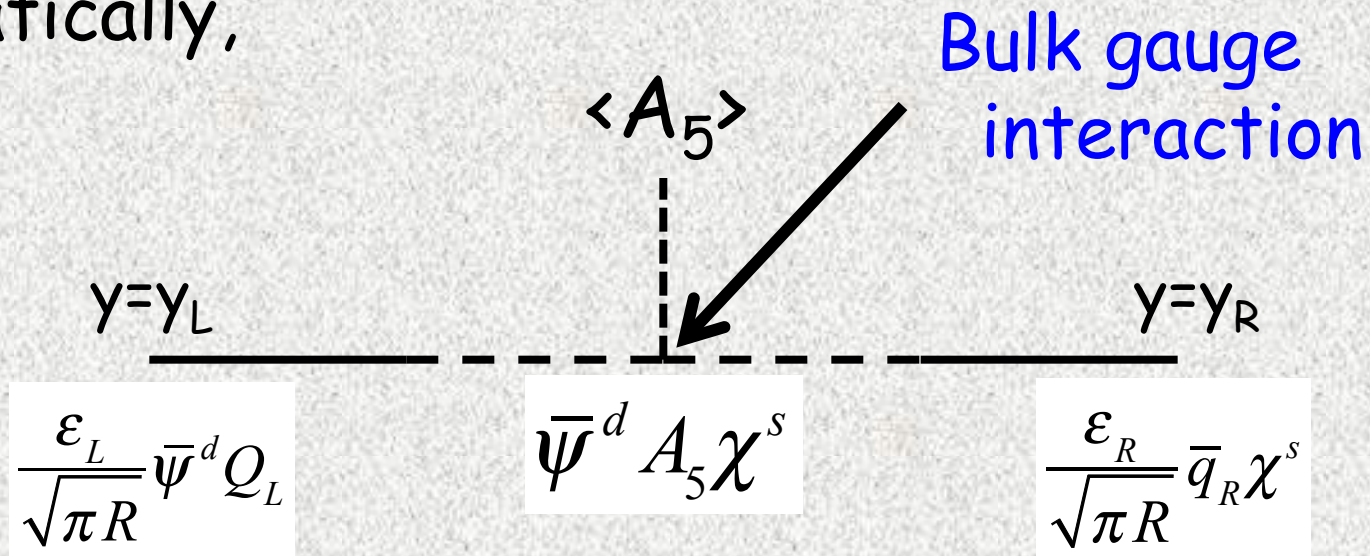
Consider the massive bulk fermion
coupling to SM fermions on the branes

$$\mathcal{L}_{Bulk} = \bar{\Psi} i \not{D} \Psi + \tilde{\bar{\Psi}} i \not{D} \tilde{\Psi} - M (\bar{\Psi} \tilde{\Psi} + \tilde{\bar{\Psi}} \Psi) \quad \Psi \supset \psi^d, \chi^s$$

$$\mathcal{L}_{Brane} = \delta(y - y_L) \left[i \bar{Q}_L \bar{\sigma}^\mu \partial_\mu Q_L + \frac{\epsilon_L}{\sqrt{\pi R}} \bar{\psi}^d Q_L + h.c. \right] + \delta(y - y_R) \left[i \bar{q}_L \bar{\sigma}^\mu \partial_\mu q_L + \frac{\epsilon_R}{\sqrt{\pi R}} \bar{q}_R \chi^s + h.c. \right]$$

Mixing mass term between bulk & brane fermions

Schematically,



2: Mixing between bulk and boundary localized fermions

Csaki, Grojean & Murayama (2002)

Consider the massive bulk fermion
coupling to SM fermions on the branes

$$\mathcal{L}_{Bulk} = \bar{\Psi} i \not{D} \Psi + \tilde{\bar{\Psi}} i \not{D} \tilde{\Psi} - M (\bar{\Psi} \tilde{\Psi} + \tilde{\bar{\Psi}} \Psi) \quad \Psi \supset \psi^d, \chi^s$$

$$\mathcal{L}_{Brane} = \delta(y - y_L) \left[i \bar{Q}_L \bar{\sigma}^\mu \partial_\mu Q_L + \frac{\varepsilon_L}{\sqrt{\pi R}} \bar{\psi}^d Q_L + h.c. \right] + \delta(y - y_R) \left[i \bar{q}_L \bar{\sigma}^\mu \partial_\mu q_L + \frac{\varepsilon_R}{\sqrt{\pi R}} \bar{q}_R \chi^s + h.c. \right]$$

Mixing mass term between bulk & brane fermions

Integrating out massive fermion generates mass term as

$$\varepsilon_L \varepsilon_R \pi M R e^{-\pi M R} \bar{q}_R e^{ig \int_0^{\pi R} dy A_y} Q_L \Rightarrow m_f \propto \varepsilon_L \varepsilon_R \pi M R e^{-\pi M R} M_W$$

Exponentially suppressed coupling

⇒ easy to generate fermion masses except for top

How do we obtain top mass???

Top mass generation

Cacciapaglia, Csaki & Park (2005)

Consider large dimensional reps,
then an upper bound on fermion mass is modified as follows

$$m_t \leq \sqrt{n} m_W \quad (n: \# \text{ of indices of rep})$$

For $m_t = 2m_W \Rightarrow$ need a **4-index** rep top is embedded
To saturate this bound, **bulk mass should be zero**

Simplest example: 

$$(15^*)_{-2/3} \rightarrow (1, 2/3)(t_R) + (2, 1/6)(t_L) \\ + (3, -1/3) + (4, -5/6) + (5, -4/3)$$

$\sqrt{N}$ enhancement

Consider a rank N symmetric tensor of $SU(3)$



Decompose it into $SU(2)$ reps as $3 = 2 + 1$
and make a singlet & a doublet

singlet

1	1	1	...	1
---	---	---	-----	---

 unique

doublet

1	1	2	...	1
---	---	---	-----	---

 etc N patterns

Canonical kinetic term $\Rightarrow 1/\sqrt{N}$

$$\text{Yukawa} = 1_R 2_L 2_H \Rightarrow N \times 1/\sqrt{N} = \sqrt{N}$$

Fermion matter content

$$3 = 2_{L1/6}(Q) + 1_{L-1/3} 2_{R1/6} + 1_{R-1/3}(d_R)$$

Down quark
sector

$$6^* = 3_{L-1/3} + 2_{L1/6}(Q) + 1_{L2/3} 3_{R-1/3} + 2_{R1/6} + 1_{R2/3}(u_R)$$

Up quark
sector
(except for top)

$$10 = 4_{L1/2} + 3_{L0} + 2_{L-1/2}(L) + 1_{L-1} 4_{R1/2} + 3_{R0} + 2_{R-1/2} + 1_{R-1}(e_R)$$

Charged lepton
sector

$$15^* = 5_{L-4/3} + 4_{L-5/6} + 3_{L-1/3} + 2_{L1/6}(Q) + 1_{L2/3} 5_{R-4/3} + 4_{R-5/6} + 3_{R-1/3} + 2_{R1/6} + 1_{R2/3}(t_R)$$

Top
quark

Unwanted massless exotics (blue reps) & two extra Qs
must be massive by brane localized mass terms

EW symmetry breaking

Electroweak symmetry breaking

In GHU, EW symmetry is dynamically broken
by the Hosotani mechanism Hosotani (1983,1989)

Higgs potential is radiatively generated
since the tree level potential is forbidden
by gauge invariance (Coleman-Weinberg potential)

$$V(A_5) = \text{---} \bigcirc \text{---} + \text{---} \bigcirc \text{---} + \text{---} \bigcirc \text{---} + \dots$$

$$V(a) = (-1)^F \frac{(\text{DOF})}{2} \int \frac{d^4 p_E}{(2\pi)^4} \frac{1}{2\pi R} \sum_n \log(p_E^2 + m_n^2)$$

↑
KK mass

Calculation of the effective potential (Adj rep)

$$I(a) \equiv \int \frac{d^4 p}{(2\pi)^4} \sum_{n=-\infty}^{\infty} \log \left[p^2 + \left(\frac{n+a}{R} \right)^2 \right]$$

$$\frac{dI(a)}{da} = \frac{2}{R} \int \frac{d^4 p}{(2\pi)^4} \sum_{n=-\infty}^{\infty} \frac{\left(\frac{n+a}{R} \right)}{p^2 + \left(\frac{n+a}{R} \right)^2} = \frac{2}{R} \int \frac{d^4 p}{(2\pi)^4} \sum_{n=-\infty}^{\infty} \left(\frac{n+a}{R} \right) \int_0^{\infty} dt \exp \left[- \left\{ p^2 + \left(\frac{n+a}{R} \right)^2 \right\} t \right]$$

$$= \frac{2}{R} \sum_{n=-\infty}^{\infty} \frac{n+a}{R} \int_0^{\infty} dt \frac{1}{(4\pi t)^2} \exp \left[- \left(\frac{n+a}{R} \right)^2 t \right]$$

$$= \frac{2}{R} \frac{1}{(4\pi)^2} \int_0^{\infty} dt \frac{1}{t^2} \sum_{n=-\infty}^{\infty} R^2 \sqrt{\frac{\pi}{t^3}} i\pi n \exp \left[- \frac{(\pi R n)^2}{t} - 2\pi i n a \right] = \frac{3R}{16(\pi R)^5} \sum_{n=1}^{\infty} \frac{1}{n^4} \sin(2\pi n a)$$

$$\Rightarrow I(a) = - \frac{3R}{32\pi^6 R^5} \sum_{n=1}^{\infty} \frac{1}{n^5} \cos(2\pi n a) + (\text{a-independent})$$

Poisson
resummation

$$\sum_{n=-\infty}^{\infty} \left(\frac{n+a}{R} \right) \exp \left[- \left(\frac{n+a}{R} \right)^2 t \right] = \sum_{m=-\infty}^{\infty} R^2 \sqrt{\frac{\pi}{t^3}} (i\pi m) \exp \left[- \frac{(\pi R m)^2}{t} - 2\pi i m a \right]$$

Ex. 5D SU(3) model on $S^1/\mathbb{Z}_2$ with N_f fundamental & N_a adjoint fermions

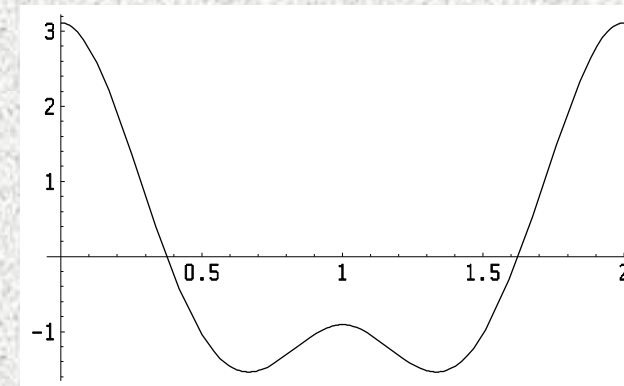
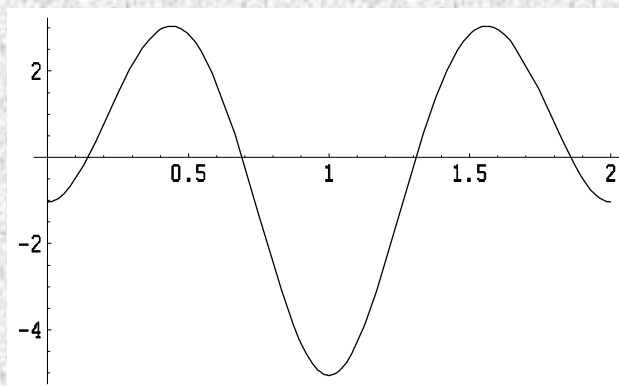
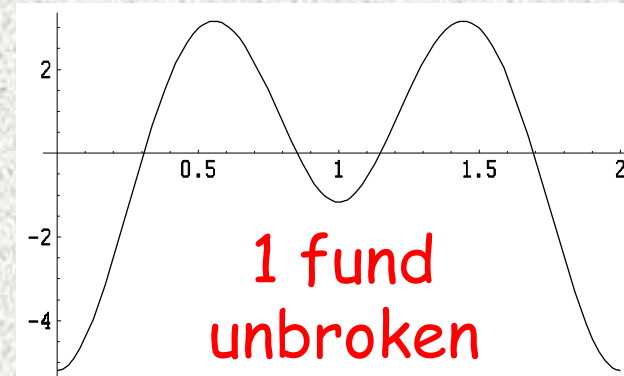
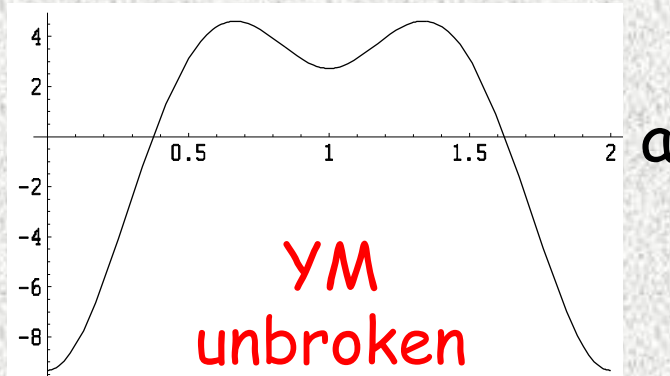
Kubo, Lim & Yamashita (2002)

$$V(a) = \frac{3}{128\pi^7 R^5} \sum_{n=1}^{\infty} \frac{1}{n^5} \left[(4N_a - 3) \underbrace{(\cos[2\pi na] + 2\cos[\pi na])}_{\text{Gauge + ghost adjoint}} + 4N_f \underbrace{\cos[\pi na]}_{\text{fund}} \right]$$

$V(a)$

Gauge + ghost adjoint

fund



Wilson line phase

$$W = \mathcal{P} \exp \left(ig \oint_{S^1} dy A_5 \right) = \begin{pmatrix} 1 & 0 & 0 \\ 0 & \cos(\pi a) & i \sin(\pi a) \\ 0 & i \sin(\pi a) & \cos(\pi a) \end{pmatrix} \quad (a \bmod 2) = \begin{cases} SU(2) \times U(1) & \text{for } a = 0 \\ U(1)' \times U(1) & \text{for } a = 1 \\ U(1)_{\text{em}} & \text{for other cases} \end{cases}$$

$$\langle A_5 \rangle = \frac{a}{gR} \frac{T^6}{2} \equiv A_5^{6(0)} \frac{T^6}{2}$$

$$T^3 = \text{diag}(1, -1, 0)$$

$$T^8 = \text{diag}(1, 1, -2)/\sqrt{3}$$

$$a = 1 : W = \text{diag}(1, -1, -1) \Rightarrow [W, T^3] = [W, T^8] = 0$$

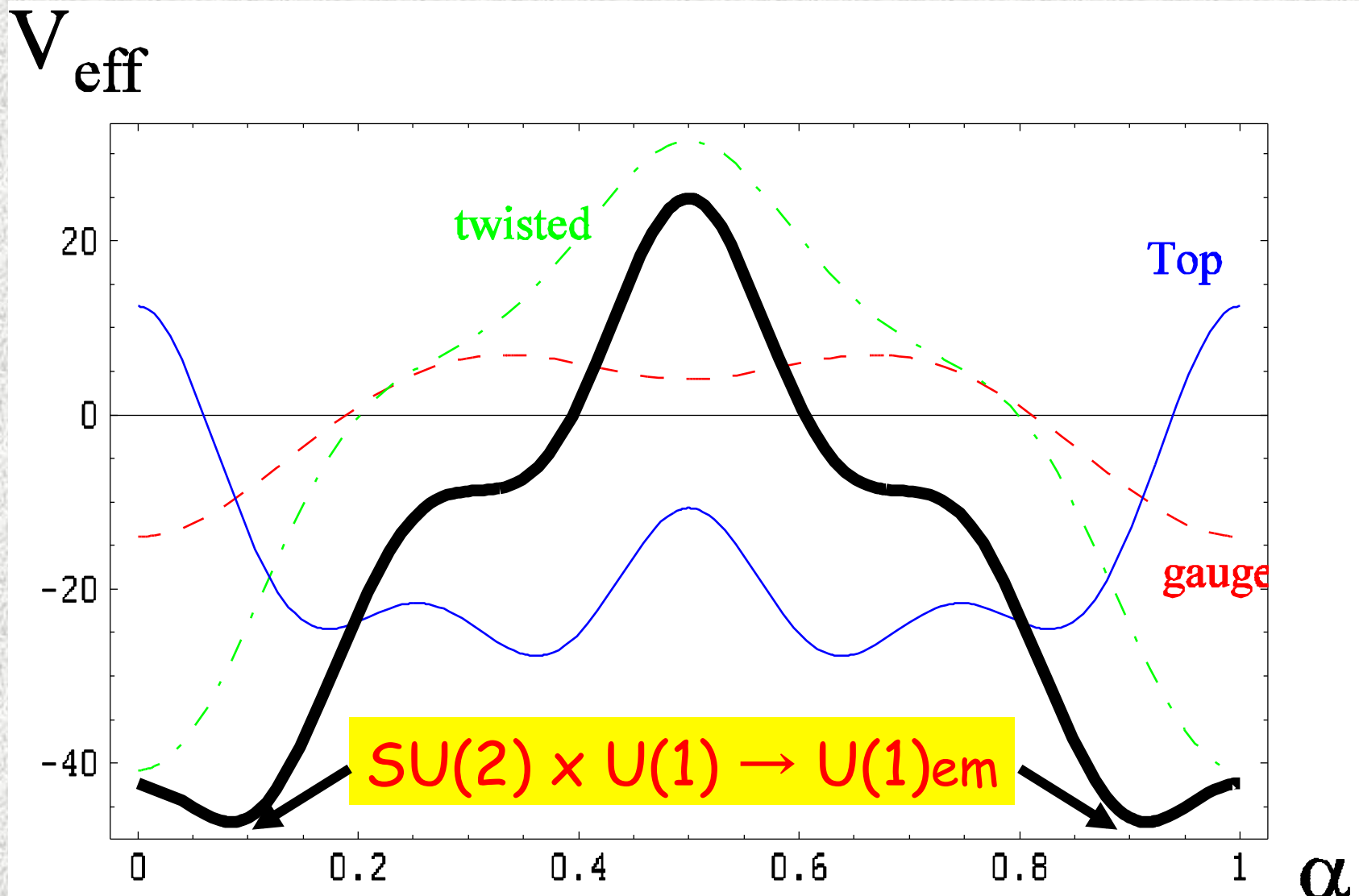
$U(1) \times U(1)'$ unbroken

$$0 < a < 1 : [W, \sqrt{3}T^3 + T^8] = [W, \sin \theta_W T^3 + \cos \theta_W T^8] = 0$$

$U(1)_{\text{em}}$ unbroken

Higgs potential (top (15*) + bottom (3) + tau (10))

Cacciapaglia, Csaki & Park (2005)



Simplified model

Adachi & NM, PRD98 (2018) 015022

$SU(3) \times U(1)$ GHU

- gauge fields A_M, B_M
- fermions $\Psi_l, \Psi_q, \Psi, \Psi_M, X_M$

Q_B, L_B

t_L, t_R
 b_L, b_R

0

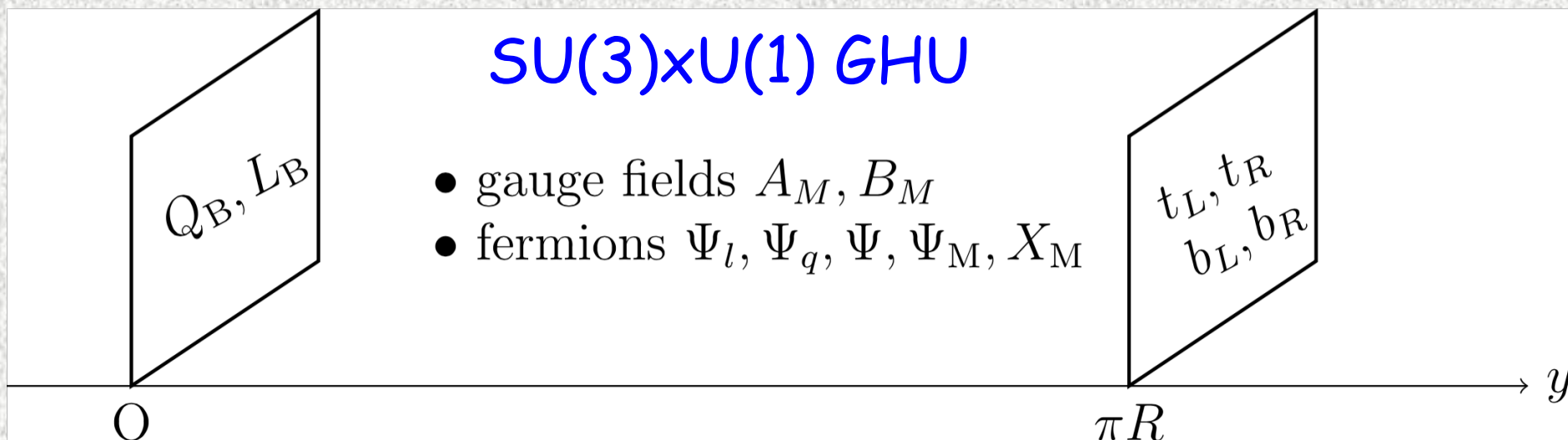
πR

y

- 3rd generation quarks: $(t_L, b_L)^T, t_R, b_R$
brane localized fermions @ $y = \pi R$
- Messenger fermions: $\Psi(3(b), 15^*(t))$
linear combination of Q_{3R} & Q_{15^*R} couple to (t_L, b_L)
 B_{3L} & T_{15^*L} couple to b_R & t_R

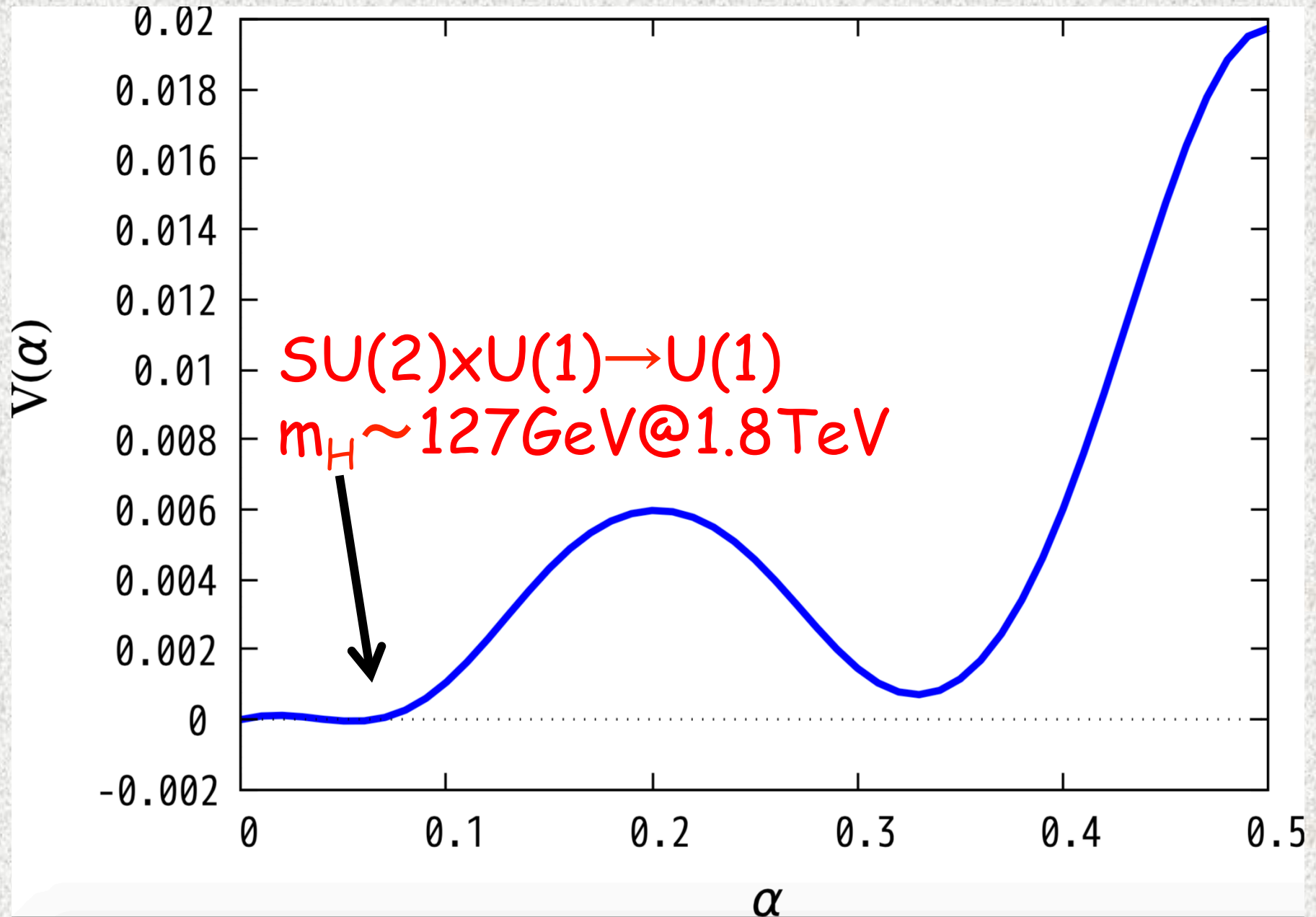
Simplified model

Adachi & NM, PRD98 (2018) 015022



- 1st & 2nd generations of q & l : bulk fields ($3, 3^*$)
 $3(Q, d_R), 3^*(Q, u_R), 3(L, e_R), 3^*(L, \nu_R)$
- Q_B, L_B : brane localized fermions @ $y=0$
to remove exotic $SU(2)$ doublets
- Mirror fermions: Ψ_M, X_M ($15^*, 15^*$) for EWSB

EW symmetry breaking & Higgs mass



Summary

- Gauge-Higgs unification is a very attractive scenario beyond the SM alternative to SUSY
- Controlled by gauge principle & very predictive
Higgs mass, potential $\rightarrow$ finite
- Yukawas except for top are easy,
but top Yukawa is hard to generate

- EWSB @loop level

Once the matter content is fixed,
 $1/R$ is a unique free parameter in Higgs potential
 $\Rightarrow$ very predictive contrary to SM case

Backup Slides

Sample points

α	$1/R$	f	m_H	m_t	m'_t
0.08	1 TeV	31% 42%	110 GeV 125 GeV	113 GeV 110 GeV	189 GeV 186 GeV
0.05	1.6 TeV	11% 14%	120 GeV 133 GeV	149 GeV 149 GeV	381 GeV 375 GeV
0.04	2 TeV	7% 9%	124 GeV 136 GeV	154 GeV 154 GeV	519 GeV 514 GeV
0.03	2.7 TeV	4% 5%	128 GeV 140 GeV	157 GeV 157 GeV	753 GeV 746 GeV
0.02	4 TeV	2% 2%	134 GeV 144 GeV	159 GeV 159 GeV	1224 GeV 1213 GeV



Fine-tuning required to obtain the potential minimum

Model A (top low)
Model B (bottom low)

Flavor Mixing

"Flavor Mixing in Gauge-Higgs Unification"

Adachi, Kurahashi, Lim and NM, JHEP1011 (2010) 015

" D^0 - D^0 bar Mixing in Gauge-Higgs Unification"

Adachi, Kurahashi, Lim and NM, JHEP1201 (2012) 047

" B^0 - B^0 bar Mixing in Gauge-Higgs Unification"

Adachi, Kurahashi, NM and Tanabe, PRD85 (2012) 096001

$$\begin{aligned}
\mathcal{L} = & -\frac{1}{4} F^{MN} F_{MN} - \frac{1}{4} B^{MN} B_{MN} - \frac{1}{4} G^{MN} G_{MN} \\
& + \bar{\psi}_3^{1,2} \left(i \not{D} - M^{1,2} \varepsilon(y) \right) \psi_3^{1,2} + \bar{\psi}_{\bar{6}}^{1,2} \left(i \not{D} - M^{1,2} \varepsilon(y) \right) \psi_{\bar{6}}^{1,2} \\
& + \bar{\psi}_3 i \not{D} \psi_3 + \bar{\psi}_{\bar{15}} i \not{D} \psi_{\bar{15}} \\
& + \delta(y) \sqrt{2\pi R} \bar{Q}_R^i(x) \left[\eta_{ij} Q_{3L}^j(x, y) + \lambda_{ij} Q_L^j(x, y) \right] (i, j = 1, 2, 3) \\
& + \text{brane mass terms for exotics} \quad Q_L = (Q_{6L}^1, Q_{6L}^2, Q_{15L})^T
\end{aligned}$$

- Brane mass matrices η, λ :
off-diagonal elements  Flavor mixing
- Brane localized fields Q_R
- $M^3 = 0$ to avoid $m_\psi \sim M_w \exp[-\pi MR]$

$$\mathcal{L}_{\text{BM}}^Q \sim \delta(y) \bar{Q}_R [\eta \quad \lambda] \begin{bmatrix} Q_3 \\ Q \end{bmatrix}_L = \delta(y) \bar{Q}'_R \begin{bmatrix} m_{\text{diag}} & 0_{3 \times 3} \end{bmatrix} \begin{bmatrix} Q_H \\ Q_{SM} \end{bmatrix}_L$$

$$\begin{bmatrix} Q_3 \\ Q \end{bmatrix}_L = \begin{bmatrix} U_1 & U_3 \\ U_2 & U_4 \end{bmatrix} \begin{bmatrix} Q_H \\ Q_{SM} \end{bmatrix}_L, \quad U^{\bar{Q}} Q_R = Q'_R$$



$$\mathcal{L}_{\text{Yukawa}} = g_5 A_y^6 \bar{d}^i Q_3^i + g_5 A_y^6 \bar{u}^i Q^i \leftarrow \text{Gauge interaction}$$

$$\rightarrow g_5 \left\langle A_y^6 \right\rangle \left(\bar{d}_R^{i(0)} Y_d^{ii} U_3^{ij} Q_{SM}^{j(0)} + \bar{u}_R^{i(0)} Y_u^{ii} U_4^{ij} Q_{SM}^{j(0)} \right)$$

Diagonalization

$$Y^{ii} = \int_{-\pi R}^{\pi R} dy f_L^{i(0)} f_R^{i(0)} \approx 2\pi R M^i e^{-\pi R M^i}$$

$$\begin{cases} \hat{Y}_d = V_{dR}^\dagger Y_d U_3 V_{dL} \\ \hat{Y}_u = V_{uR}^\dagger Y_u U_4 V_{uL} \end{cases} \quad V_{\text{CKM}} = V_{uL}^\dagger V_{dL} \quad (U_3^\dagger U_3 + U_4^\dagger U_4 = 1_{3 \times 3})$$

$M_{3,6} \propto 1$ ($Y_{u,d} \propto 1$) case (flavor symmetry restored)

$$\begin{aligned} \begin{cases} \hat{Y}_d = V_{dR}^\dagger Y_d U_3 V_{dL} \rightarrow V_{dR}^\dagger U_3 V_{dL} \Rightarrow \hat{Y}_d^\dagger \hat{Y}_d = V_{dL}^\dagger U_3^\dagger U_3 V_{dL} \\ \hat{Y}_u = V_{uR}^\dagger Y_u U_4 V_{uL} \rightarrow V_{uR}^\dagger U_4 V_{uL} \Rightarrow \hat{Y}_u^\dagger \hat{Y}_u = V_{uL}^\dagger U_4^\dagger U_4 V_{uL} \end{cases} \\ \xrightarrow{U_3^\dagger U_3 + U_4^\dagger U_4 = 1} V_{uL} \propto V_{dL} \\ \Rightarrow V_{CKM} = V_{uL}^\dagger V_{dL} \propto V_{dL}^\dagger V_{dL} = 1 \text{ (No mixing)} \end{aligned}$$

Lesson

To get flavor mixing,
we need non-degenerate bulk masses
as well as the off-diagonal brane masses
(specific to gauge-Higgs unification)

Parametrization of Unitary matrices $U_{3,4}$ (CP violation ignored)

$$U_4 = R_u \begin{pmatrix} a_1 & 0 & 0 \\ 0 & a_2 & 0 \\ 0 & 0 & a_3 \end{pmatrix}, U_3 = R_d \begin{pmatrix} \sqrt{1-a_1^2} & 0 & 0 \\ 0 & \sqrt{1-a_2^2} & 0 \\ 0 & 0 & \sqrt{1-a_3^2} \end{pmatrix}$$

$$R_u = \begin{pmatrix} 1 & 0 & 0 \\ 0 & \cos\theta'_2 & \sin\theta'_2 \\ 0 & \sin\theta'_2 & \cos\theta'_2 \end{pmatrix} \begin{pmatrix} \cos\theta'_3 & 0 & \sin\theta'_3 \\ 0 & 1 & 0 \\ -\sin\theta'_3 & 0 & \cos\theta'_3 \end{pmatrix} \begin{pmatrix} \cos\theta'_1 & -\sin\theta'_1 & 0 \\ \sin\theta'_1 & \cos\theta'_1 & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

$$R_d = \begin{pmatrix} 1 & 0 & 0 \\ 0 & \cos\theta_2 & \sin\theta_2 \\ 0 & \sin\theta_2 & \cos\theta_2 \end{pmatrix} \begin{pmatrix} \cos\theta_3 & 0 & \sin\theta_3 \\ 0 & 1 & 0 \\ -\sin\theta_3 & 0 & \cos\theta_3 \end{pmatrix} \begin{pmatrix} \cos\theta_1 & -\sin\theta_1 & 0 \\ \sin\theta_1 & \cos\theta_1 & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

Physical observables: 6 quark masses + 3 CKM angles

of parameters: $a_{1,2,3}, b_{1,2,3} = I_{RL}^{1,2(00)}, \theta_{1,2,3}, \theta'_{1,2,3}$

$\Rightarrow 11-9=2$ free parameters (6-5=1 for 2 generations)

Parameter fitting

Numerical results reproducing quark masses & mixings
(2 parameter scan technically hard)

$\Rightarrow$ 3 angles fixed & m_t unfixed)

(i) No up-type mixing case

$$R_u = 1_{3 \times 3} : a_1^2 \approx 0.1023, b_1^2 \approx 4.335 \times 10^{-9}, \sin \theta_1 \approx -2.587 \times 10^{-2}$$
$$a_2^2 \approx 0.9887, b_2^2 \approx 1.302 \times 10^{-4}, \sin \theta_2 \approx 2.224 \times 10^{-2}$$
$$a_3^2 \approx 0.9966, \sin \theta_3 \approx 2.112 \times 10^{-4}$$

(ii) No down-type mixing case

$$R_d = 1_{3 \times 3} : a_1^2 \approx 0.0650, b_1^2 \approx 3.973 \times 10^{-9}, \sin \theta'_1 \approx 0.6704$$
$$a_2^2 \approx 0.9931, b_2^2 \approx 2.235 \times 10^{-4}, \sin \theta'_2 \approx -3.936 \times 10^{-2}$$
$$a_3^2 \approx 0.9966, \sin \theta'_3 \approx 1.773 \times 10^{-2}$$

FCNC @tree level

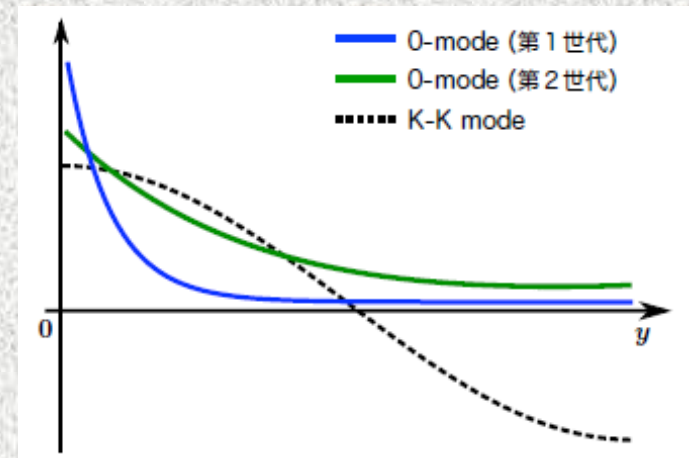
FCNC@tree level even in QCD sector

$$\begin{aligned}\mathcal{L}_{strong} \supset & \frac{g_s}{\sqrt{2\pi R}} G_\mu^{(0)} \left(\bar{\psi}_R^{i(0)} \gamma^\mu \psi_R^{i(0)} + \bar{\psi}_L^{i(0)} \gamma^\mu \psi_L^{i(0)} \right) \\ & + g_s G_\mu^{(n)} \bar{\psi}_R^{i(0)} \gamma^\mu \psi_R^{j(0)} \left(V_{dR}^\dagger I_{RR}^{(0n0)} V_{dR} \right)_{ij} \\ & + g_s G_\mu^{(n)} \bar{\psi}_L^{i(0)} \gamma^\mu \psi_L^{j(0)} \left[V_{dL}^\dagger \left(U_3^\dagger I_{LL}^{(0n0)} U_3 + U_4^\dagger I_{LL}^{(0n0)} U_4 \right) V_{dL} \right]_{ij}\end{aligned}$$

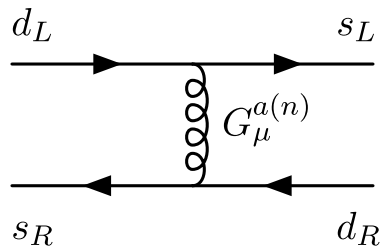
0 mode sector: No mixing O.K.

Nonzero KK gluon couplings
induce nontrivial flavor mixing

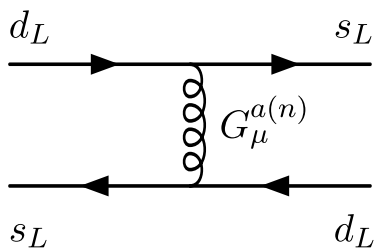
$\Rightarrow$ flavor mixing@tree level



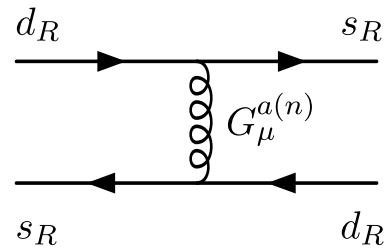
$$K^0 - \bar{K}^0$$



(i) LR type

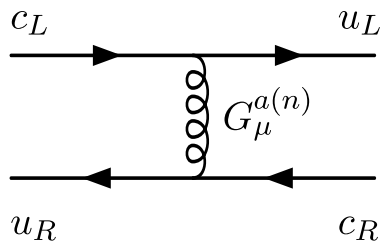


(ii) LL type

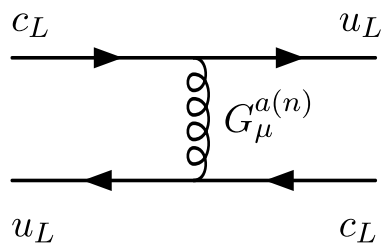


(iii) RR type

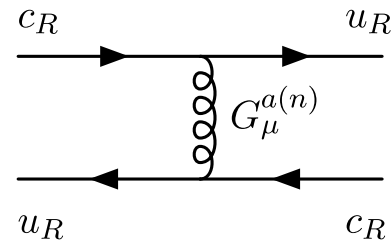
$$D^0 - \bar{D}^0$$



(i) LR type



(ii) LL type

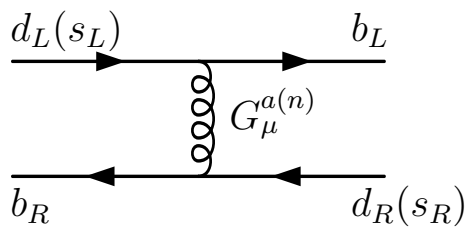


(iii) RR type

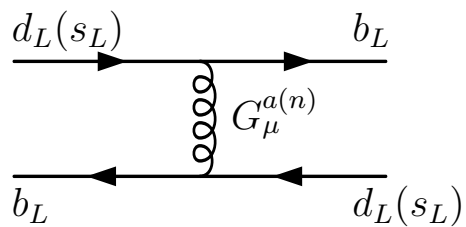
$$B_d^0 - \bar{B}_d^0$$

&

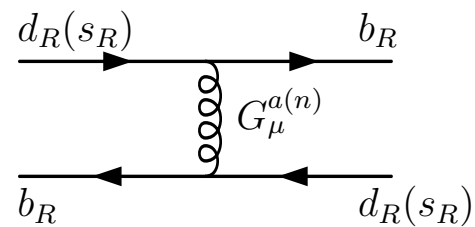
$$B_s^0 - \bar{B}_s^0$$



(i) LR type



(ii) LL type



(iii) RR type

K_L - K_S mass difference

$$\Delta m_K(KK) = 2 \langle \bar{K} | \mathcal{L}_{eff}^{\Delta S=2} | K \rangle \approx \alpha_s C B_1 R^2 f_K^2 m_K \sum_n \frac{1}{n^2} \left[I_{RR}^{1(0n0)} - I_{RR}^{2(0n0)} \right]^2$$
$$I_{RR}^{i(0n0)} = \frac{1}{\sqrt{\pi R}} \int_{-\pi R}^{\pi R} \left(f_R^{(0)}(y) \right)^2 \cos\left(\frac{n}{R} y\right)$$

Bag parameter: $B_1=0.57$, $f_K \sim 1.23 f_\pi$, $m_K \sim 497 \text{ MeV}$

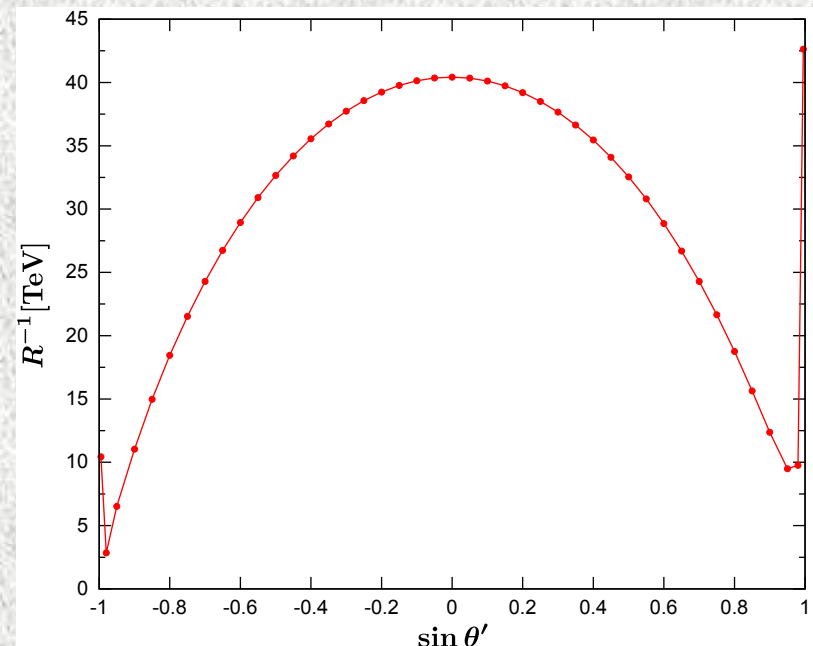
Mode sum is finite

Exp. constraint:

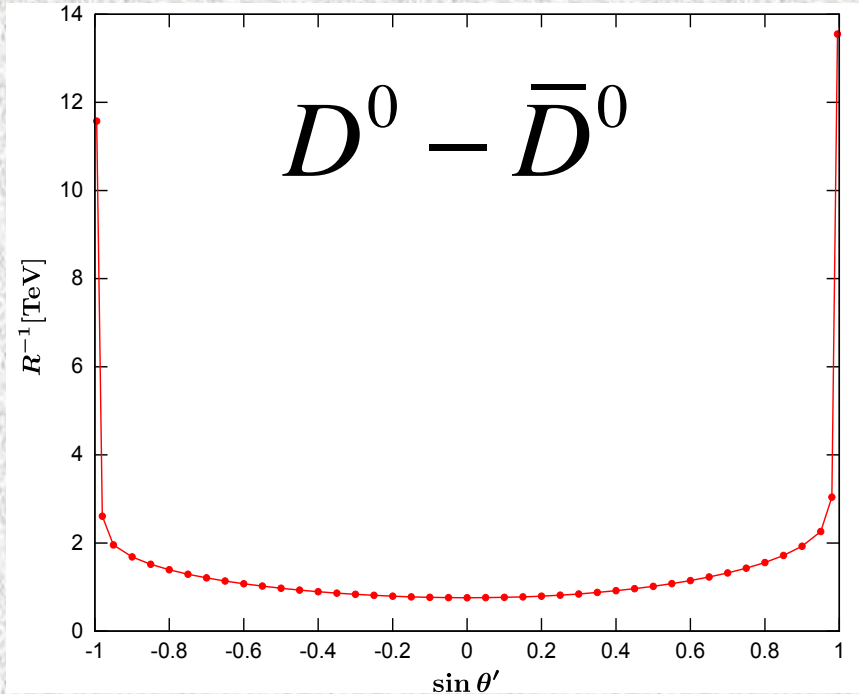
$$|\Delta m_K(NP)| < 3.48 \times 10^{-12} \text{ MeV}$$



$$R^{-1} \geq 2.8 \text{ TeV} \sim 43 \text{ TeV}$$



Similar analysis applied to D & B systems



Lower bounds for
compactification scale

$$R^{-1} \geq 0.8 \text{TeV} \sim 14 \text{TeV}$$

$$B^0 - \bar{B}^0$$

$$R_u = 1_{3 \times 3} : R^{-1} \geq 1.71 \text{TeV} (B_d^0 - \bar{B}_d^0)$$

$$R^{-1} \geq 2.54 \text{TeV} (B_s^0 - \bar{B}_s^0)$$

$$R_d = 1_{3 \times 3} : R^{-1} \geq 0.92 \text{TeV} (B_d^0 - \bar{B}_d^0)$$

$$R^{-1} \geq 1.79 \text{TeV} (B_s^0 - \bar{B}_s^0)$$

Results

$$K^0 - \bar{K}^0 : \mathcal{O}(10) TeV$$

$$D^0 - \bar{D}^0 : \mathcal{O}(1) TeV$$

$$B_d^0 - \bar{B}_d^0, B_s^0 - \bar{B}_s^0 : \mathcal{O}(1) TeV$$

“GIM-like” mechanism

The above results is smaller than that from naive order estimate

$$\frac{1}{M_{KK}^2} \bar{\psi}\psi\bar{\psi}\psi \Rightarrow \begin{cases} M_{KK} \geq 1000 \text{TeV} (K^0 - \bar{K}^0, D^0 - \bar{D}^0) \\ M_{KK} \geq 400 \text{TeV} (B_d^0 - \bar{B}_d^0) \\ M_{KK} \geq 70 \text{TeV} (B_s^0 - \bar{B}_s^0) \end{cases}$$

This apparent discrepancy can be understood since the “GIM-like” mechanism works in GHU

i.e. FCNC processes are automatically suppressed for 1st & 2nd generation of quarks

In the large bulk mass limit,
the KK mode sum can be approximated as follows

$$S_{KK}^{LR} = \pi R \sum_{n=1}^{\infty} \frac{(-1)^n}{n^2} \left(I_{RR}^{1(0n0)} - I_{RR}^{2(0n0)} \right)^2$$

$$\simeq -\frac{\pi^2}{2} \left(e^{-2\pi R M^1} + e^{-2\pi R M^2} \right)$$

$$-\frac{\pi}{2R} \frac{\left(M^1 \right)^2 - M^1 M^2 + \left(M^2 \right)^2}{M^1 M^2 \left(M^1 - M^2 \right)} \left(e^{-2\pi R M^1} - e^{-2\pi R M^2} \right) \left(\pi R M^i \gg 1 \right)$$

exponential
suppression!!

$$e^{-2\pi R M^i} \Leftrightarrow \frac{m_{q^i}^2}{m_W^2}$$

similar to
GIM suppression

$$\frac{m_c^2 - m_u^2}{m_W^2}$$

$$S_{KK}^{LL(RR)} = \pi R \sum_{n=1}^{\infty} \frac{1}{n^2} \left(I_{RR}^{1(0n0)} - I_{RR}^{2(0n0)} \right)^2 \simeq \frac{\pi}{8R} \frac{\left(M^1 - M^2 \right)^2}{M^1 M^2 \left(M^1 + M^2 \right)}$$

Power suppression

More intuitive understanding of “GIM-like” suppression

FCNC is controlled by the factor

$$\left(I_{RR}^{1(0n0)} - I_{RR}^{2(0n0)} \right)^2$$

$$I_{RR}^{i(0n0)} = \frac{1}{\sqrt{\pi R}} \int_{-\pi R}^{\pi R} dy \frac{M^i}{e^{2\pi R M^i} - 1} e^{2M^i y} \cos\left(\frac{n}{R} y\right)$$

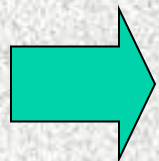
In $\pi MR \gg 1$ limit & for small mode index n

Width “ $1/M$ ” of
0 mode function

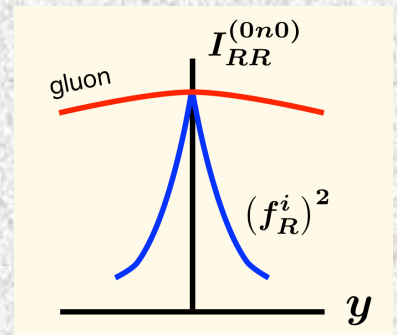
=

Period $2\pi R/n$ of
KK gluon mode function

Almost flat KK gluon mode function for
fast exponential dumping 0 mode fermions



Almost flavor universal
(similar to 0 mode sector)



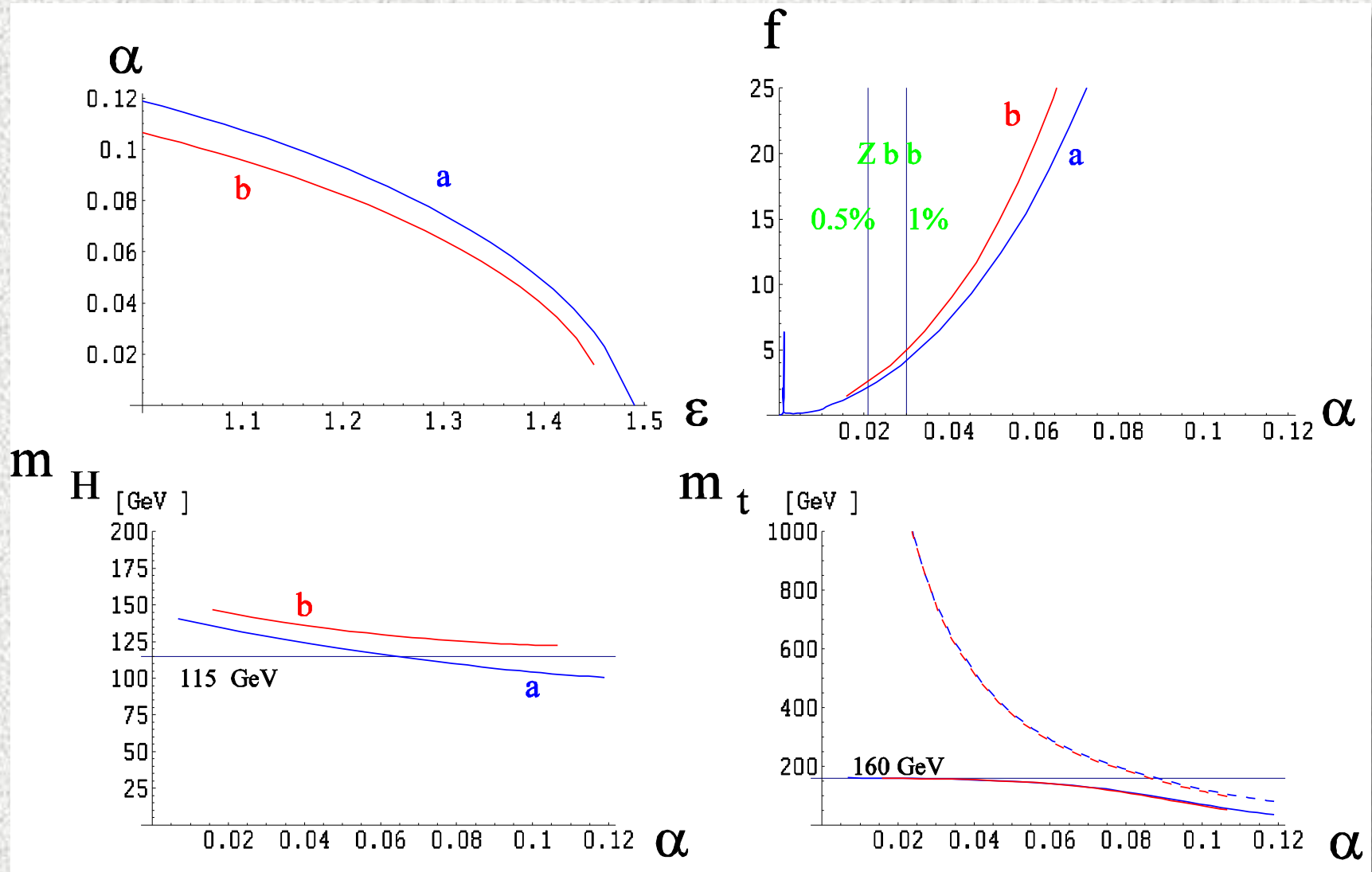
As for the 3rd generation,
GIM-like mechanism does not work

$\therefore M^3=0$ for top mass



Suppressed FCNC due to small mixing
between 1-3 & 2-3 generations

Higgs mass, top mass,...etc



Model a: $b(3), \tau(10)$

Model b: $b(6), \tau(3)$

top

Calculation of the effective potential (Adj rep)

$$\begin{aligned}
 I(a) &\equiv \int \frac{d^4 p}{(2\pi)^4} \sum_{n=-\infty}^{\infty} \log \left[p^2 + \left(\frac{n+a}{R} \right)^2 \right] \\
 \frac{dI(a)}{da} &= \frac{2}{R} \int \frac{d^4 p}{(2\pi)^4} \sum_{n=-\infty}^{\infty} \frac{\left(\frac{n+a}{R} \right)}{p^2 + \left(\frac{n+a}{R} \right)^2} = \frac{2}{R} \int \frac{d^4 p}{(2\pi)^4} \sum_{n=-\infty}^{\infty} \left(\frac{n+a}{R} \right) \int_0^{\infty} dt \exp \left[- \left\{ p^2 + \left(\frac{n+a}{R} \right)^2 \right\} t \right] \\
 &= \frac{2}{R} \sum_{n=-\infty}^{\infty} \frac{n+a}{R} \int_0^{\infty} dt \frac{1}{(4\pi t)^2} \exp \left[- \left(\frac{n+a}{R} \right)^2 t \right] \\
 &= \frac{2}{R} \frac{1}{(4\pi)^2} \int_0^{\infty} dt \frac{1}{t^2} \sum_{n=-\infty}^{\infty} R^2 \sqrt{\frac{\pi}{t^3}} i\pi n \exp \left[- \frac{(\pi R n)^2}{t} - 2\pi i n a \right] = \frac{3R}{16(\pi R)^5} \sum_{n=1}^{\infty} \frac{1}{n^4} \sin(2\pi n a) \\
 \Rightarrow I(a) &= - \frac{3R}{32\pi^6 R^5} \sum_{n=1}^{\infty} \frac{1}{n^5} \cos(2\pi n a) + (\text{a-independent})
 \end{aligned}$$

Poisson
resummation

We have investigated the structure of divergence
for 1-loop contributions to S & T parameters
in the gauge-Higgs unification

Results:

In 5D case, S & T are both finite

In more than 5D case, both divergent

⇒ Natural from the power counting argument

However, the gauge-Higgs unification predicts

$S - 4 \cos\theta_w T$ becomes finite in 6D cases

because S & T are related by the higher dim. gauge inv.

Substituting KK mode expansions

$$A_{\mu,5}^{(+,+)}(x,y) = \frac{1}{\sqrt{2\pi R}} \left[A_{\mu,5}^{(0)}(x) + \sqrt{2} \sum_{n=1}^{\infty} A_{\mu,5}^{(n)} \cos\left(\frac{n}{R}y\right) \right],$$

$$A_{\mu,5}^{(-,-)}(x,y) = \frac{1}{\sqrt{\pi R}} \sum_{n=1}^{\infty} A_{\mu,5}^{(n)} \sin\left(\frac{n}{R}y\right),$$

$$\Psi_{1L,2L,3R}^{(+,+)}(x,y) = \frac{1}{\sqrt{2\pi R}} \left[\psi_{1L,2L,3R}^{(0)}(x) + \sqrt{2} \sum_{n=1}^{\infty} \psi_{1L,2L,3R}^{(n)} \cos\left(\frac{n}{R}y\right) \right],$$

$$\Psi_{1R,2R,3L}^{(-,-)}(x,y) = \frac{1}{\sqrt{\pi R}} \sum_{n=1}^{\infty} \psi_{1R,2R,3L}^{(n)} \sin\left(\frac{n}{R}y\right)$$

and integrating out 5th coordinate "y",
 making a chiral rotation $\psi_{1,2,3} \rightarrow e^{-i\pi\gamma_5/4} \psi_{1,2,3}$ to remove $i\gamma_5$,

we obtain 4D effective Lagrangian

$$\begin{aligned}
L_{\text{fermion}}^{(4D)} = & \sum_{n=1}^{\infty} \left[i \left(\bar{\psi}_1^{(n)}, \bar{\psi}_2^{(n)}, \bar{\psi}_3^{(n)} \right) \gamma^\mu \partial_\mu \begin{pmatrix} \psi_1^{(n)} \\ \psi_2^{(n)} \\ \psi_3^{(n)} \end{pmatrix} \right. \\
& + \frac{g}{2} \left(\bar{\psi}_1^{(n)}, \bar{\psi}_2^{(n)}, \bar{\psi}_3^{(n)} \right) \begin{pmatrix} W_\mu^3 + \frac{B_\mu}{\sqrt{3}} & \sqrt{2} W_\mu^+ & 0 \\ \sqrt{2} W_\mu^- & -W_\mu^3 + \frac{B_\mu}{\sqrt{3}} & 0 \\ 0 & 0 & -\frac{2}{\sqrt{3}} B_\mu \end{pmatrix} \gamma^\mu \begin{pmatrix} \psi_1^{(n)} \\ \psi_2^{(n)} \\ \psi_3^{(n)} \end{pmatrix} \\
& \left. - \left(\bar{\psi}_1^{(n)}, \bar{\psi}_2^{(n)}, \bar{\psi}_3^{(n)} \right) \begin{pmatrix} m_n & 0 & 0 \\ 0 & m_n & -m \\ 0 & -m & m_n \end{pmatrix} \begin{pmatrix} \psi_1^{(n)} \\ \psi_2^{(n)} \\ \psi_3^{(n)} \end{pmatrix} \right] \\
& + i \bar{t}_L \gamma^\mu \partial_\mu t_L + \bar{b} \left(i \gamma^\mu \partial_\mu - m \right) b + \frac{g}{\sqrt{2}} \left(\bar{t} \gamma^\mu L b W_\mu^+ + \bar{b} \gamma^\mu L t W_\mu^- \right) + \frac{g}{2} \left(\bar{t} \gamma^\mu L t + \bar{b} \gamma^\mu L b \right) W_\mu^3 \\
& + \frac{\sqrt{3}g}{6} \left(\bar{t} \gamma^\mu L t + \bar{b} \gamma^\mu L b - 2 \bar{b} \gamma^\mu R b \right) B_\mu \\
& L \equiv \frac{1}{2} (1 - \gamma_5), R \equiv \frac{1}{2} (1 + \gamma_5), m_n = \frac{n}{R}, g = \frac{g_5}{\sqrt{2\pi R}}, m = \frac{1}{2} g v (= M_W)
\end{aligned}$$

Mixing occurs between SU(2) **doublet** component
& **singlet** component

Each of mass eigenvalues has a **periodicity**
with respect to m

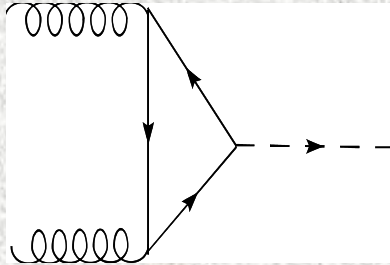
$$m_n \pm \left(m + \frac{1}{R} \right) = m_{n\pm 1} \pm m$$

Characteristic feature of gauge-Higgs unification

(c.f. $\sqrt{m_n^2 + m^2}$ for UED)

SM contributions

$gg \rightarrow H$

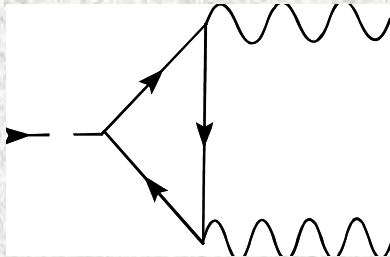


$$\mathcal{L}_{eff} = C_{glue}^{SM} h G^{a\mu\nu} G_{\mu\nu}^a$$

$$C_{glue}^{SM} = -\frac{m_t}{v} \times \frac{\alpha_s F_{1/2}(4m_t^2/m_h^2)}{8\pi m_t} \times \frac{1}{2}$$

$$F_{1/2}(x) = -2x \left[1 + (1-x) \left(\sin^{-1}(1/\sqrt{x}) \right)^2 \right] \rightarrow -\frac{4}{3} (x \gg 1)$$

$H \rightarrow \gamma\gamma$



$$\mathcal{L}_{eff} = C_{\gamma}^{SM} h G^{a\mu\nu} G_{\mu\nu}^a$$

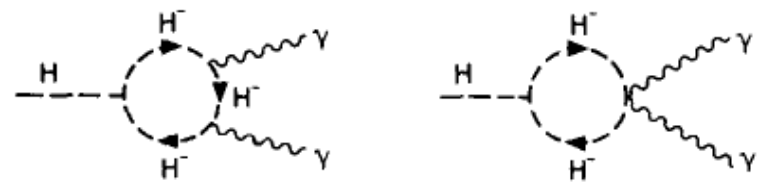
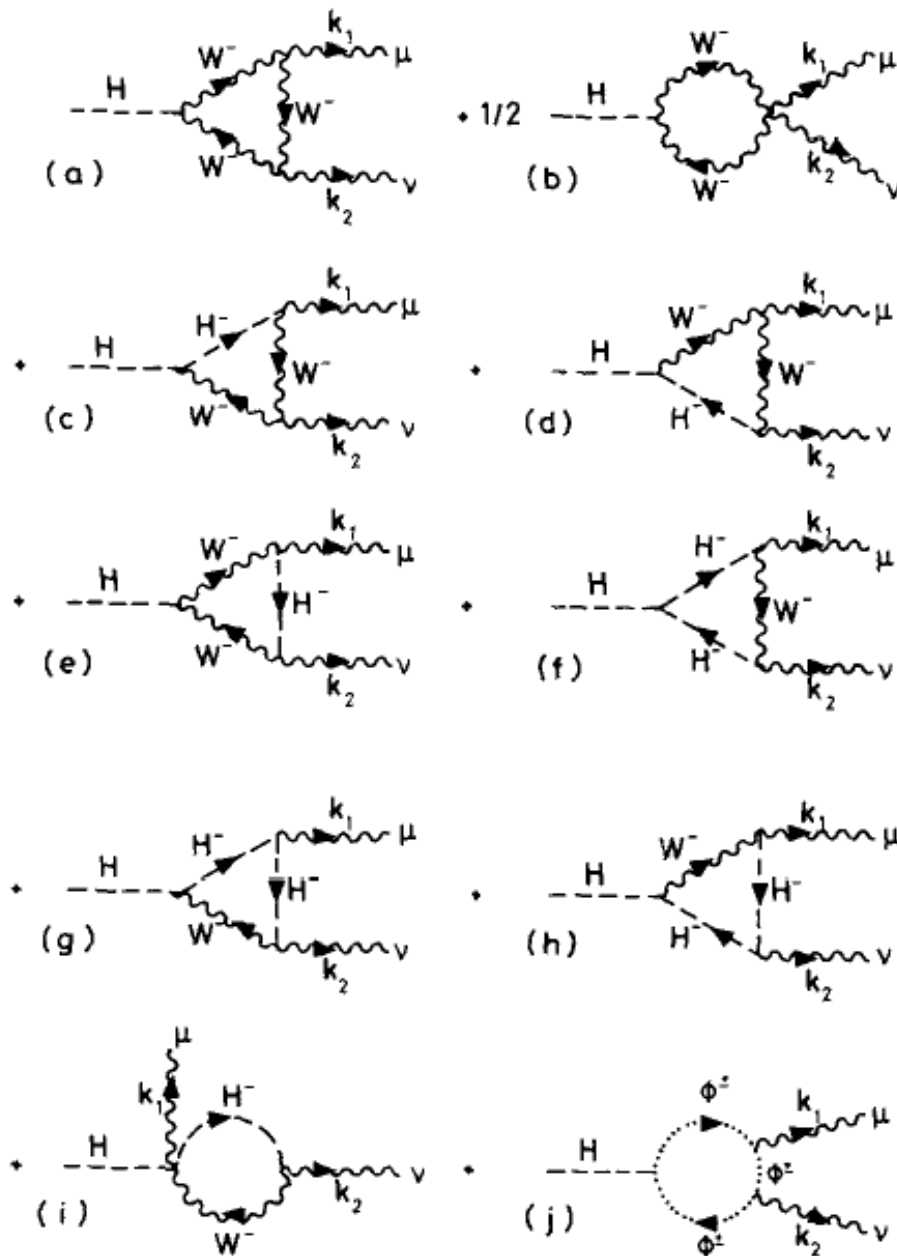
$$C_{\gamma}^{SM} = C_{\gamma}^t + C_{\gamma}^W$$

$$C_{\gamma}^t = -\frac{m_t}{v} \times \frac{\alpha_{em} F_{1/2}(4m_t^2/m_h^2)}{8\pi m_t} \times \frac{4}{3}, \quad C_{\gamma}^W = -\frac{m_t}{v} \times \frac{\alpha_{em} F_1(4m_W^2/m_h^2)}{8\pi m_t}$$

$$F_1(x) = 2 + 3x + 3x(2-x) \left(\sin^{-1}(1/\sqrt{x}) \right)^2 \rightarrow 7 (x \gg 1)$$

W-boson loop effects of 2 photon decay in SM

Ellis, Gaillard & Nanopoulos,
NPB106 (1976) 292



Collider Physics in Gauge-Higgs Unification



Nobuhito Maru
(Osaka City University
& NITEP)



2019/06/07 Lecture@E-ken

References

- 125 GeV Higgs Boson and TeV Scale Colored Fermions
in Gauge-Higgs Unification, arXiv:1310.3348
- Diphoton and Z photon Decays of Higgs Boson
in Gauge-Higgs Unification: A Snowmass white paper
arXiv: 1307:8181
- $H \rightarrow Z\gamma$ in Gauge-Higgs Unification, PRD88 037701 (2013)
- Diphoton Decay Excess and 125 GeV Higgs Mass
in Gauge-Higgs Unification, PRD87 095019 (2013)
- Gauge-Higgs Unification at CERN LHC,
PRD77 055010 (2008)

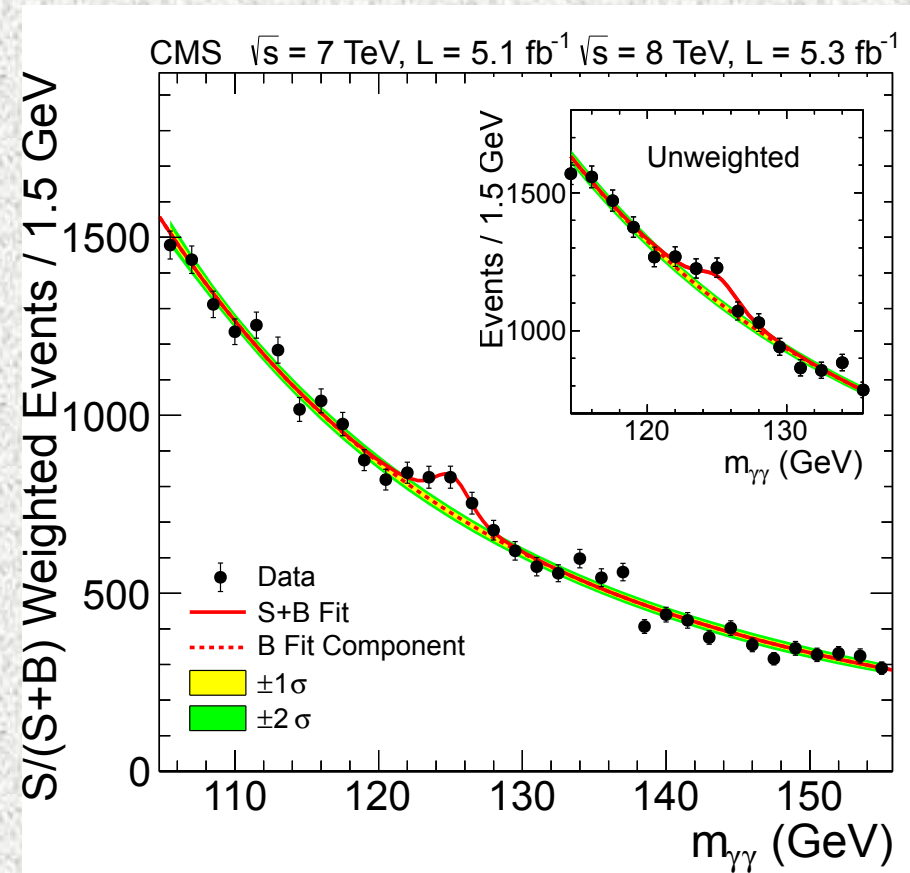
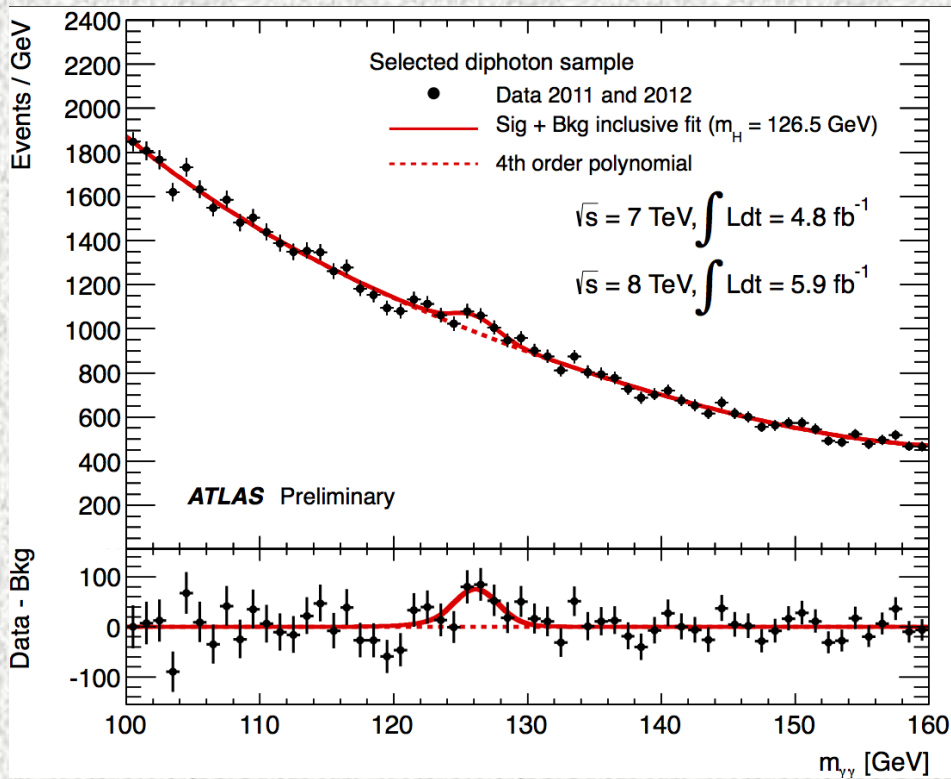
All papers with Nobuchika Okada

PLAN

- Introduction
- A Model of GHU
- $gg \rightarrow H$ & $H \rightarrow \gamma \gamma$ in GHU
- $H \rightarrow Z \gamma$
- Summary

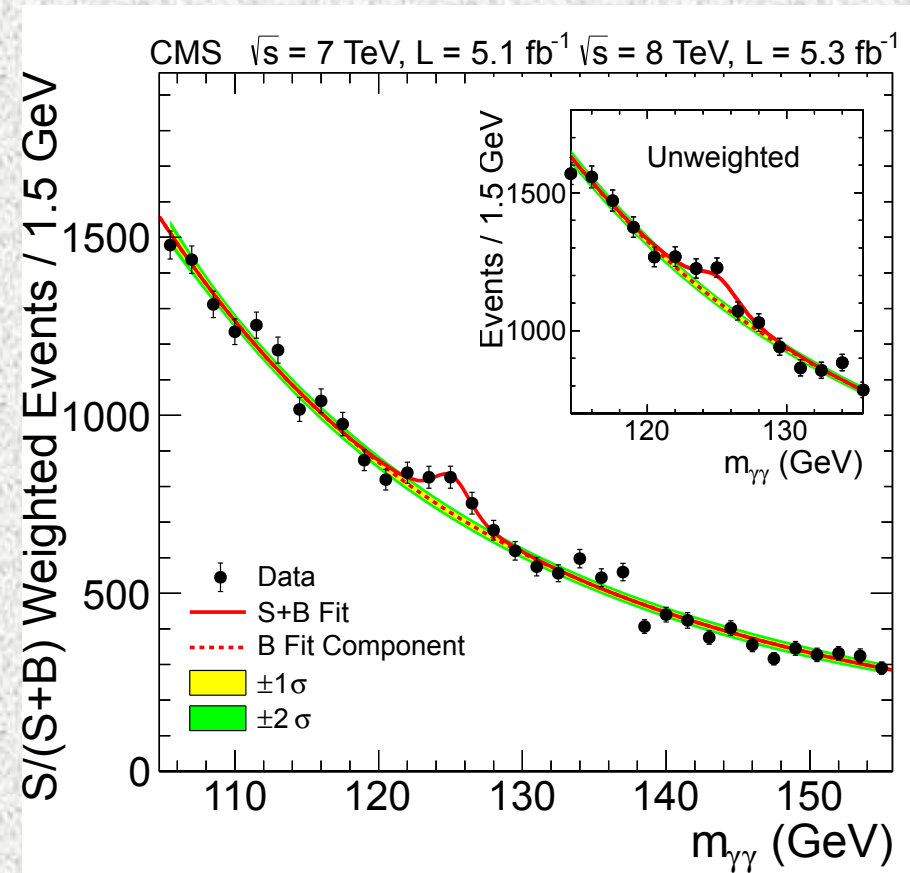
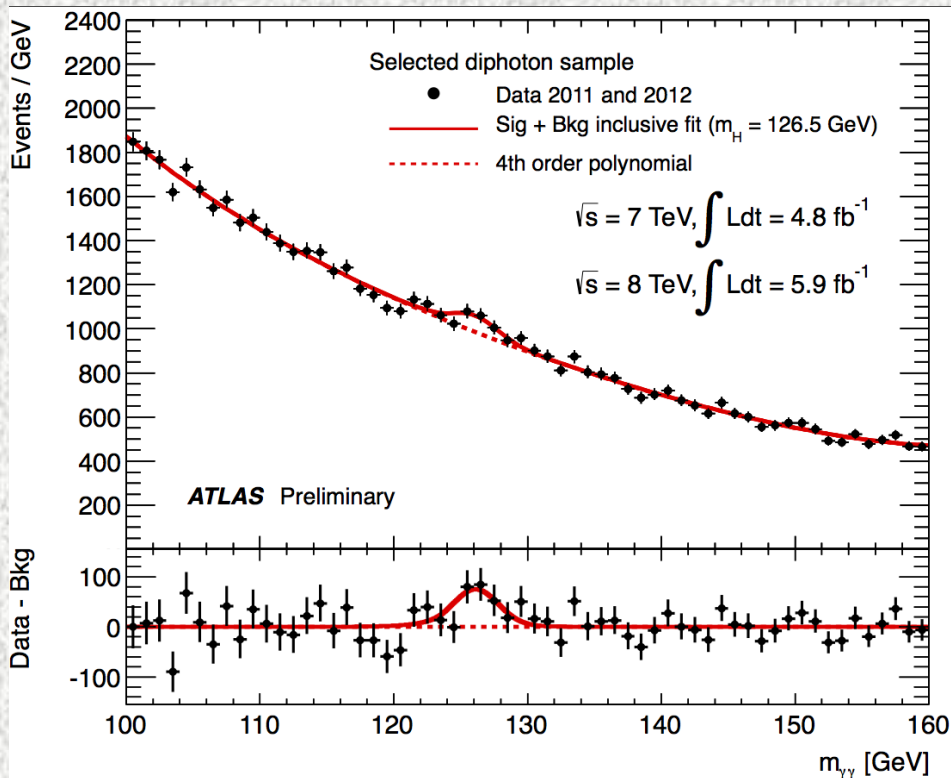
Introduction

A Higgs boson was discovered!!



Introduction

Still unclear, the origin of Higgs ??



Which Higgs?

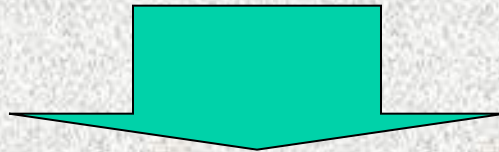
UnHiggs? Private Higgs? Guralnik's Higgs?
Gaugephobic Higgs? Kibble's Higgs? Little Higgs?
Buried Higgs? Littlest Higgs?
Composite Higgs? Intermediate Higgs? Slim Higgs?
Fat Higgs? Higgsless?
Portal Higgs? Peter's Higgs? Brout-Englert's Higgs?
Gauge-Higgs? Lone Higgs?
Twin Higgs?
Simplest Higgs? Phantom Higgs?

In the gauge-Higgs unification,

1: New structure in the Higgs sector

2: Coupling of new particles to Higgs boson

controlled by higher dimensional gauge invariance



Deviations from the SM predictions &
Collider signatures specific to GHU
are expected!!

A Model of GHU

Lagrangian

5D $SU(3) \times U(1)'$ model on S^1/Z_2

$$\begin{aligned}\mathcal{L} = & -\frac{1}{2} \text{Tr} \left(F_{MN} F^{MN} \right) + \bar{\Psi}_3^{i=1,2,3} \left(i\Gamma^M D_M - M_d^i \varepsilon(y) \right) \Psi_3^{i=1,2,3} \\ & + \bar{\Psi}_{\bar{6}}^{i=1,2} \left(i\Gamma^M D_M - M_u^i \varepsilon(y) \right) \Psi_{\bar{6}}^{i=1,2} + \bar{\Psi}_{\bar{15}} i\Gamma^M D_M \Psi_{\bar{15}} \\ & + \bar{\Psi}_{10}^{i=1,2,3} \left(i\Gamma^M D_M - M_l^i \varepsilon(y) \right) \Psi_{10}^{i=1,2,3} \quad \Gamma^M = (\gamma^\mu, i\gamma^5)\end{aligned}$$

Boundary conditions:

$$S^1: \Psi(y+2\pi R) = \psi(y), \quad Z_2: \Psi(-y) = \pm \psi(y)$$

$$A_\mu = \begin{pmatrix} (+,+) & (+,+) & (-,-) \\ (+,+) & (+,+) & (-,-) \\ (-,-) & (-,-) & (+,+) \end{pmatrix}, \quad A_5 = \begin{pmatrix} (-,-) & (-,-) & (+,+) \\ (-,-) & (-,-) & (+,+) \\ (+,+) & (+,+) & (-,-) \end{pmatrix}$$

Lagrangian

5D $SU(3) \times U(1)'$ model on S^1/Z_2

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Boundary conditions:

(+,+) only has
massless mode

(+,+): $\cos(ny/R)$
(-,-): $\sin(ny/R)$

$$A_\mu = \begin{pmatrix} (+,+) & (+,+) & (-,-) \\ (+,+) & (+,+) & (-,-) \\ (-,-) & (-,-) & (+,+) \end{pmatrix}, A_5 = \begin{pmatrix} (-,-) & (-,-) & (+,+) \\ (-,-) & (-,-) & (+,+) \\ (+,+) & (+,+) & (-,-) \end{pmatrix}$$

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$$SU(3) \times U(1)' \rightarrow SU(2) \times U(1)_Y \times U(1)_X$$

Lagrangian

5D $SU(3) \times U(1)'$ model on S^1/Z_2

$$\begin{aligned} \mathcal{L} = & -\frac{1}{2} \text{Tr} \left(F_{MN} F^{MN} \right) + \bar{\Psi}_3^{i=1,2,3} \left(i\Gamma^M D_M - M_d^i \varepsilon(y) \right) \Psi_3^{i=1,2,3} \\ & + \bar{\Psi}_{\bar{6}}^{i=1,2} \left(i\Gamma^M D_M - M_u^i \varepsilon(y) \right) \Psi_{\bar{6}}^{i=1,2} + \bar{\Psi}_{\bar{15}} i\Gamma^M D_M \Psi_{\bar{15}} \\ & + \bar{\Psi}_{10}^{i=1,2,3} \left(i\Gamma^M D_M - M_l^i \varepsilon(y) \right) \Psi_{10}^{i=1,2,3} \quad \Gamma^M = (\gamma^\mu, i\gamma^5) \end{aligned}$$

Boundary conditions:

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$$A_\mu = \begin{pmatrix} (+,+) & (+,+) & (-,-) \\ (+,+) & (+,+) & (-,-) \\ (-,-) & (-,-) & (+,+) \end{pmatrix}, A_5 = \begin{pmatrix} (-,-) & (-,-) & (+,+) \\ (-,-) & (-,-) & (+,+) \\ (+,+) & (+,+) & (-,-) \end{pmatrix}$$

0 mode of A_5 = SM Higgs

Fermion matter content

$$3 = 2_{L1/6}(Q) + 1_{L-1/3} 2_{R1/6} + 1_{R-1/3}(d_R)$$

Down quark sector

$$6^* = 3_{L-1/3} + 2_{L1/6}(Q) + 1_{L2/3} 3_{R-1/3} + 2_{R1/6} + 1_{R2/3}(u_R)$$

Up quark sector
(except for top)

$$10 = 4_{L1/2} + 3_{L0} + 2_{L-1/2}(L) + 1_{L-1} 4_{R1/2} + 3_{R0} + 2_{R-1/2} + 1_{R-1}(e_R)$$

Charged lepton sector

$$15^* = 5_{L-4/3} + 4_{L-5/6} + 3_{L-1/3} + 2_{L1/6}(Q) + 1_{L2/3} 5_{R-4/3} + 4_{R-5/6} + 3_{R-1/3} + 2_{R1/6} + 1_{R2/3}(t_R)$$

Top quark

Unwanted massless exotics (blue reps) & two extra Qs must be massive by brane localized mass terms

Big
Hurdle

In the gauge-Higgs unification,
Yukawa coupling = gauge coupling

How can we get fermion mass hierarchy???



Localizing fermions@different point in 5th direction

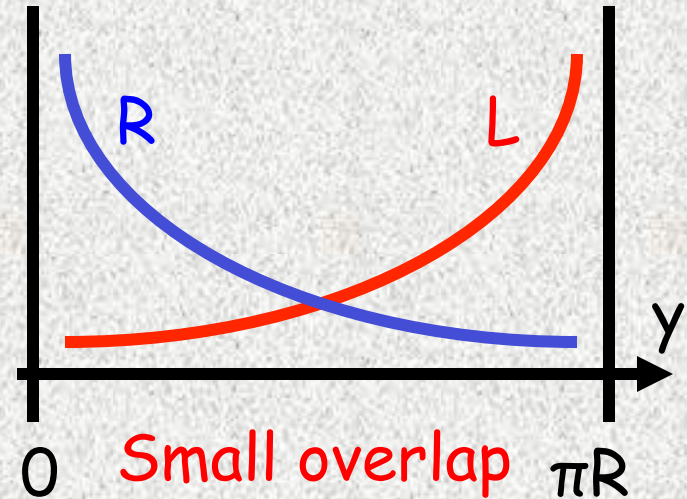
**Yukawa $\sim$ exponentially suppressed
overlap integral of wave functions**

Arkani-Hamed & Schmaltz (1999)

Zero mode wave functions

$$0 = [\partial_y - M\varepsilon(y)] f_L^{(0)}(y) \rightarrow f_L^{(0)}(y) = \sqrt{\frac{M}{e^{2\pi MR} - 1}} e^{M|y|}$$

$$0 = [\partial_y + M\varepsilon(y)] f_R^{(0)}(y) \rightarrow f_R^{(0)}(y) = \sqrt{\frac{M}{1 - e^{-2\pi MR}}} e^{-M|y|}$$



4D effective Yukawa coupling

$$Y = g_4 \int_{-\pi R}^{\pi R} dy f_L^{(0)}(y) f_R^{(0)}(y) = g_4 \int_{-\pi R}^{\pi R} dy \sqrt{\frac{M^2}{(1 - e^{-2\pi MR})(e^{2\pi MR} - 1)}}$$

$$\approx 2\pi MR g_4 e^{-\pi MR} \leq g_4 (\pi MR \gg 1) \Leftrightarrow m_f \leq m_W$$

Fermion masses **except top** is easy to obtain by tuning M
 Top in 15* rep $\Rightarrow$ factor "2" enhancement

Martinelli, Salvatori, Scrucra & Silvestrini (2005)

$\sqrt{N}$ enhancement

Consider a rank N symmetric tensor of $SU(3)$



Decompose it into $SU(2)$ reps as $3 = 2 + 1$
and make a singlet & a doublet

singlet



unique

doublet



etc N patterns

Canonical kinetic term $\Rightarrow 1/\text{sqrt}[N]$

$$\text{Yukawa} = 1_R 2_L 2_H \Rightarrow N \times 1/\text{sqrt}[N] = \text{sqrt}[N]$$

Essential Points for calculation (Specific form of KK masses in GHU)

Mass splitting & Coupling to Higgs

KK top

$$m_t^{+(n)} = \frac{n}{R} + m_t - \frac{m_t}{v} h$$

$$m_t^{-(n)} = \frac{n}{R} - m_t + \frac{m_t}{v} h$$

KK W

$$m_W^{+(n)} = \frac{n}{R} + m_W + \frac{m_W}{v} h$$

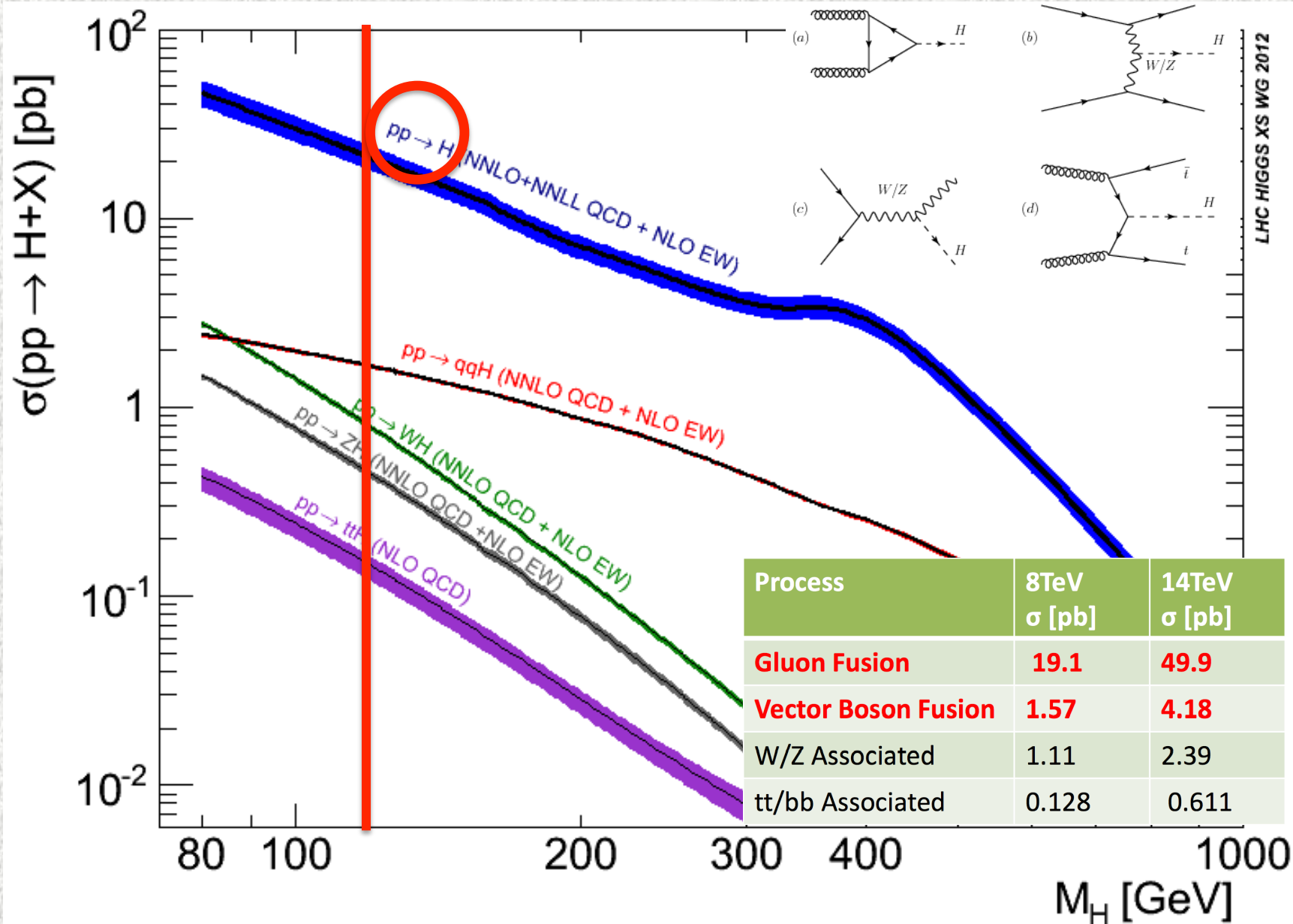
$$m_W^{-(n)} = \frac{n}{R} - m_W - \frac{m_W}{v} h$$

Characteristic predictions to
"finite" $gg \rightarrow H, H \rightarrow \gamma\gamma$ amplitudes

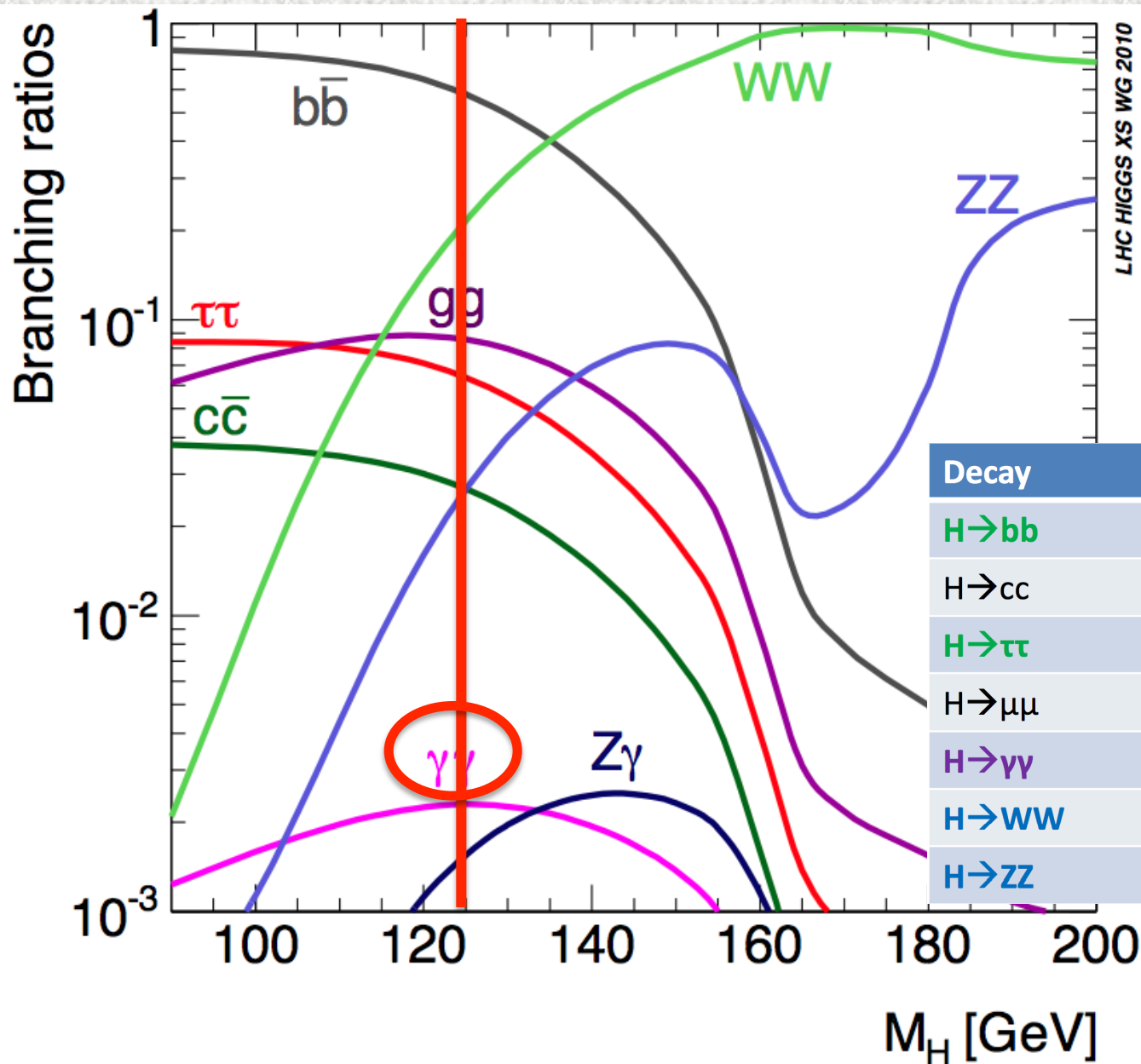
MN & Okada (2007), NM(2008)

$gg \rightarrow H \not\rightarrow \gamma \gamma$
in GHU

Higgs production



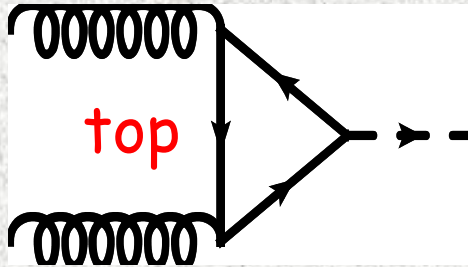
Decay rate of Higgs boson



Decay	Branch in percent
$H \rightarrow b\bar{b}$	57 %
$H \rightarrow c\bar{c}$	2.9%
$H \rightarrow \tau\tau$	6.2%
$H \rightarrow \mu\mu$	0.02%
$H \rightarrow \gamma\gamma$	0.23%
$H \rightarrow WW$	22%
$H \rightarrow ZZ$	2.8%

SM contributions

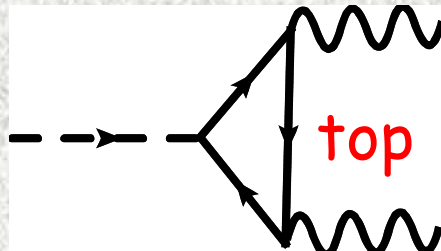
$gg \rightarrow H$



$$\mathcal{L}_{eff} = C_{gg}^{SM} h G^{a\mu\nu} G_{\mu\nu}^a$$

$$C_{gg}^{SM} = \frac{\alpha_s}{8\pi v} b_3^t \frac{\partial \ln m_t}{\partial \ln v} = \frac{\alpha_s}{12\pi v}$$

$H \rightarrow \gamma\gamma$

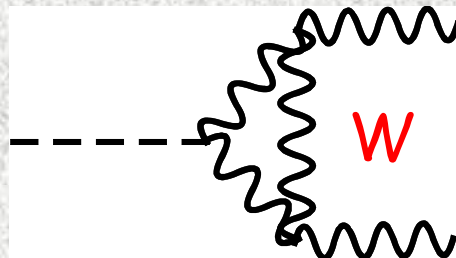


$$\mathcal{L}_{eff} = C_{\gamma\gamma}^{SM} h F^{\mu\nu} F_{\mu\nu}$$

$$C_{\gamma\gamma}^{SM} = C_{\gamma\gamma}^{top} + C_{\gamma\gamma}^W = -\frac{47\alpha_{em}}{72\pi v}$$

$$C_{\gamma\gamma}^{top} = \frac{\alpha_{em}}{6\pi v} \frac{4}{3} \frac{\partial \ln m_t}{\partial \ln v} = \frac{2\alpha_{em}}{9\pi v}$$

$$C_{\gamma\gamma}^W = \frac{\alpha_{em}}{8\pi v} (-7) \frac{\partial \ln m_W}{\partial \ln v} = -\frac{7\alpha_{em}}{8\pi v}$$



Higgs Low Energy Theorem

Coefficient of dim 5 operator $hG_{\mu\nu}^a G^{a\mu\nu}$ can be extracted from 1-loop RGE of gauge coupling

Gauge kinetic term

$$\mathcal{L} = -\frac{1}{4g^2(\mu)} G_{\mu\nu}^a G^{a\mu\nu}$$

β -function coefficient below and above $M(v)$

1-loop RGE

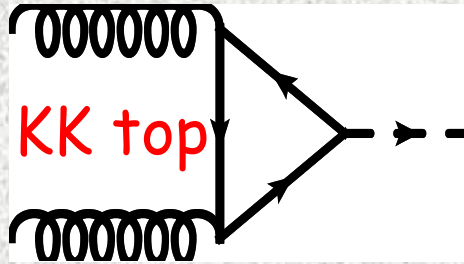
$$\frac{1}{g^2(\mu)} = \frac{1}{g^2(\Lambda)} + \frac{b_3}{8\pi^2} \ln \frac{\Lambda}{\mu} + \frac{\Delta b_3}{8\pi^2} \ln \frac{\Lambda}{M(v)}$$

Higgs VEV dependent threshold

Under $v \rightarrow v + h$, and extracting $O(h)$ term, we find

$$\mathcal{L}_{\text{eff}} = \frac{\Delta b_3}{32\pi^2} \left(\frac{\partial}{\partial v} \ln M(v) \right) h G_{\mu\nu}^a G^{a\mu\nu}$$

KK mode contributions: $gg \rightarrow H$



$$\mathcal{L}_{eff} = C_{gg}^{KK} h G^{a\mu\nu} G_{\mu\nu}^a$$

$$\begin{aligned} C_{gg}^{KKtop} &= \frac{\alpha_s}{8\pi v} \frac{2}{3} \sum_{n=1}^{\infty} \frac{\partial}{\partial \ln v} \left[\ln(m_n + m_t) + \ln(m_n - m_t) \right] \\ &\quad \text{m}_n = n/R \\ &= \frac{\alpha_s}{12\pi v} \sum_{n=1}^{\infty} \left[\frac{m_t}{m_n + m_t} - \frac{m_t}{m_n - m_t} \right] \quad \text{log}\infty - \text{log}\infty \\ &\quad \quad \quad = \text{finite} \\ &\cong -\frac{\alpha_s}{12\pi v} 2 \sum_{n=1}^{\infty} \frac{m_t^2}{m_n^2} \left(m_t^2 \ll m_n^2 \right) = -\frac{\alpha_s}{12\pi v} \frac{1}{3} (\pi m_t R)^2 \end{aligned}$$

Opposite sign to SM $\Rightarrow$ destructive

KK mode contributions: $H \rightarrow \gamma\gamma$

$$\mathcal{L}_{eff} = C_{\gamma}^{KK} h F^{\mu\nu} F_{\mu\nu} \quad \text{---} \rightarrow \text{---} \left(\begin{array}{c} \text{KK} \\ \text{top} \end{array} \right) + \text{---} \rightarrow \text{---} \left(\begin{array}{c} \text{KK} \\ W \end{array} \right)$$

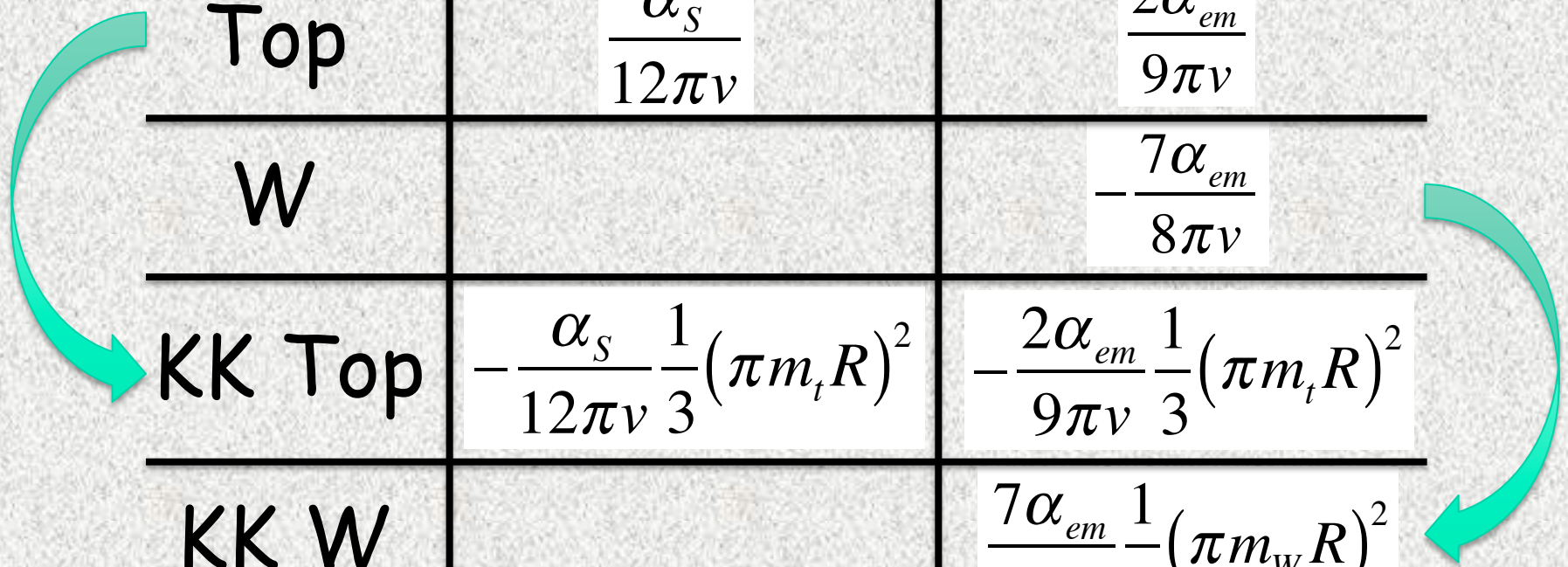
$$C_{\gamma}^{KKtop} = \frac{\alpha_{em}}{6\pi\nu} \frac{4}{3} \sum_{n=1}^{\infty} \frac{\partial}{\partial \ln \nu} \left[\ln(m_n + m_t) + \ln(m_n - m_t) \right]$$

$$\cong -\frac{2\alpha_{em}}{9\pi\nu} \frac{1}{3} (\pi m_t R)^2 \quad \text{Opposite sign to SM}$$

$$C_{\gamma}^{KKW} = \frac{\alpha_{em}}{8\pi\nu} (-7) \sum_{n=1}^{\infty} \frac{\partial}{\partial \ln \nu} \left[\ln(m_n + m_W) + \ln(m_n - m_W) \right]$$

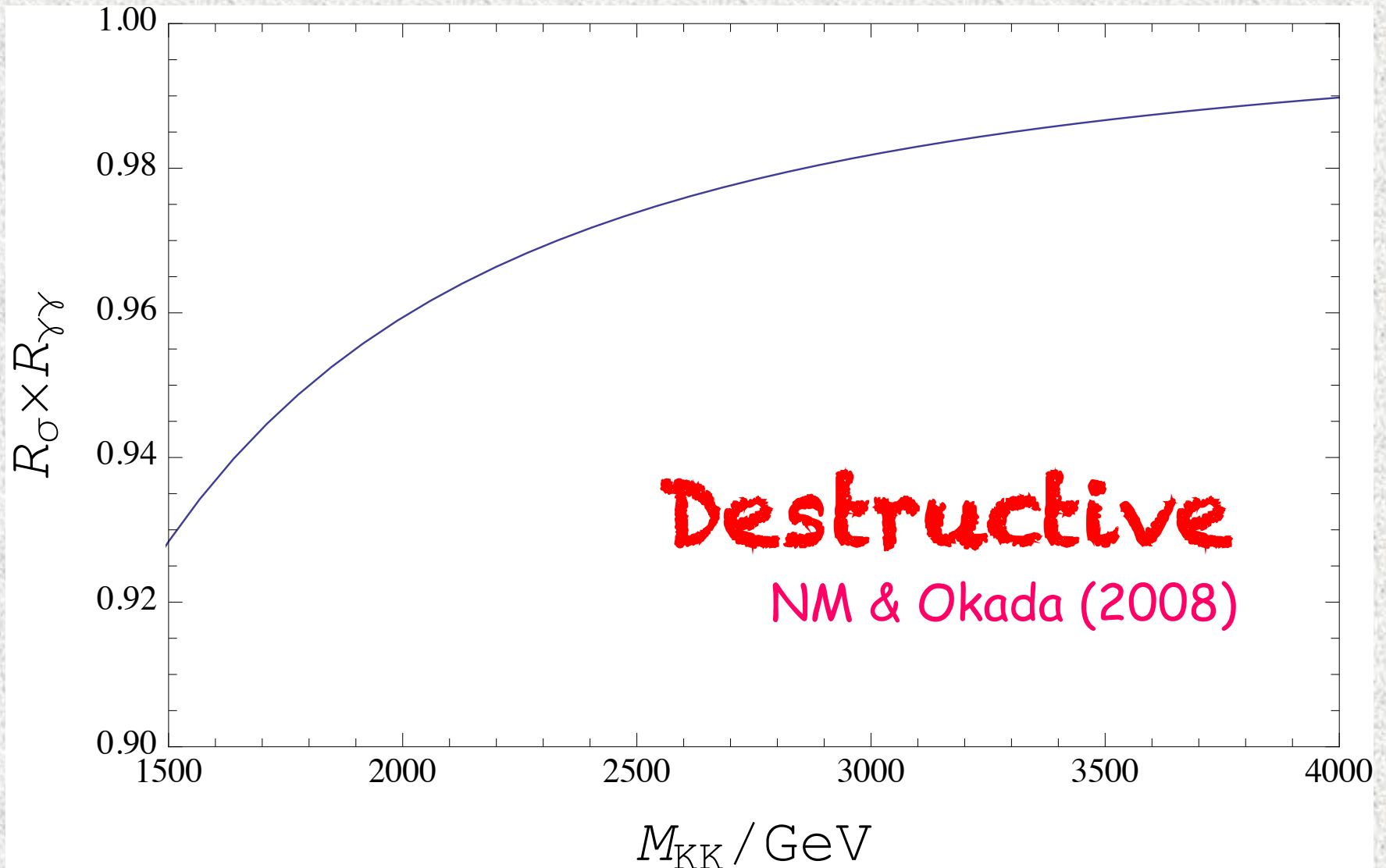
$$\cong +\frac{7\alpha_{em}}{8\pi\nu} \frac{1}{3} (\pi m_W R)^2 \quad \text{Opposite sign to SM}$$

	$gg \rightarrow H$	$H \rightarrow \gamma\gamma$
Top	$\frac{\alpha_s}{12\pi v}$	$\frac{2\alpha_{em}}{9\pi v}$
W		$-\frac{7\alpha_{em}}{8\pi v}$
KK Top	$-\frac{\alpha_s}{12\pi v} \frac{1}{3} (\pi m_t R)^2$	$-\frac{2\alpha_{em}}{9\pi v} \frac{1}{3} (\pi m_t R)^2$
KK W		$\frac{7\alpha_{em}}{8\pi v} \frac{1}{3} (\pi m_W R)^2$
GHU/SM	$1 - \frac{1}{3} (\pi m_t R)^2$	$1 + \frac{1}{141} (\pi m_W R)^2$

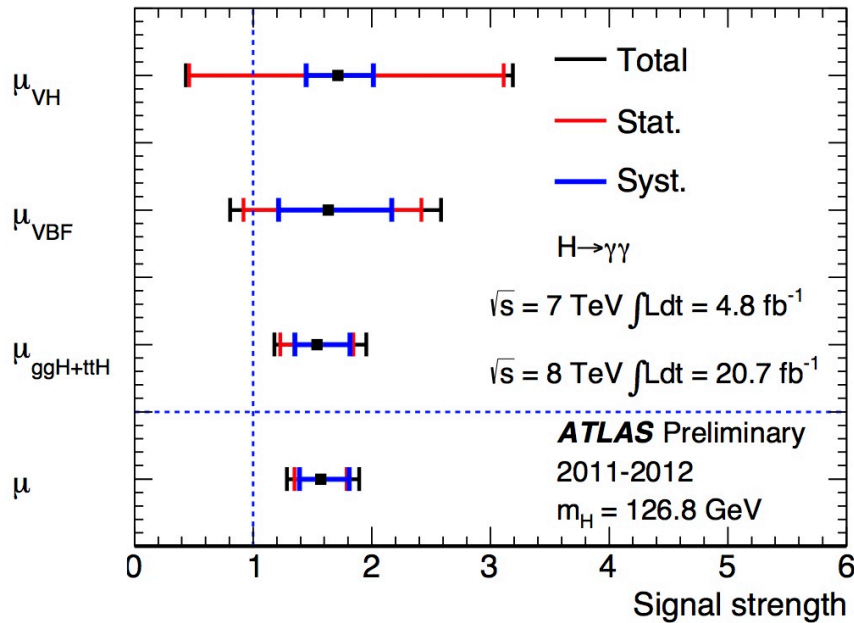


KK mode contributions: opposite sign!!

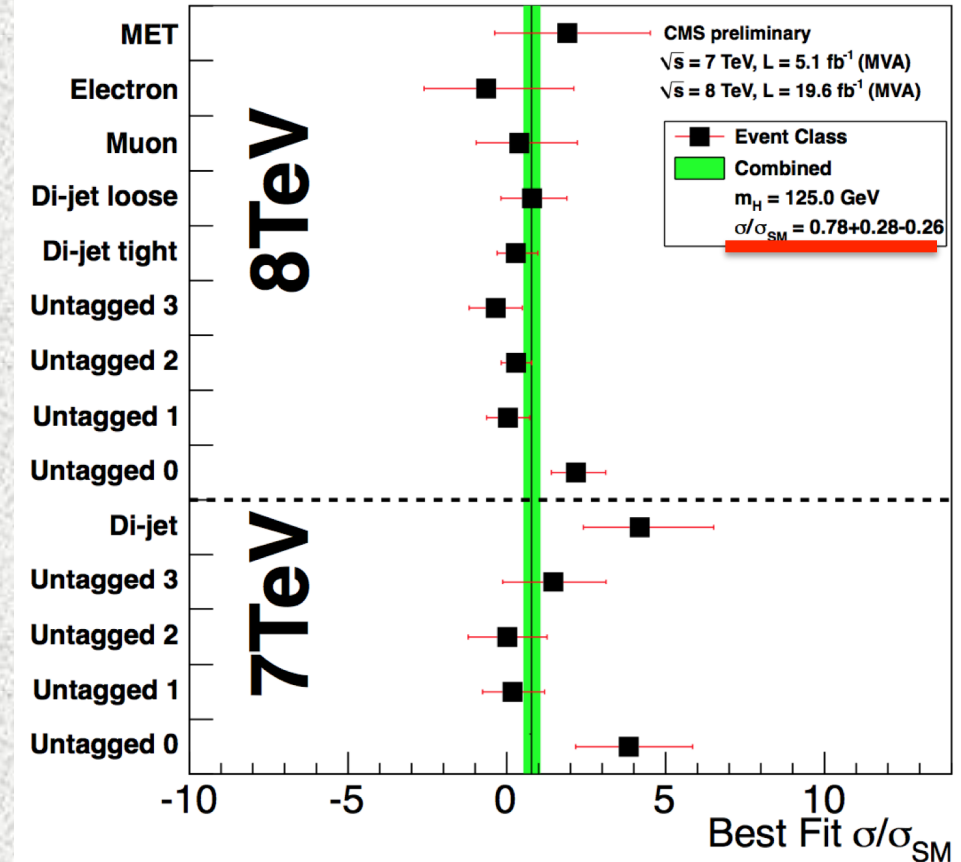
$$(gg \rightarrow H \rightarrow \gamma\gamma)_{GHU} / (gg \rightarrow H \rightarrow \gamma\gamma)_{SM}$$



Diphoton decay data



$$\sigma/\sigma_{SM} = 1.57 \pm 0.22(\text{stat}) + 0.24 - 0.18(\text{syst})$$



Extension is required

Two extensions

1: Color Singlet Fermions
&
2: Colored Fermions

Why fermions??

∴ KK fermion contributions to

$$(H \rightarrow \gamma\gamma)_{\text{KK fermions}} / (H \rightarrow \gamma\gamma)_{\text{SM}} > 0$$

	$gg \rightarrow H$	$H \rightarrow \gamma\gamma$
Top	$\frac{\alpha_s}{12\pi v}$	$\frac{2\alpha_{em}}{9\pi v}$
W		$-\frac{7\alpha_{em}}{8\pi v}$
KK Top	$-\frac{\alpha_s}{12\pi v} \frac{1}{3} (\pi m_t R)^2$	$-\frac{2\alpha_{em}}{9\pi v} \frac{1}{3} (\pi m_t R)^2$
KK W		$\frac{7\alpha_{em}}{8\pi v} \frac{1}{3} (\pi m_W R)^2$
GHU/SM	$1 - \frac{1}{3} (\pi m_t R)^2$	$1 + \frac{1}{141} (\pi m_W R)^2$




KK mode contributions: opposite sign!!

Color Singlet Fermions

"Diphoton Decay Excess and 125 GeV Higgs Boson
in Gauge-Higgs Unification"
NM and Nobuchika Okada
PRD87 095019 (2013)

Simplest extension:

"Extra Leptons"

(colored particles greatly affect $gg \rightarrow H$, but discuss later)

Two examples:

10, 15 reps. of $SU(3)$ with bulk mass
& half-periodic BC $\psi(y+2\pi R) = -\psi(y)$

No unwanted massless fermions

1st KK mass = $1/(2R)$

$\Rightarrow$ Higgs mass enhancement

Helpful to
adjust
125 GeV Higgs

Diphoton decay from 10 & 15

$$C_{\gamma\gamma}^{KKlepton10} = (Q-1)^2 F(3m_W) + (Q-1)^2 F(m_W) + Q^2 F(2m_W) + (Q+1)^2 F(m_W)$$

$$C_{\gamma\gamma}^{KKlepton15} = (Q-4/3)^2 F(4m_W) + (Q-4/3)^2 F(2m_W) + (Q-1/3)^2 F(3m_W) \\ + (Q-1/3)^2 F(m_W) + (Q+2/3)^2 F(2m_W) + (Q+5/3)^2 F(m_W)$$

Q: U(1)' charge

$$F(m_W) = \frac{\alpha_{em}}{6\pi\nu} N_f m_W \sum_{n=0}^{\infty} \left[\frac{\frac{n+1/2}{R} + m_W}{M^2 + \left(\frac{n+1/2}{R} + m_W \right)^2} - \frac{\frac{n+1/2}{R} - m_W}{M^2 + \left(\frac{n+1/2}{R} - m_W \right)^2} \right] \\ \cong -\frac{\alpha_{em}}{3\pi\nu} N_f m_W^2 \sum_{n=0}^{\infty} \frac{\left(\frac{n+1/2}{R} \right)^2 - M^2}{\left[\left(\frac{n+1/2}{R} \right)^2 + M^2 \right]^2} \left(m_W^2 \ll m_n^2 \right) = -\frac{\alpha_{em}}{6\pi\nu} N_f \frac{(\pi m_W R)^2}{\cosh(\pi MR)}$$

Negative

Mass eigenvalues & charges of 10 & 15

$$10 = 1_{-1} + 2_{-1/2} + 3_0 + 4_{1/2} \leftarrow \begin{matrix} \text{U(1) charge of} \\ \text{SU(2) \times U(1)} \end{matrix}$$

"elemag charge"
of SU(2) × U(1)

$$\left(m_{n,-1}^{(\pm)}\right)^2 = \left(m_{n+1/2}^{(\pm)} \pm 3m_W\right)^2 + M^2, \left(m_{n+1/2}^{(\pm)} \pm m_W\right)^2 + M^2$$

$$\left(m_{n,0}^{(\pm)}\right)^2 = \left(m_{n+1/2}^{(\pm)} \pm 2m_W\right)^2 + M^2, m_{n+1/2}^2 + M^2$$

$$\left(m_{n,+1}^{(\pm)}\right)^2 = \left(m_{n+1/2}^{(\pm)} \pm m_W\right)^2 + M^2, \left(m_{n,+2}^{(\pm)}\right)^2 = m_{n+1/2}^2 + M^2$$

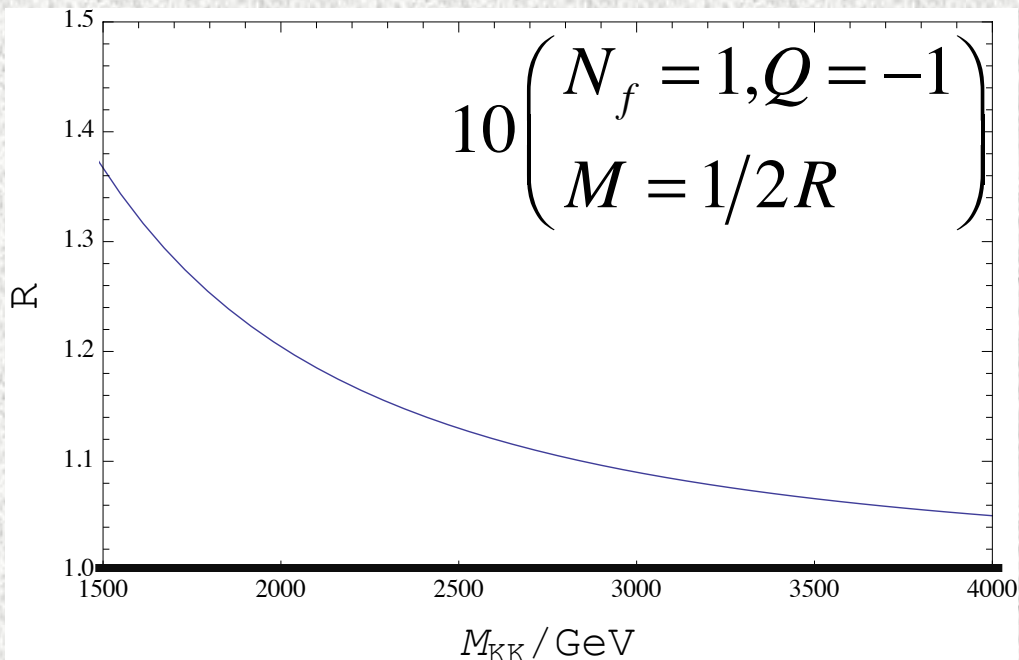
$$15 = 1_{-4/3} + 2_{-5/6} + 3_{-1/3} + 4_{1/6} + 5_{2/3}$$

$$\left(m_{n,-4/3}^{(\pm)}\right)^2 = \left(m_{n+1/2}^{(\pm)} \pm 4m_W\right)^2 + M^2, \left(m_{n+1/2}^{(\pm)} \pm 2m_W\right)^2 + M^2, m_{n+1/2}^2 + M^2$$

$$\left(m_{n,-1/3}^{(\pm)}\right)^2 = \left(m_{n+1/2}^{(\pm)} \pm 3m_W\right)^2 + M^2, \left(m_{n+1/2}^{(\pm)} \pm m_W\right)^2 + M^2$$

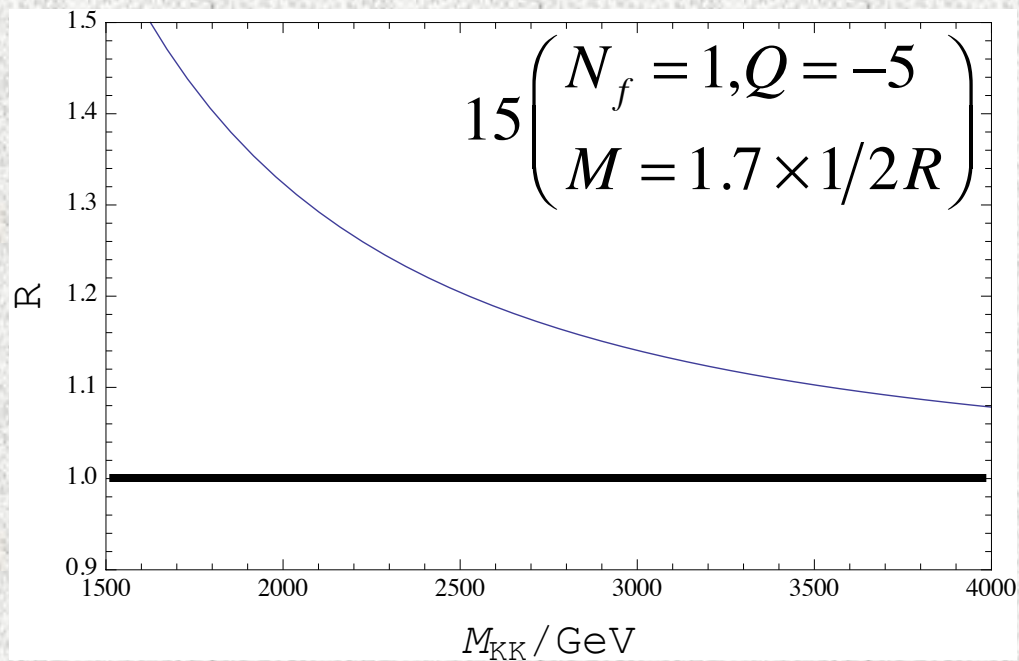
$$\left(m_{n,2/3}^{(\pm)}\right)^2 = \left(m_{n+1/2}^{(\pm)} \pm 2m_W\right)^2 + M^2, m_{n+1/2}^2 + M^2$$

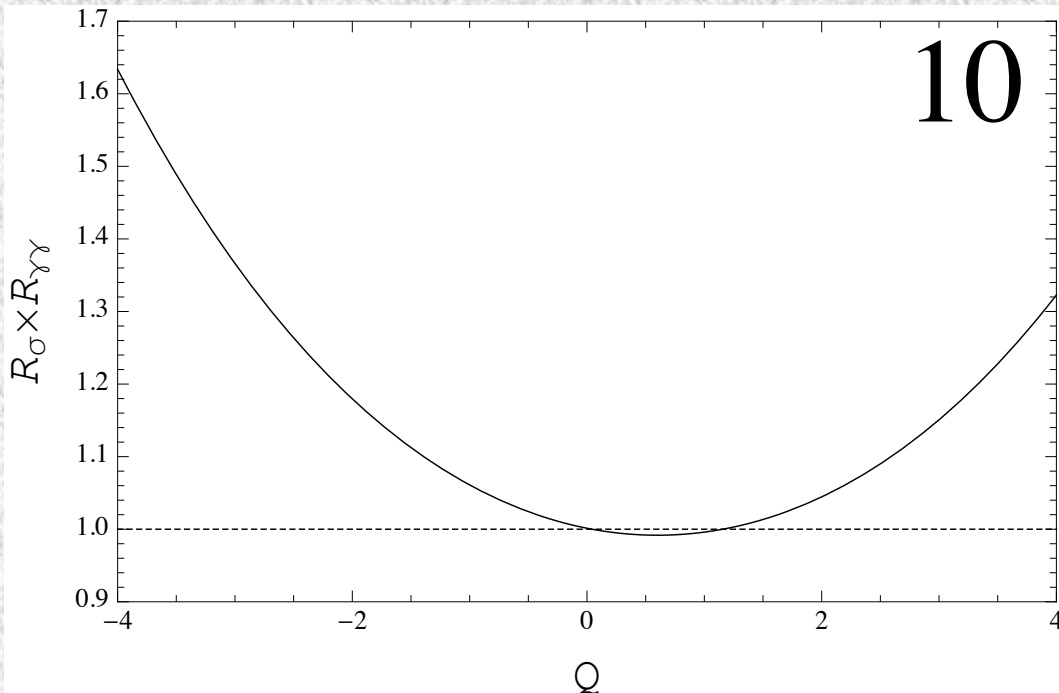
$$\left(m_{n,5/3}^{(\pm)}\right)^2 = \left(m_{n+1/2}^{(\pm)} \pm m_W\right)^2 + M^2, \left(m_{n,8/3}^{(\pm)}\right)^2 = m_{n+1/2}^2 + M^2$$



$$R = \frac{\sigma(gg \rightarrow H)_{GHU+10} \times BR(H \rightarrow \gamma\gamma)_{GHU+10}}{\sigma(gg \rightarrow H)_{SM} \times BR(H \rightarrow \gamma\gamma)_{SM}}$$

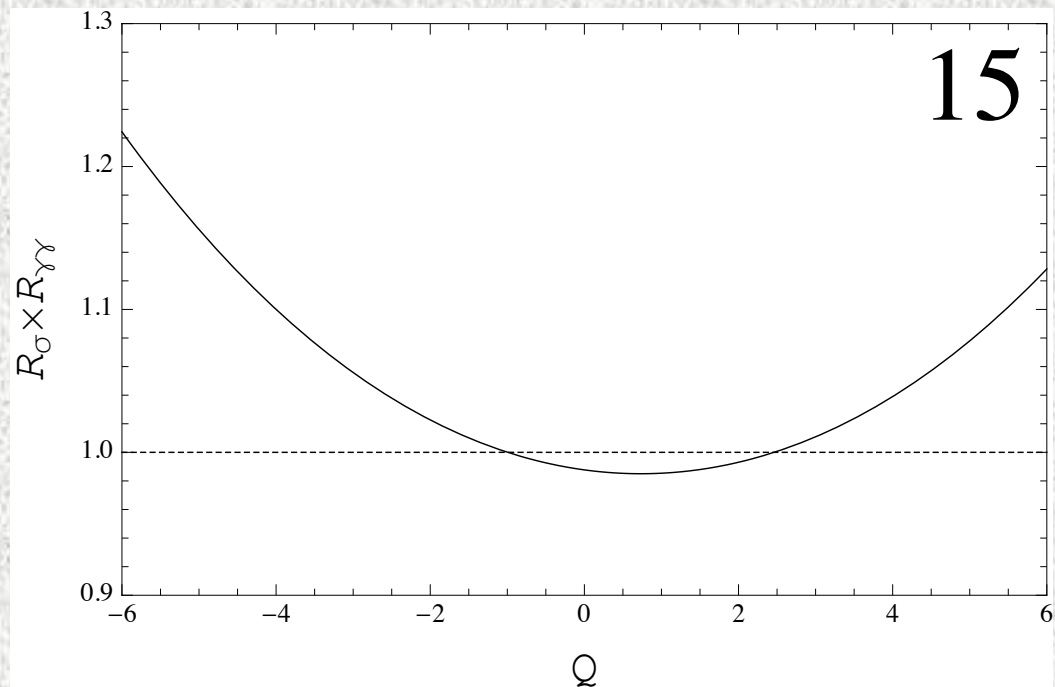
$$R = \frac{\sigma(gg \rightarrow H)_{GHU+15} \times BR(H \rightarrow \gamma\gamma)_{GHU+15}}{\sigma(gg \rightarrow H)_{SM} \times BR(H \rightarrow \gamma\gamma)_{SM}}$$





$$R = \frac{\sigma(gg \rightarrow H)_{GHU} \times BR(H \rightarrow \gamma\gamma)_{GHU}}{\sigma(gg \rightarrow H)_{SM} \times BR(H \rightarrow \gamma\gamma)_{SM}}$$

$1/R = 3\text{TeV fixed}$



Higgs mass analysis

Higgs mass analysis by 4D EFT approach

In GHU, m_H likely to be small $\because$ loop generated

Instead of 5D Higgs potential minimization,
solve 1-loop RGE for Higgs quartic coupling λ
by imposing BC $\lambda=0@1/R$ "gauge-Higgs condition"

Haba, Matsumoto, Okada & Yamashita (2006, 2008)

Natural realization of GHU in 4D viewpoint:

$V_H = 0$ above $1/R$ by 5D gauge invariance

Furthermore, NO vacuum instability

This approach greatly simplifies Higgs mass study

1-loop RGE for λ

$$\frac{d\lambda}{d\ln\mu} = \frac{1}{16\pi^2} \left[12\lambda^2 - \left(\frac{9}{5}g_1^2 + 9g_2^2 \right) \lambda + \frac{9}{4} \left(\frac{3}{25}g_1^4 + \frac{2}{5}g_1^2g_2^2 + g_2^4 \right) \right. \\ \left. + 4 \left(3y_t^2 + N_f C_q(R) (\sqrt{2}g_2)^2 \right) \lambda - 4 \left(3y_t^4 + N_f C_f(R) (\sqrt{2}g_2)^4 \right) \right] (\mu \geq M)$$

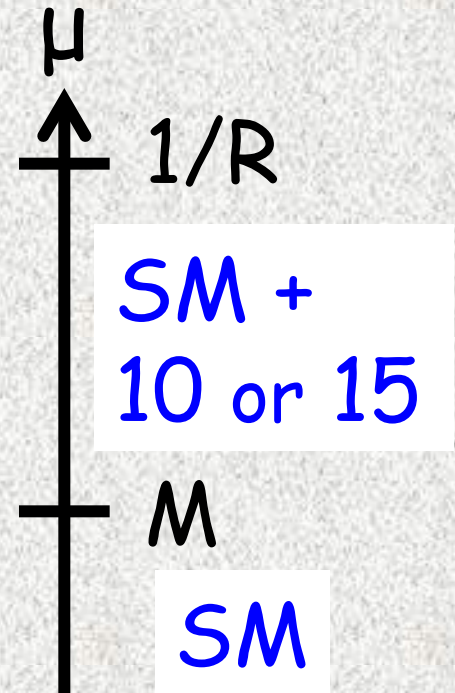
$$\frac{d\lambda}{d\ln\mu} = \frac{1}{16\pi^2} \left[12\lambda^2 - \left(\frac{9}{5}g_1^2 + 9g_2^2 \right) \lambda + \frac{9}{4} \left(\frac{3}{25}g_1^4 + \frac{2}{5}g_1^2g_2^2 + g_2^4 \right) + 12y_t^2\lambda - 12y_t^4 \right] (\mu < M)$$

$$C_f(10) = 2 \left[\left(\frac{3}{2} \right)^4 + \left(\frac{1}{2} \right)^4 + 1^4 + \left(\frac{1}{2} \right)^4 \right]$$

$$C_q(10) = 2 \left[\left(\frac{3}{2} \right)^2 + \left(\frac{1}{2} \right)^2 + 1^2 + \left(\frac{1}{2} \right)^2 \right]$$

$$C_f(15) = 2 \left[2^4 + 1^4 + \left(\frac{3}{2} \right)^4 + \left(\frac{1}{2} \right)^4 + 1^4 + \left(\frac{1}{2} \right)^4 \right]$$

$$C_q(15) = 2 \left[2^2 + 1^2 + \left(\frac{3}{2} \right)^2 + \left(\frac{1}{2} \right)^2 + 1^2 + \left(\frac{1}{2} \right)^2 \right]$$

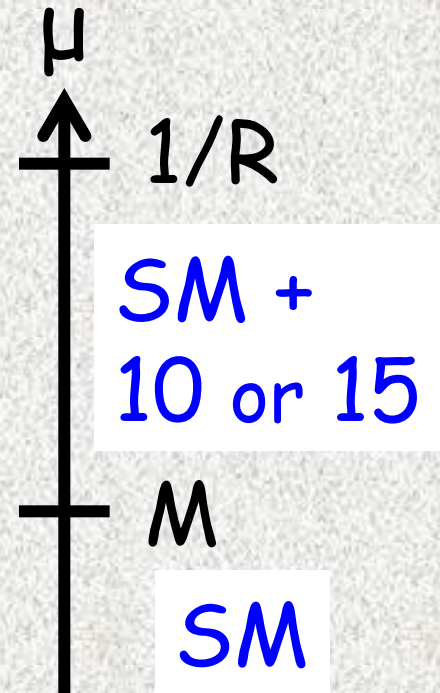


1-loop RGE for λ

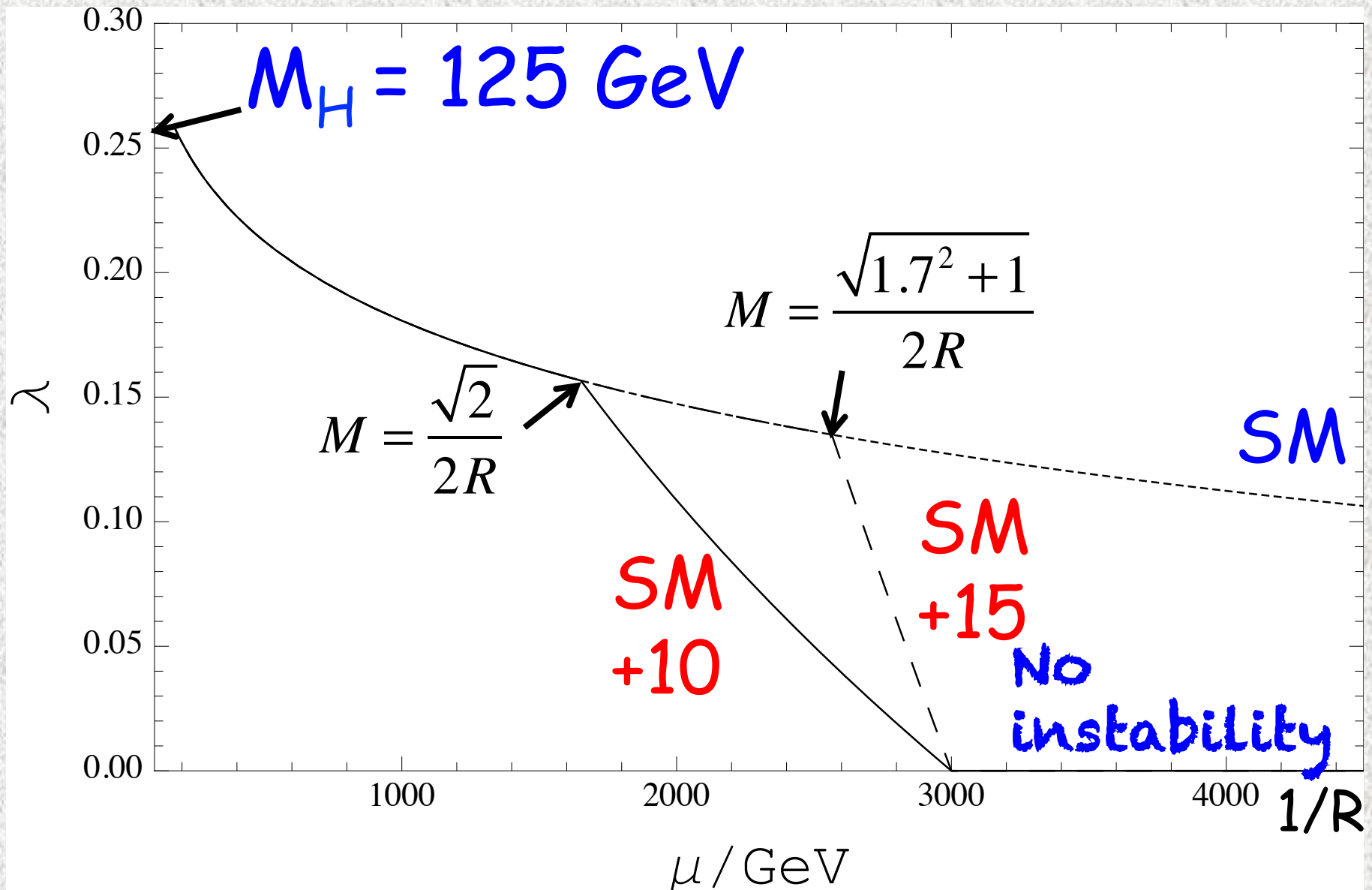
$$\frac{d\lambda}{d\ln\mu} = \frac{1}{16\pi^2} \left[12\lambda^2 - \left(\frac{9}{5}g_1^2 + 9g_2^2 \right) \lambda + \frac{9}{4} \left(\frac{3}{25}g_1^4 + \frac{2}{5}g_1^2g_2^2 + g_2^4 \right) + 4 \left(3y_t^2 + N_f C_q(R) (\sqrt{2}g_2)^2 \right) \lambda - 4 \left(3y_t^4 + N_f C_f(R) (\sqrt{2}g_2)^4 \right) \right] (\mu \geq M)$$

$$\frac{d\lambda}{d\ln\mu} = \frac{1}{16\pi^2} \left[12\lambda^2 - \left(\frac{9}{5}g_1^2 + 9g_2^2 \right) \lambda + \frac{9}{4} \left(\frac{3}{25}g_1^4 + \frac{2}{5}g_1^2g_2^2 + g_2^4 \right) + 12y_t^2\lambda - 12y_t^4 \right] (\mu < M)$$

- Contributions from **1st KK mode with mass $1/(2R)$** in 10 or 15 rep.
- 2nd term $\propto (\sqrt{2}g_2)^4$ dominated
- Negative sign**



Numerical results for 1-loop RGE of λ



TeV Scale Colored Fermions

"125 GeV Higgs Boson & TeV Scale Colored Fermion
in Gauge-Higgs Unification"

NM and Nobuchika Okada

arXiv: 1310.3348

Another possibility to realize 125 GeV Higgs mass
by introducing extra colored fermions

Colored fermions contribute to $gg \rightarrow H$ destructively
 $\Rightarrow$ LHC Data put a lower bound for KK masses
 $\Rightarrow$ It would be interesting
if the lower bound is within a detectable range
w/o contradicting 125 GeV Higgs mass

Result:

$$M_{KK} = 2-3 \text{ TeV for } \sigma(gg \rightarrow H)/\sigma_{SM} \sim 0.9-0.95$$

Extra colored fermions have half-periodic BC

⇒ the lightest KK particle (LKP) is stable,
but **stable colored particle is**
cosmologically disfavor

⇒ introduce **the mixing btw the LKP & SM quarks**
on the brane for decay to SM quarks

⇒ **$U(1)'$ charge fixed to be**
 $-1/3$ ($2/3$) for down(up)-type quarks

⇒ **$Q=2/3, 5/3$ for 10-plet, $Q=1, 2$ for 15-plet**
($Q-1=-1/3, 2/3$ for 10, $Q-4/3=-1/3$ or $2/3$)

Mass eigenvalues & charges of 10 & 15

$$10 = 1_{-1} + 2_{-1/2} + 3_0 + 4_{1/2} \leftarrow \begin{matrix} \text{U(1) charge of} \\ \text{SU(2) \times U(1)} \end{matrix}$$

"elemag charge"
of SU(2) × U(1)

$$\left(m_{n,-1}^{(\pm)}\right)^2 = \left(m_{n+1/2}^{(\pm)} \pm 3m_W\right)^2 + M^2, \left(m_{n+1/2}^{(\pm)} \pm m_W\right)^2 + M^2$$

$$\left(m_{n,0}^{(\pm)}\right)^2 = \left(m_{n+1/2}^{(\pm)} \pm 2m_W\right)^2 + M^2, m_{n+1/2}^2 + M^2$$

$$\left(m_{n,+1}^{(\pm)}\right)^2 = \left(m_{n+1/2}^{(\pm)} \pm m_W\right)^2 + M^2, \left(m_{n,+2}^{(\pm)}\right)^2 = m_{n+1/2}^2 + M^2$$

$$15 = 1_{-4/3} + 2_{-5/6} + 3_{-1/3} + 4_{1/6} + 5_{2/3}$$

$$\left(m_{n,-4/3}^{(\pm)}\right)^2 = \left(m_{n+1/2}^{(\pm)} \pm 4m_W\right)^2 + M^2, \left(m_{n+1/2}^{(\pm)} \pm 2m_W\right)^2 + M^2, m_{n+1/2}^2 + M^2$$

$$\left(m_{n,-1/3}^{(\pm)}\right)^2 = \left(m_{n+1/2}^{(\pm)} \pm 3m_W\right)^2 + M^2, \left(m_{n+1/2}^{(\pm)} \pm m_W\right)^2 + M^2$$

$$\left(m_{n,2/3}^{(\pm)}\right)^2 = \left(m_{n+1/2}^{(\pm)} \pm 2m_W\right)^2 + M^2, m_{n+1/2}^2 + M^2$$

$$\left(m_{n,5/3}^{(\pm)}\right)^2 = \left(m_{n+1/2}^{(\pm)} \pm m_W\right)^2 + M^2, \left(m_{n,8/3}^{(\pm)}\right)^2 = m_{n+1/2}^2 + M^2$$

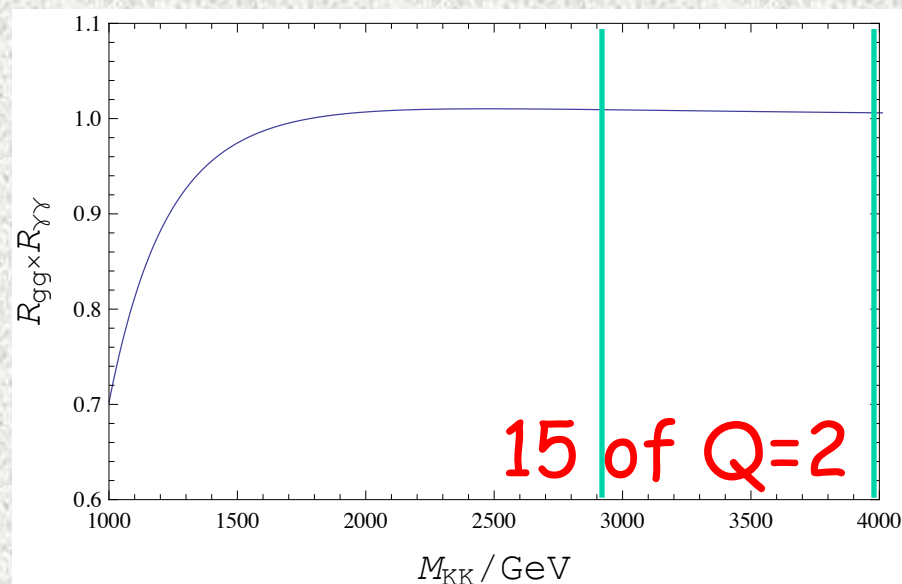
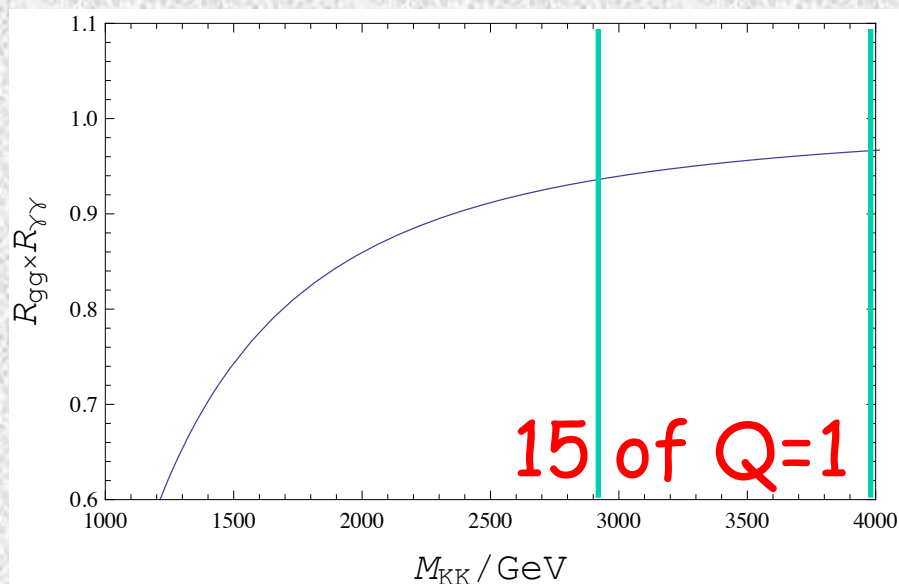
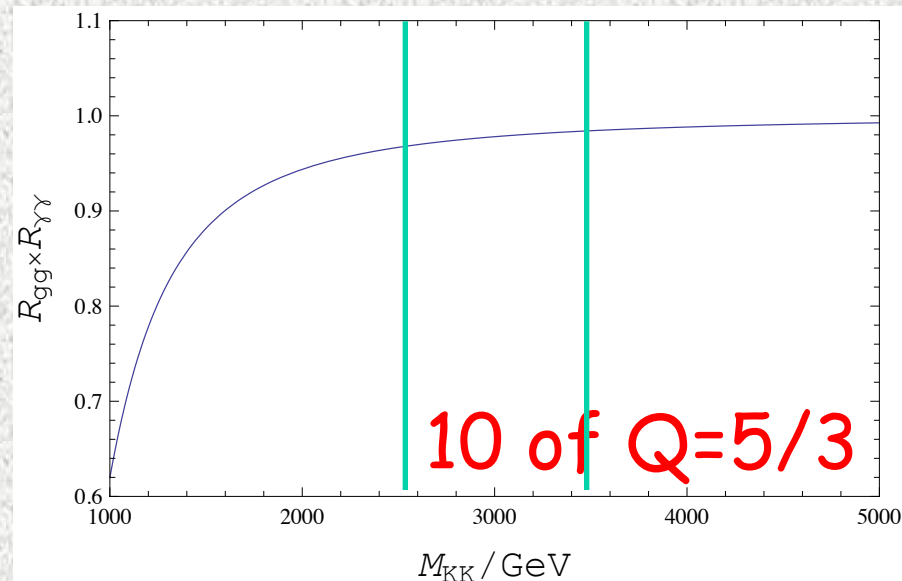
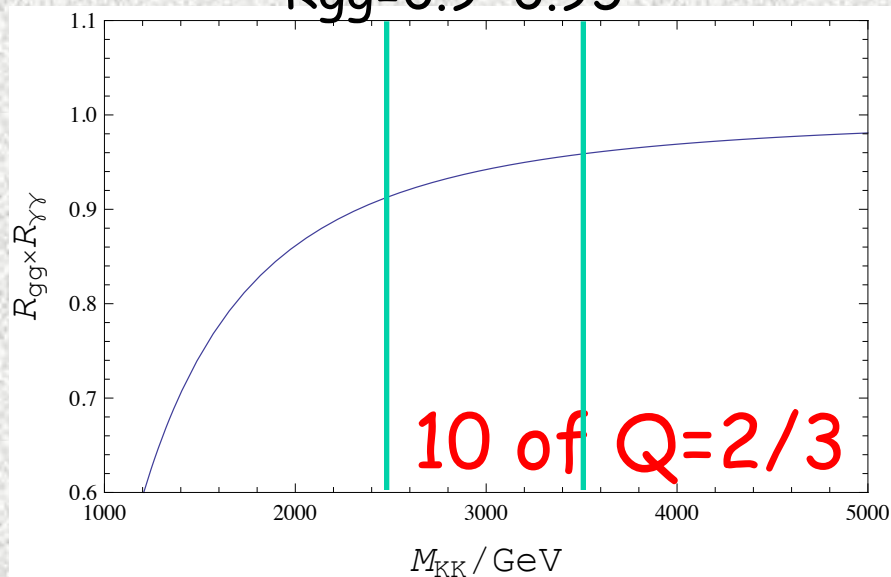
Lower bound on KK scale etc from gluon fusion

10-plet	$R_{gg} = 0.9$	$R_{gg} = 0.95$
M_{KK} (TeV)	2.54	3.45
$m_0^{(\pm)}$ (TeV)	2.05	2.91
m_{lightest} (TeV)	1.91	2.77

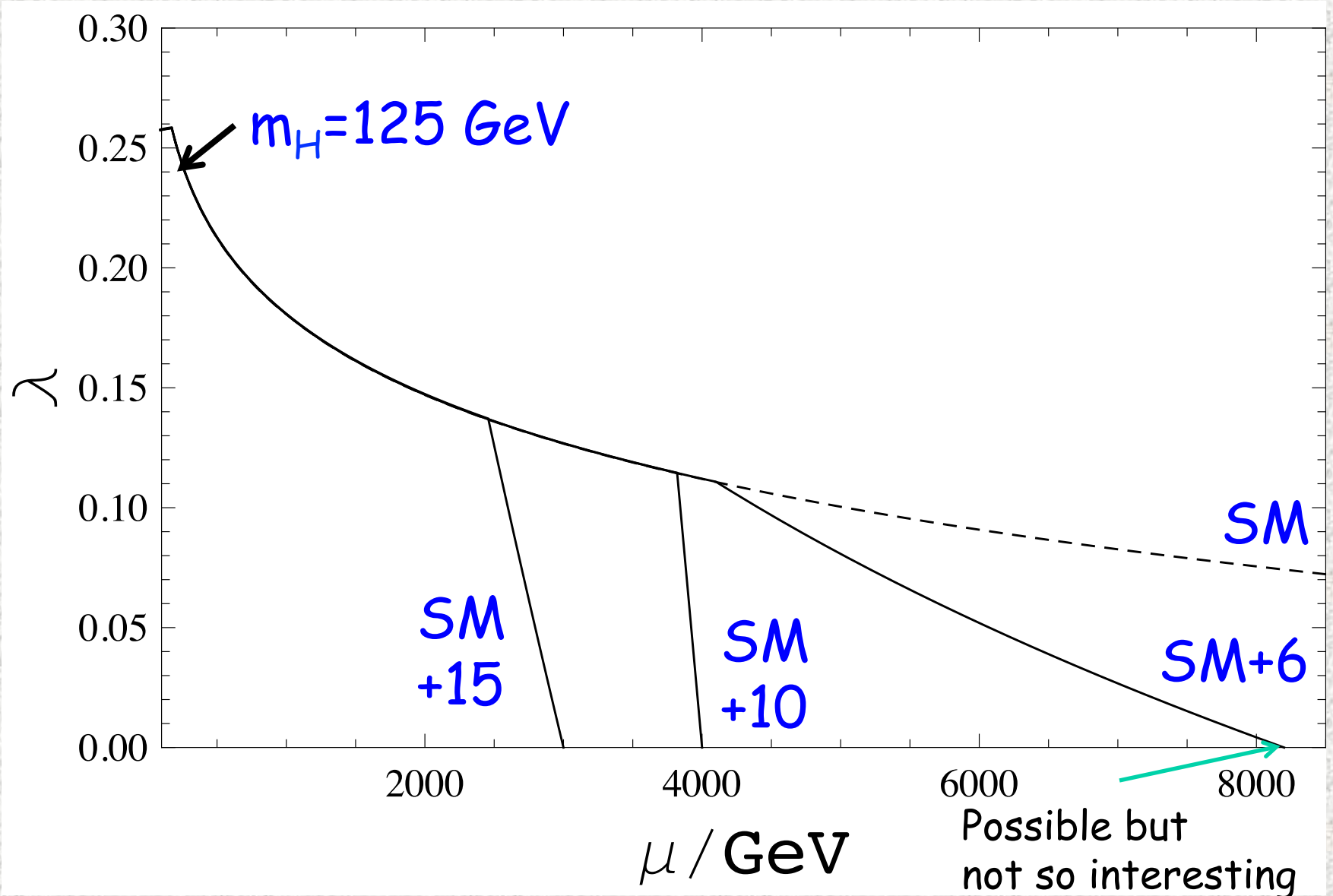
15-plet	$R_{gg} = 0.9$	$R_{gg} = 0.95$
M_{KK} (TeV)	2.88	4.05
$m_0^{(\pm)}$ (TeV)	2.73	3.87
m_{lightest} (TeV)	2.57	3.71

Diphoton Decay Signal Strength

$R_{gg}=0.9$ 0.95



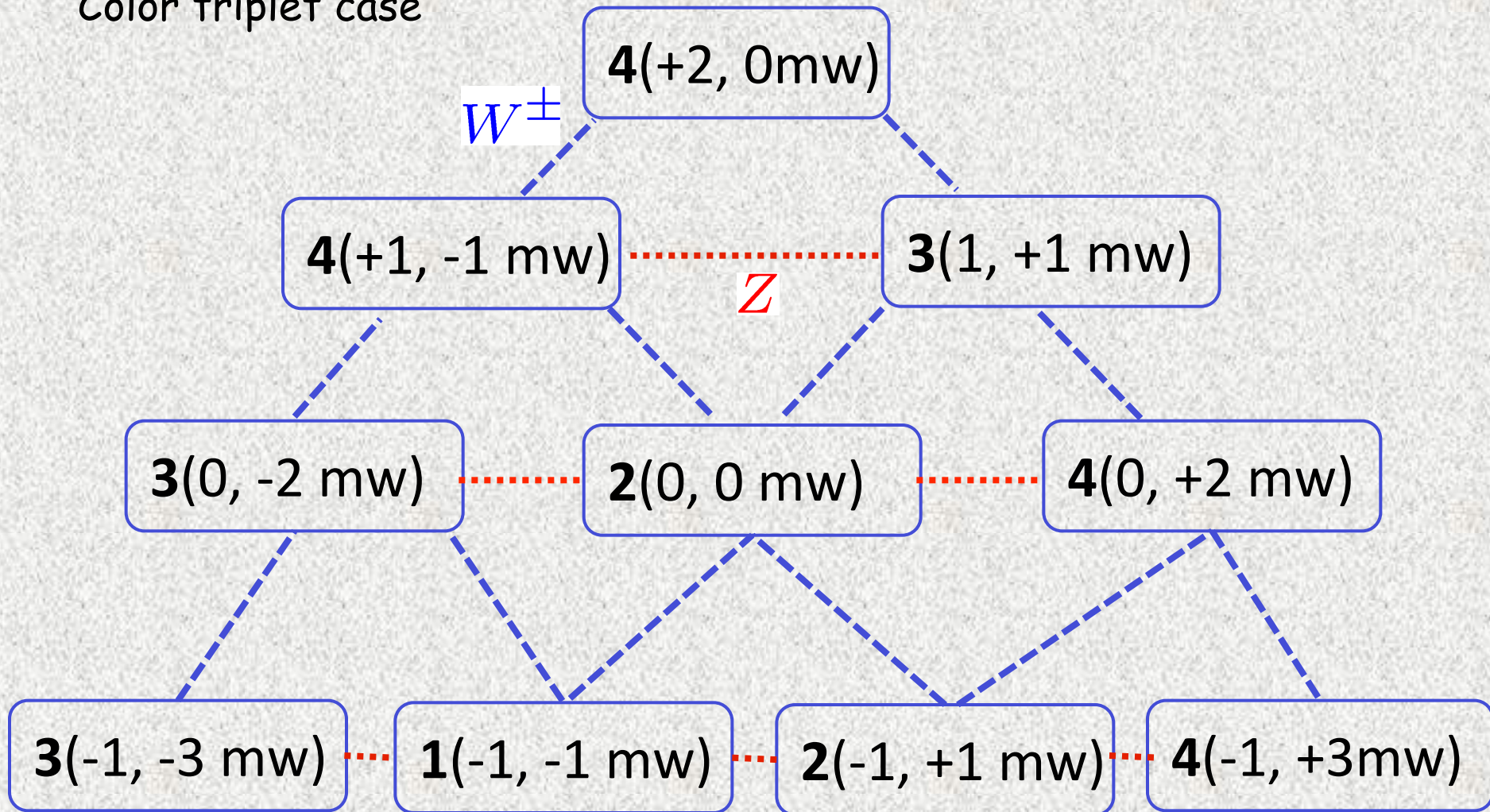
1-loop RGE for Higgs quartic coupling



Interactions between KK modes & W, Z

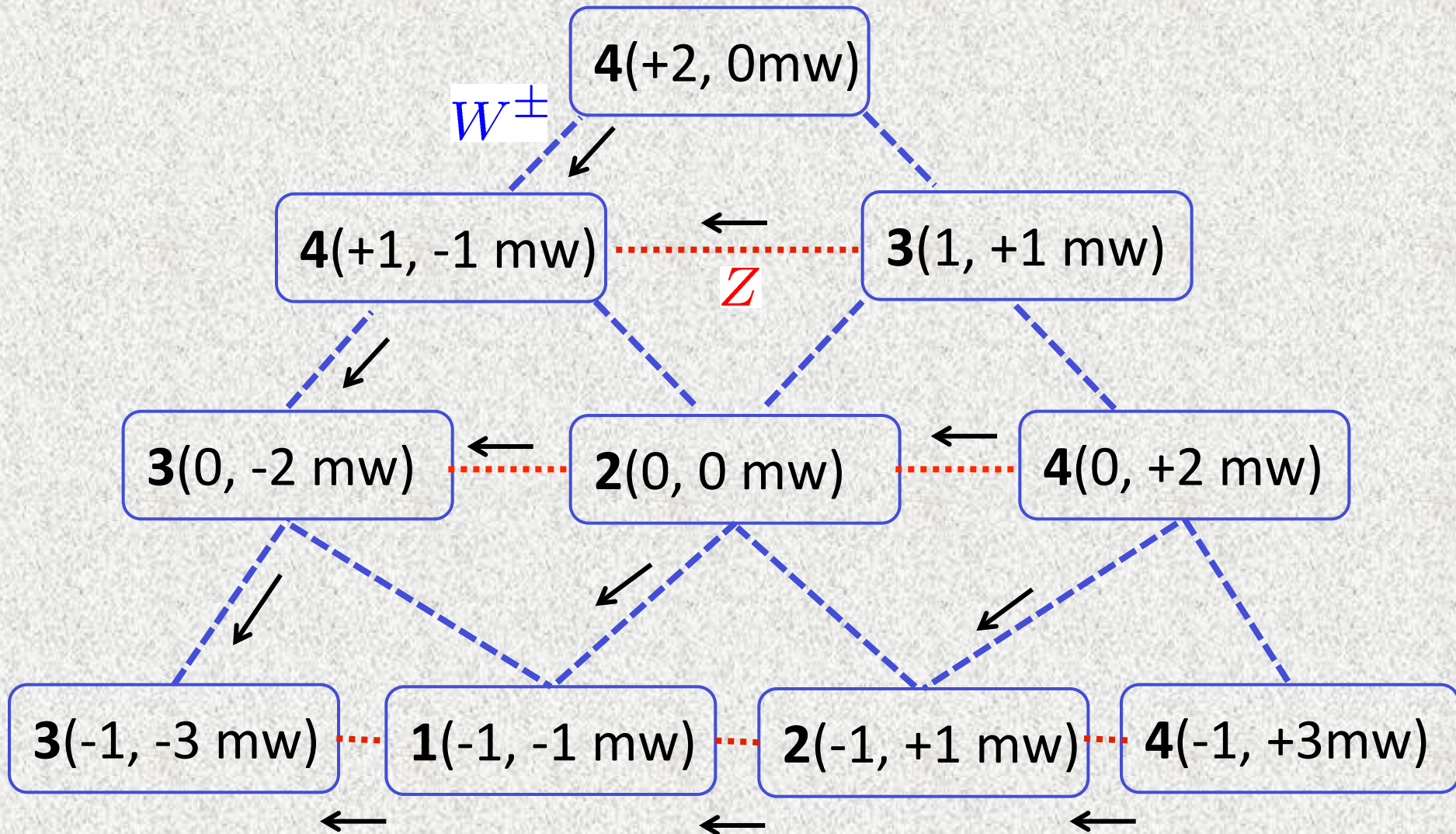
Ex: 10-plet: $10 = 1_{-1} \oplus 2_{-1/2} \oplus 3_0 \oplus 4_{1/2}$

Color triplet case



Heavy fermion cascades

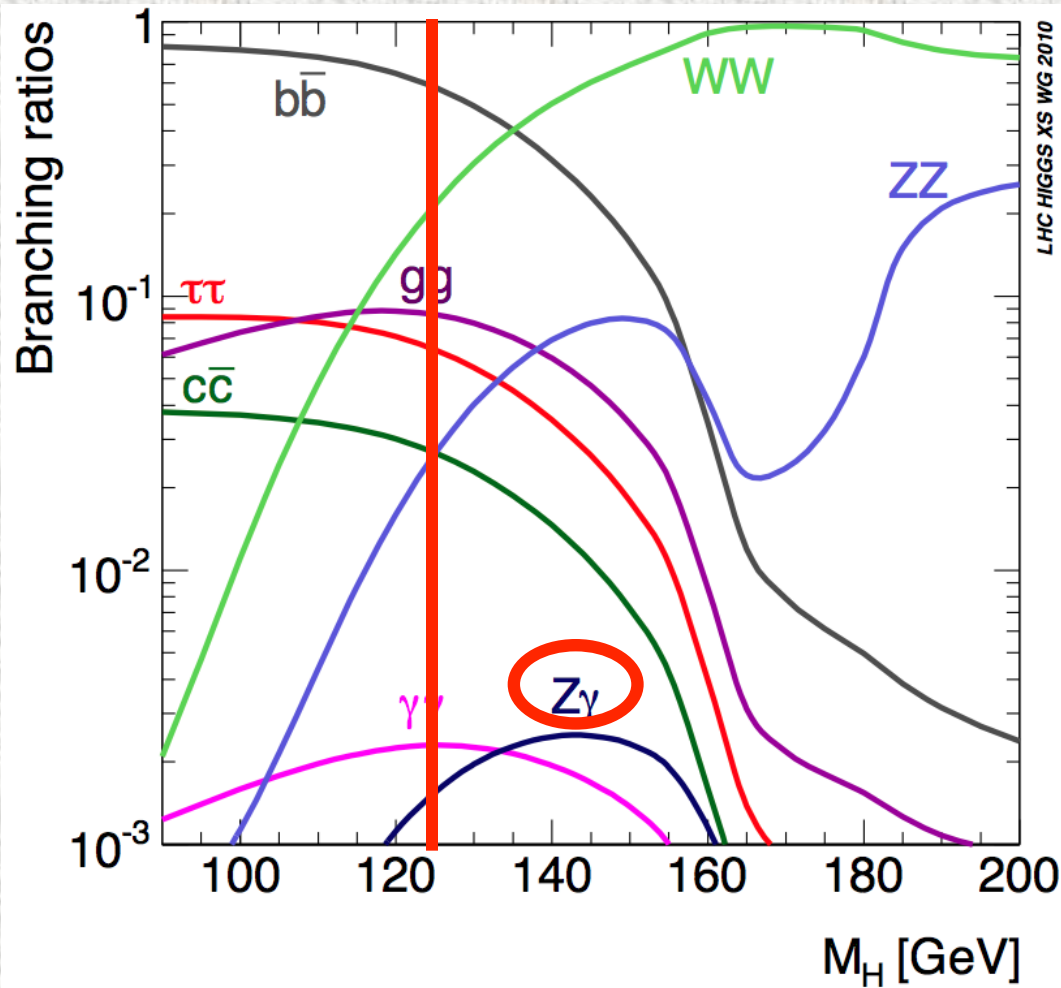
Ex: 10-plet: $10 = 1_{-1} \oplus 2_{-1/2} \oplus 3_0 \oplus 4_{1/2}$



$$H \rightarrow Z \gamma$$

"H to Z Gamma in Gauge-Higgs Unification"
NM & Nobuchika Okada,
PRD88 (2013) 037701

A Comment on $H \rightarrow Z\gamma$



KK modes have
EW charges



Naturally, a deviation
of $Z\gamma$ decay from
the SM prediction
expected

Model dep.
Correlation btw $\gamma\gamma$ & $Z\gamma$
is interesting

No KK mode contributions to $Z\gamma$ decay@1-loop

Simple reason: in the mass eigenstates, H and γ couples to
KK modes with same mass eigenstates,
but **Z does not**

Fermion
coupling

$$(\bar{\psi}_0^{(n)}, \bar{\psi}_+^{(n)}, \bar{\psi}_-^{(n)}) \begin{pmatrix} 2\gamma_\mu/\sqrt{3} & W_\mu^+ & W_\mu^+ \\ W_\mu^- & -\gamma_\mu/\sqrt{3} & -Z_\mu \\ W_\mu^- & -Z_\mu & -\gamma_\mu/\sqrt{3} \end{pmatrix} \gamma^\mu \begin{pmatrix} \psi_0^{(n)} \\ \psi_+^{(n)} \\ \psi_-^{(n)} \end{pmatrix}, \psi_{0,\pm}^{(n)} : \frac{n}{R}, \frac{n}{R} \pm m_f$$

ZW^nW^n
coupling

$$Z_\mu (W_{\mu\nu+}^{\mp(n)}, W_{\mu\nu-}^{\mp(n)}) \begin{pmatrix} 0 & \mp i \\ \pm i & 0 \end{pmatrix} (W_{\nu+}^{\pm(n)}, W_{\nu-}^{\pm(n)})$$

$$W_{\mu\pm}^{(n)} : n/R \pm m_W, W_{\mu\nu} \equiv \partial_\mu W_\nu - \partial_\nu W_\mu$$

$Z\gamma W^nW^n$
coupling

$$Z^\mu \gamma_\nu (W_{\mu+}^{\mp(n)}, W_{\mu-}^{\mp(n)}) \begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix} \begin{pmatrix} W_+^{\pm\nu(n)} \\ W_-^{\pm\nu(n)} \end{pmatrix} + 2Z_\mu \gamma^\mu (W_{\nu+}^{\mp(n)}, W_{\nu-}^{\mp(n)}) \begin{pmatrix} 0 & i \\ -i & 0 \end{pmatrix} \begin{pmatrix} W_+^{\pm\nu(n)} \\ W_-^{\pm\nu(n)} \end{pmatrix}$$

In SU(3) GHU, No H-Z- γ coupling@1-loop found

Correct even in $SU(3) \times U(1)$ GHU??

- In Hasegawa & Lim (2016), it was shown that $d=6$ operator describing $H \rightarrow Z\gamma$ is **(not) forbidden** in $SU(3)(\times U(1))$ GHU
- Hosotani et al. (2015) has obtained nonzero but tiny value of $H \rightarrow Z\gamma$ in warped $SO(5) \times U(1)$ GHU

We investigated $H \rightarrow Z\gamma$ @ 1-loop in $SU(3) \times U(1)$ GHU and verified a nonzero result
NM & Onogi (in progress)

Summary

- We have calculated KK mode contributions to $gg \rightarrow H$ & $H \rightarrow \gamma\gamma$ @LHC in 5D $SU(3) \times U(1)'$ GHU
- Simplest model cannot explain the data
- Extra fermions can enhance $H \rightarrow \gamma\gamma$ as we like
by adjusting $U(1)'$ charges
ex. Color singlet & Colored fermions
in 10 & 15 reps. of $SU(3)$ w/ bulk mass
& half-periodic BC
- These fermions also help to enhance Higgs mass

Summary

- 1-loop RGE analysis of Higgs quartic coupling
with GH condition $\lambda=0@M_{KK}$
 $\Rightarrow$ No instability
- Extra fermions are (some kind of) Z_2 odd
& stable due to the half-periodic BC
 - (i) Color singlet case $\Rightarrow$ LKP can be DM candidate
in case of vanishing electric charge
(10: 2TeV, 15: 3TeV)
 - (ii) Colored case $\Rightarrow$ TeV scale LKP decay to
the SM quark by the mixing

Backup Slides

Lagrangian

5D $SU(3) \times U(1)'$ model on S^1/Z_2

$$\begin{aligned} \mathcal{L} = & -\frac{1}{2} \text{Tr} \left(F_{MN} F^{MN} \right) + \bar{\Psi}_3^{i=1,2,3} \left(i\Gamma^M D_M - M_d^i \varepsilon(y) \right) \Psi_3^{i=1,2,3} \\ & + \bar{\Psi}_{\bar{6}}^{i=1,2} \left(i\Gamma^M D_M - M_u^i \varepsilon(y) \right) \Psi_{\bar{6}}^{i=1,2} + \bar{\Psi}_{15} i\Gamma^M D_M \Psi_{15} \\ & + \bar{\Psi}_{10}^{i=1,2,3} \left(i\Gamma^M D_M - M_l^i \varepsilon(y) \right) \Psi_{10}^{i=1,2,3} \quad \Gamma^M = (\gamma^\mu, i\gamma^5) \end{aligned}$$

Boundary conditions:

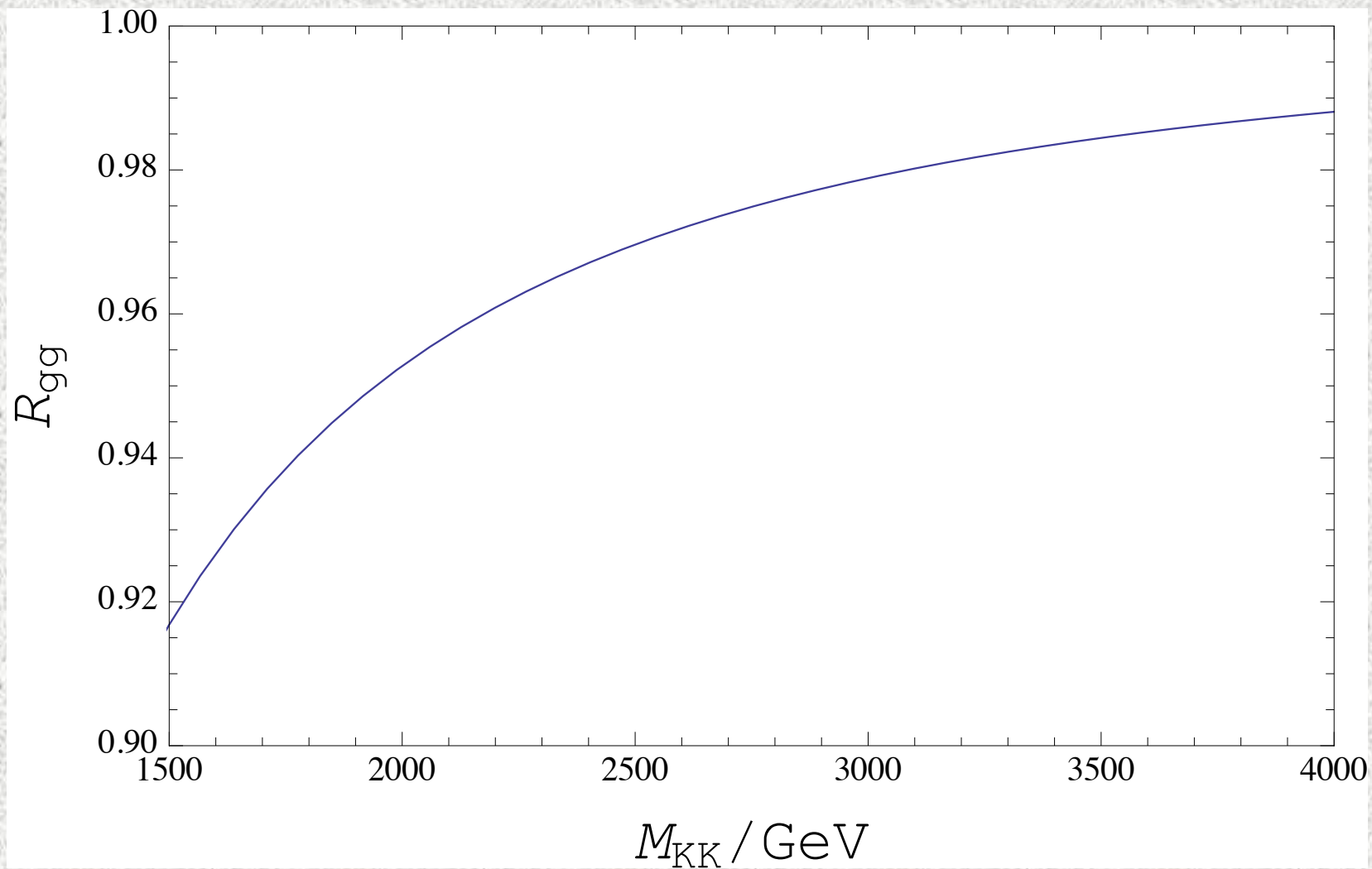
(+,+) only has
massless mode

(+,+): $\cos(ny/R)$
(-,-): $\sin(ny/R)$

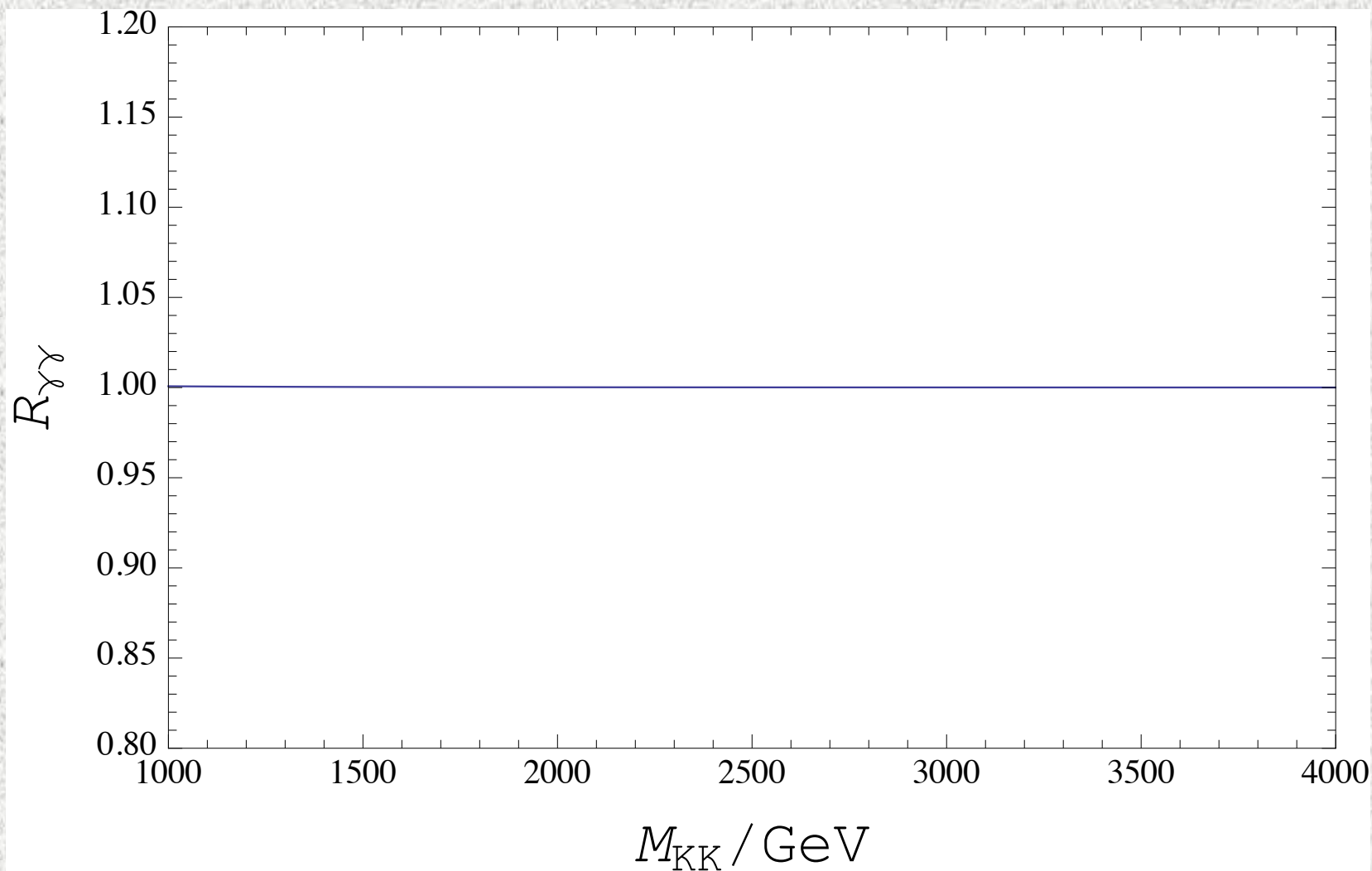
$$A_\mu^{(0)} = \frac{1}{2} \begin{pmatrix} W_\mu^3 + B_\mu/\sqrt{3} & \sqrt{2}W_\mu^+ & 0 \\ \sqrt{2}W_\mu^- & -W_\mu^3 & 0 \\ 0 & 0 & -2B_\mu/\sqrt{3} \end{pmatrix}, A_5^{(0)} = \frac{1}{2} \begin{pmatrix} 0 & 0 & H^+ \\ 0 & 0 & H^0 \\ H^- & H^{0*} & 0 \end{pmatrix}$$

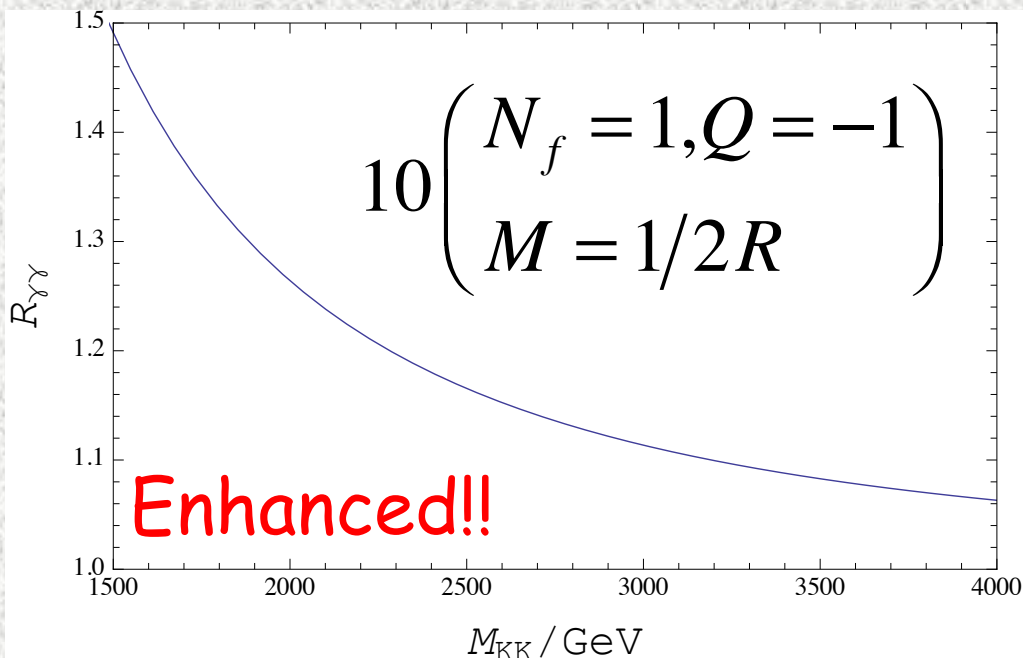
$$M_{W_n} = \frac{n+\alpha}{R}, M_{Z_n} = \frac{n+2\alpha}{R}, M_{\gamma_n} = \frac{n}{R}, \left\langle A_5^{(0)} \right\rangle = \frac{\alpha}{g_5 R}$$

$$\sigma(gg \rightarrow H)_{GHU} / \sigma(gg \rightarrow H)_{SM}$$



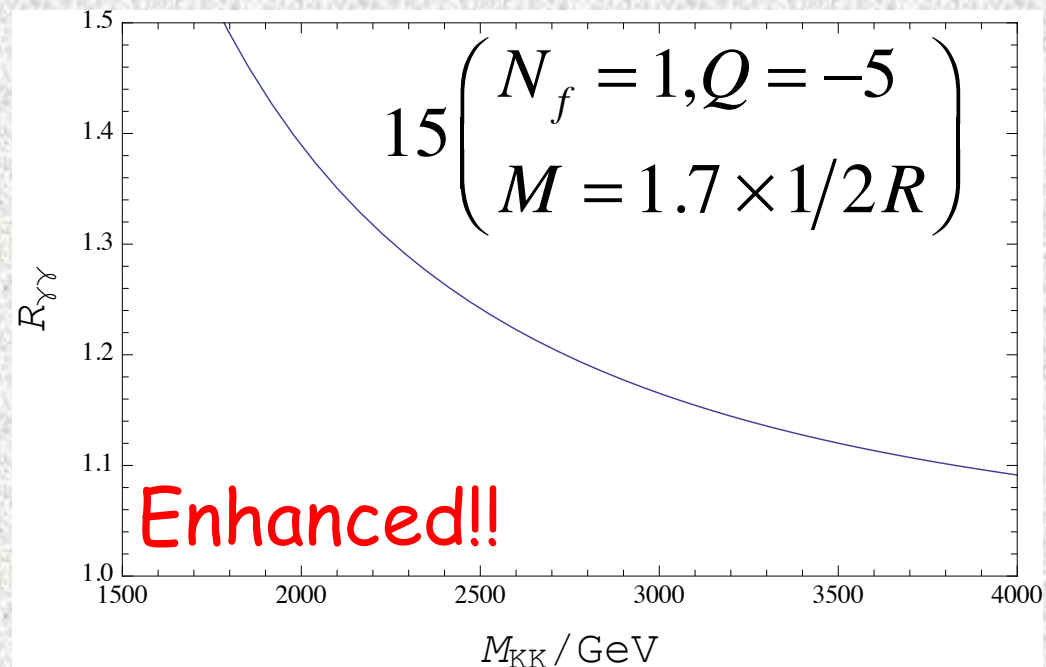
$$\Gamma(H \rightarrow \gamma\gamma)_{GHU} / \Gamma(H \rightarrow \gamma\gamma)_{SM}$$



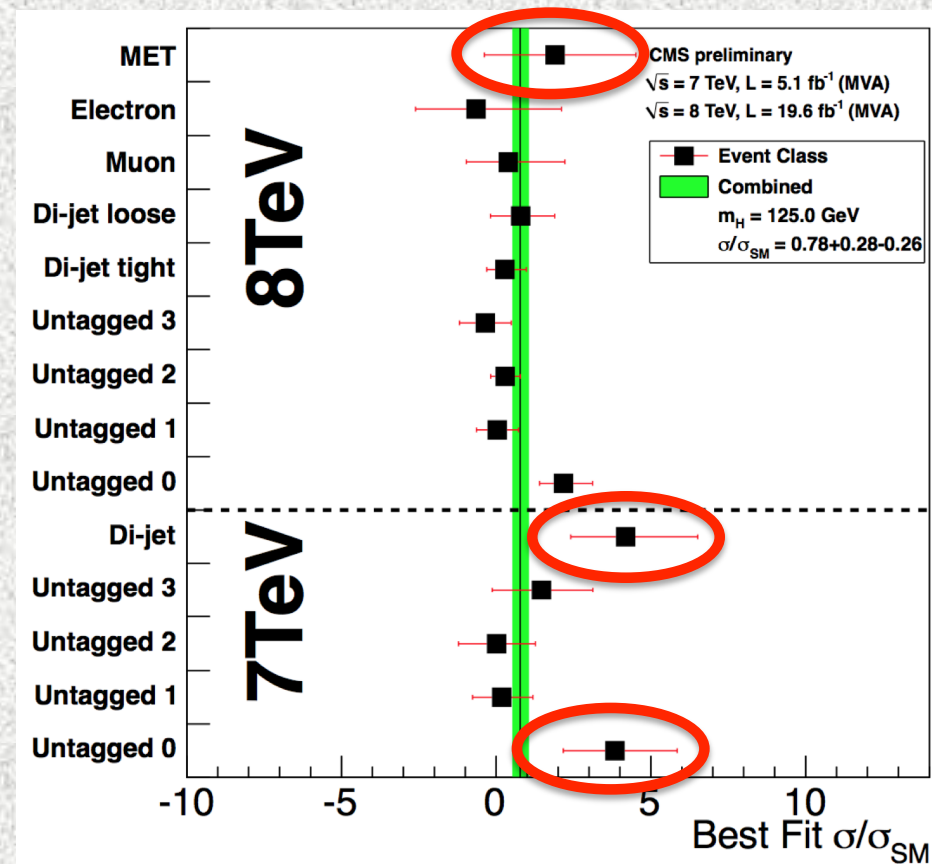
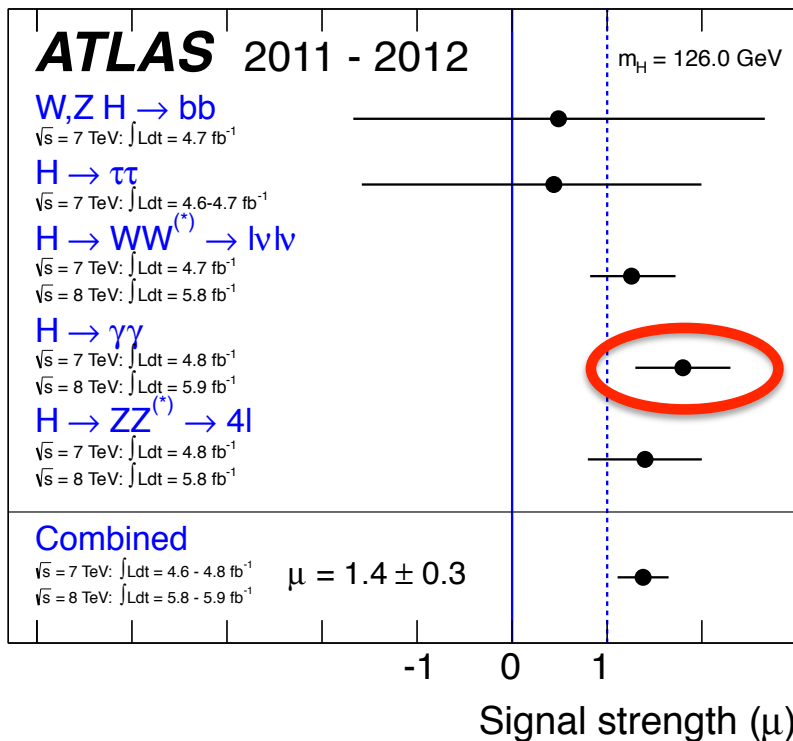


$$R_{\gamma\gamma} = \frac{BR(H \rightarrow \gamma\gamma)_{GHU+10}}{BR(H \rightarrow \gamma\gamma)_{SM}}$$

$$R_{\gamma\gamma} = \frac{BR(H \rightarrow \gamma\gamma)_{GHU+15}}{BR(H \rightarrow \gamma\gamma)_{SM}}$$

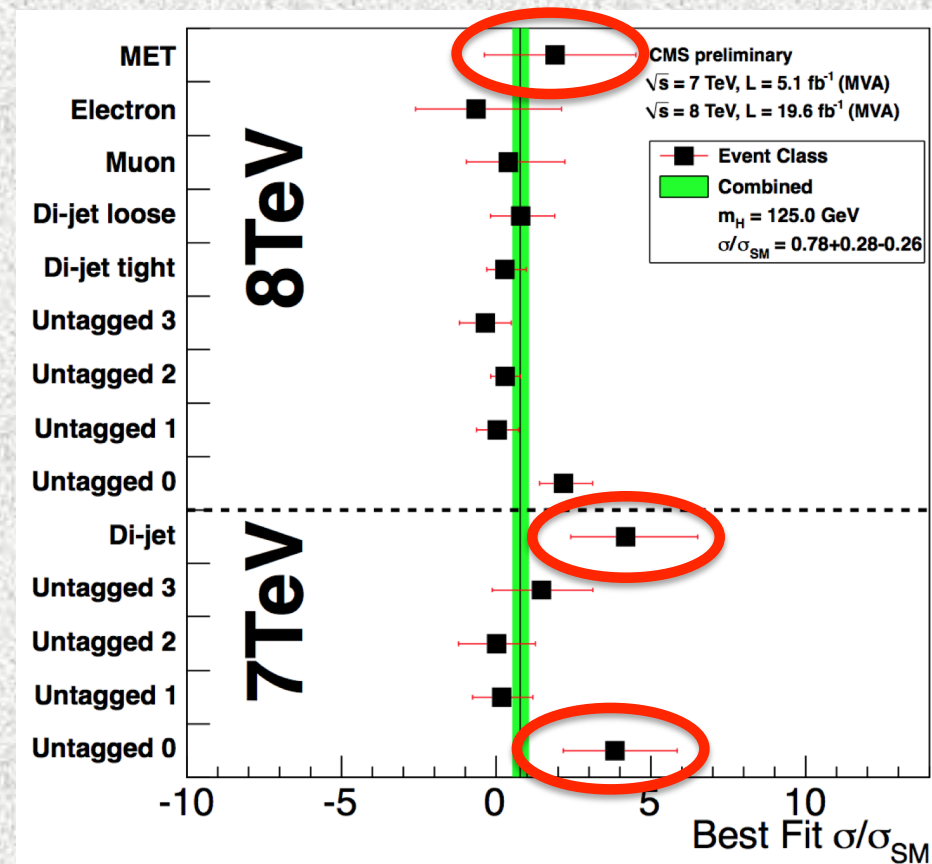
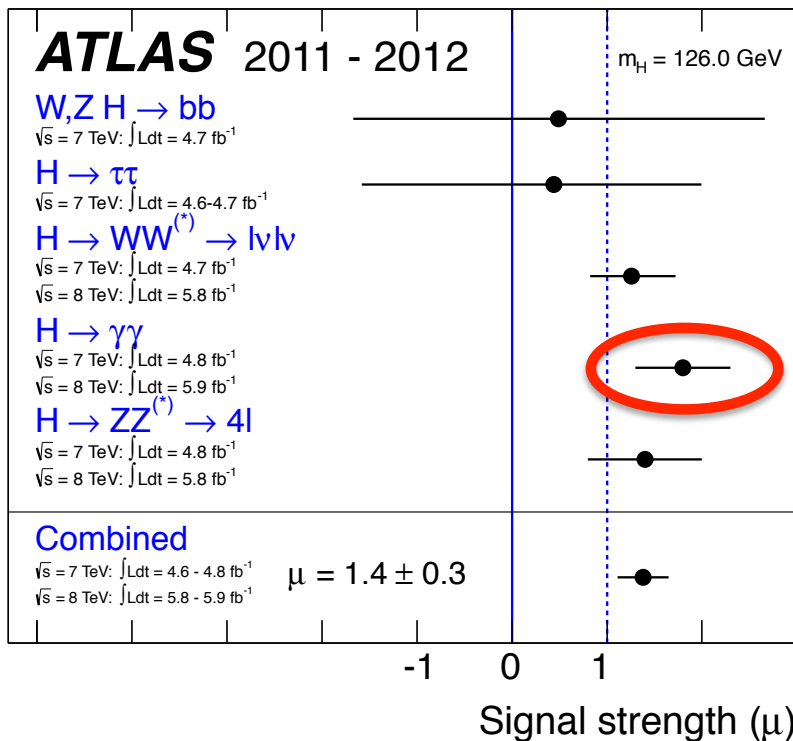


Diphoton decay excess



Hint for New Physics??

Diphoton decay excess



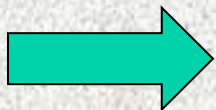
In this talk, we show that the diphoton decay excess can be explained in gauge-Higgs unification

Weinberg angle in GHU

If $SU(3) \rightarrow SU(2)_L \times U(1)_Y$: $U(1)$ normalization fixed

Check the hypercharge of Higgs doublet

$$\begin{aligned}\delta_{U(1)} A_5^{(0)} &= g [T^8, A_5^{(0)}] = \frac{g}{2\sqrt{3}} \left[\begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & -2 \end{pmatrix}, \begin{pmatrix} 0 & 0 & H^+ \\ 0 & 0 & H^0 \\ H^- & H^{0*} & 0 \end{pmatrix} \right] \\ &= \frac{g}{2\sqrt{3}} \begin{pmatrix} 0 & 0 & H^+ \\ 0 & 0 & H^0 \\ -2H^- & -2H^{0*} & 0 \end{pmatrix} - \frac{g}{2\sqrt{3}} \begin{pmatrix} 0 & 0 & -2H^+ \\ 0 & 0 & -2H^0 \\ H^- & H^{0*} & 0 \end{pmatrix} = \frac{g\sqrt{3}}{2} \begin{pmatrix} 0 & 0 & H^+ \\ 0 & 0 & H^0 \\ -H^- & -H^{0*} & 0 \end{pmatrix}\end{aligned}$$



$$\sin^2 \theta_W = \frac{g_Y^2}{g^2 + g_Y^2} = \frac{(\sqrt{3}g)^2}{g^2 + (\sqrt{3}g)^2} = \frac{3}{4} \gg 0.23(\text{Exp})$$

Too Big!!

Way out to get a correct Weinberg angle

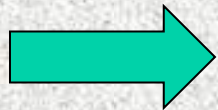
Additional U(1)

$$SU(3) \times U(1)' \rightarrow SU(2)_L \times U(1)_Y \times U(1)_X$$

Scrucca, Serone & Silvestrini (2003)

$$A_Y = \frac{g' A_8 + \sqrt{3} g A'}{\sqrt{3g^2 + g'^2}}, A_X = \frac{\sqrt{3} g A_8 - g' A'}{\sqrt{3g^2 + g'^2}}$$

$$\Rightarrow \delta_{U(1)} A_5^{(0)} = \frac{g'}{\sqrt{3g^2 + g'^2}} g [T^8, A_5^{(0)}] \Rightarrow g_Y = \frac{\sqrt{3} g g'}{\sqrt{3g^2 + g'^2}}$$



$$\sin^2 \theta_W = \frac{g_Y^2}{g^2 + g_Y^2} = \frac{3}{4 + 3g^2/g'^2}$$

Adjustable
by g'

Electroweak symmetry breaking

$$SU(2)_L \times U(1)_Y \rightarrow U(1)_{em}$$

is radiatively triggered by **nonzero** $\langle A_5 \rangle$
(Hosotani mechanism)

Ratio of GHU to SM

$$R_{\sigma} \equiv \left(1 + \frac{C_{gg}^{KKtop}}{C_{gg}^{top}} \right)^2 \cong \left(1 - \frac{1}{3} (\pi m_t R)^2 \right)^2 \cong \left(1 - \frac{4}{3} (\pi m_W R)^2 \right)^2$$

$$R_{\gamma} \equiv \left(1 + \frac{C_{\gamma}^{KKtop} + C_{\gamma}^{KKW}}{C_{\gamma}^{top} + C_{\gamma}^W} \right)^2 \cong \left(1 + \frac{1}{141} (\pi m_W R)^2 \right)^2$$

$$R \equiv R_{\sigma} \times R_{\gamma} \cong \left(1 - \frac{187}{141} (\pi m_W R)^2 \right)^2$$

ATLAS

$m_H = 125.5 \text{ GeV}$

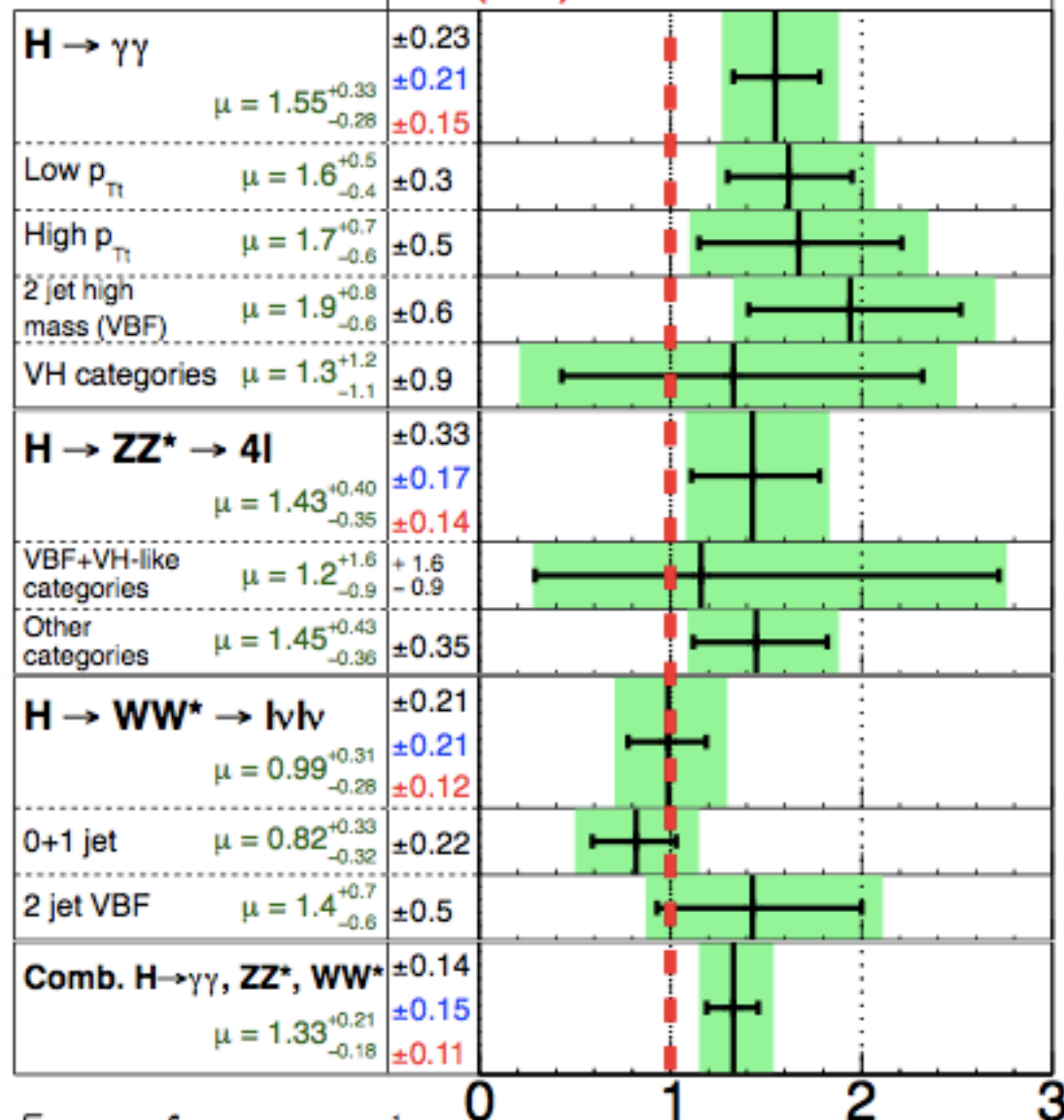
$\sigma(\text{stat})$

$\sigma(\text{sys})$

$\sigma(\text{theo})$

Total uncertainty

$\pm 1\sigma$ on μ

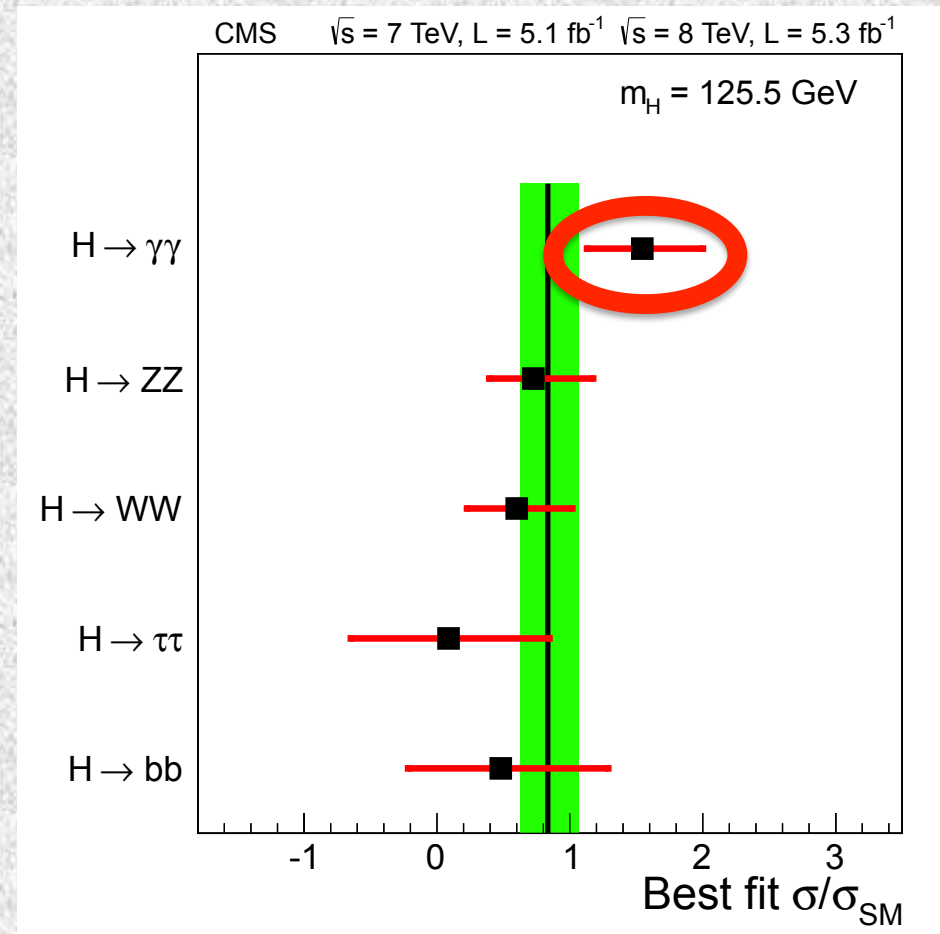
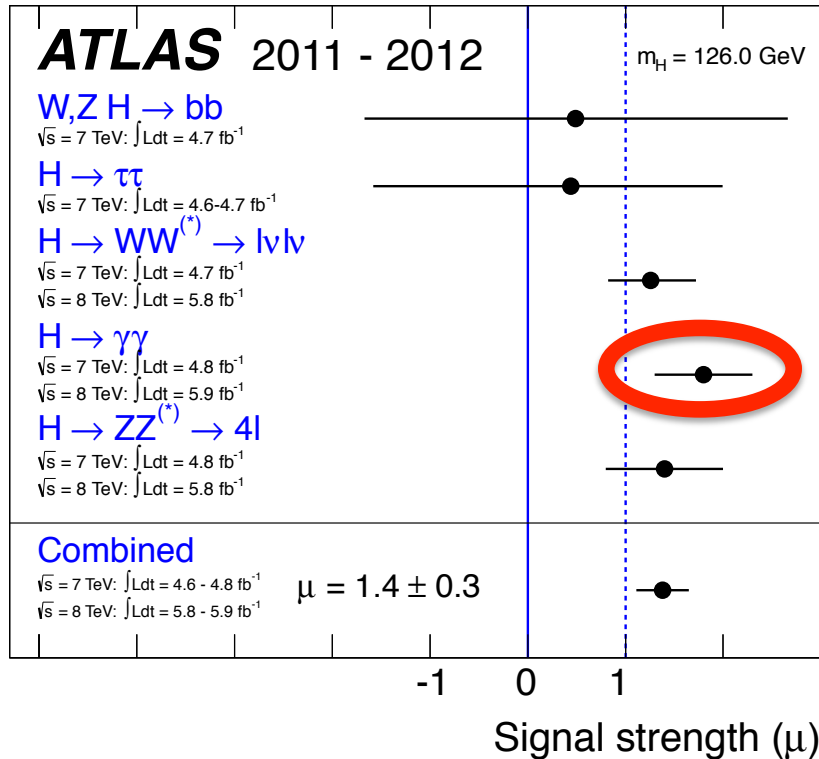


$\sqrt{s} = 7 \text{ TeV } \int L dt = 4.6\text{-}4.8 \text{ fb}^{-1}$

$\sqrt{s} = 8 \text{ TeV } \int L dt = 20.7 \text{ fb}^{-1}$

「標準模型らしさ」

LHC data: $R = R_\sigma \times R_{\gamma\gamma} = 1.5-2.0$

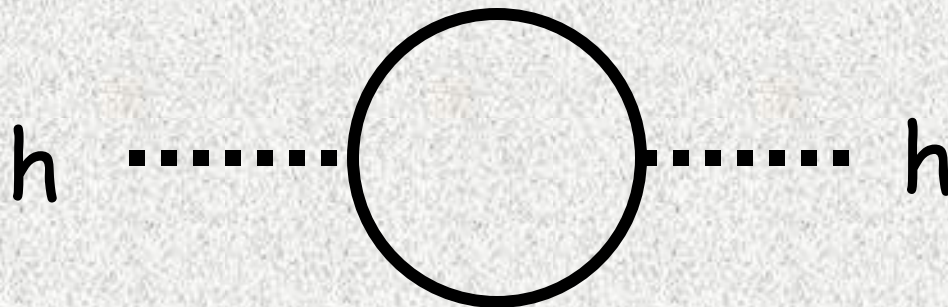


Extension is required

Similar type of deviations from the SM are also seen in
SUSY, Little Higgs

Common feature among GH, SUSY & LH:
Quadratic divergence in m_h^2 canceled
by top partner effects

This can be seen diagrammatically as follows

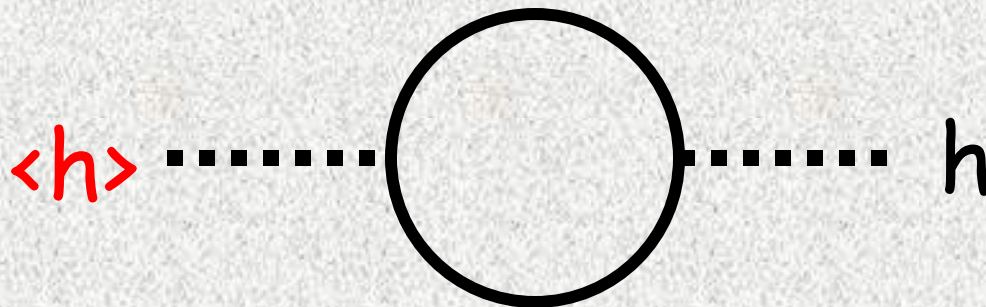


Start with Higgs self-energy diagram

Similar type of deviations from the SM are also seen in
SUSY, Little Higgs

Common feature among GH, SUSY & LH:
Quadratic divergence in m_h^2 canceled
by top partner effects

This can be seen diagrammatically as follows

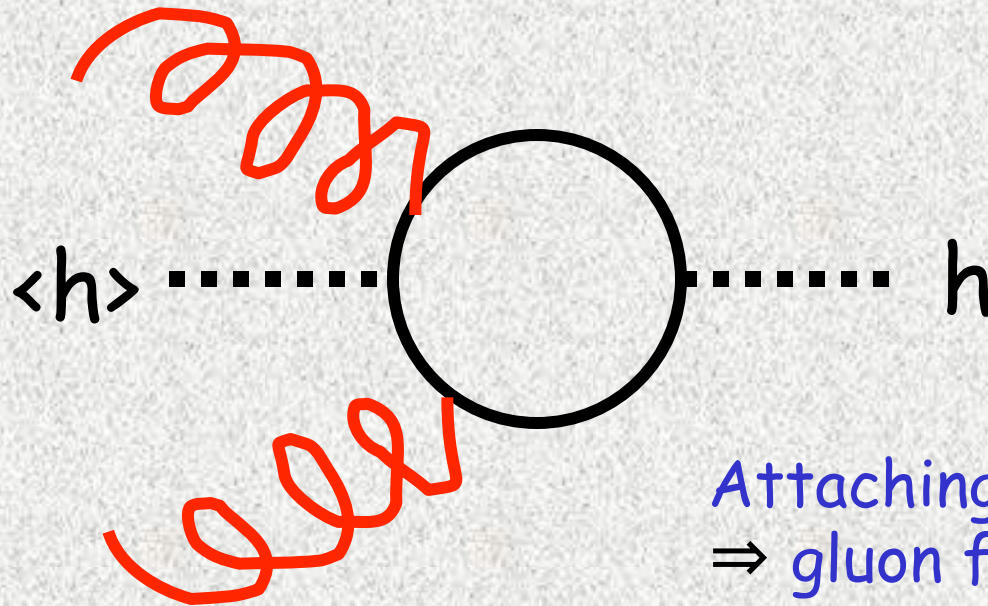


Replace one of the Higgs
with VEV!

Similar type of deviations from the SM are also seen in
SUSY, Little Higgs

Common feature among GH, SUSY & LH:
Quadratic divergence in m_h^2 canceled
by top partner effects

This can be seen diagrammatically as follows



Attaching 2 gluon lines
 $\Rightarrow$ gluon fusion diagram!!