Topological order in the color-flavor locked phase of (3+1)-dimensional $U(N)$ gauge-Higgs system

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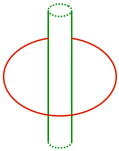
2019. 11. 14

Nagoya U. E-lab seminar

Based on Y. Hidaka, Y Hirono, M. Nitta, Y. Tanizaki, RY,

1903.06389

Overview of this talk



$$= \exp \frac{2\pi i k}{3k+1} \in \mathbb{Z}_{3k+1}$$

(for $N = 3$)

Color-flavor locked phase of a $U(N)$ gauge theory with N-Higgs fields is **topologically ordered** if the Higgs fields have non-trivial $U(1)$ charge k.

- Non-Abelian vortex and Wilson loop have a $\mathbb{Z}_{Nk+1}$ fractional linking phase.
- There are $\mathbb{Z}_{Nk+1}$ **1- and 2-form symmetries**, and both of them are spontaneously broken.

Motivation

Understanding phases of gauge theories

- Gauge theories in many contexts

  SM, BSM, superconductor (SC), cosmology, string theory, ...

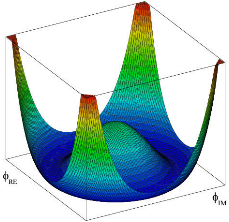
- Classified by behavior of potentials at **IR (long range)**

  Coulomb, confined, **Higgs**,... (e.g. [Intriligator & Seiberg '95])

Q. Can we classify each of the phases more precisely?

Today's talk: We focus on the Higgs phase

Higgs phase



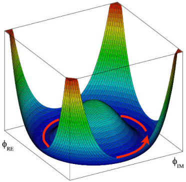
(from wikipedia)

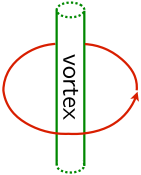
- SM, GUTs, superconductor, ...
- Local (perturbative) properties
  - NG bosons can be eaten by gauge fields
  - Gauge fields become massive

Higgs phases can have **global (topological)** properties

Global (topological) properties [Abrikosov '56; Nielsen & Olesen '73]

Higgs phases can admit **vortex strings with topological charge**



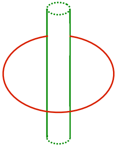


$$\Phi \sim e^{i\theta}$$

- Classical solution in Abelian Higgs model
- **Topological charge**: winding number around vortices
- Applied to magnetic flux in SC, cosmic string, ...

Higgs phases can be further classified by topology

Topologically ordered phase [Wen '89, '91]



$$= \exp \frac{2\pi i}{k} \in \mathbb{Z}_k$$

- Phase classified by **topology** of **non-local order parameters**
- Order parameters: Wilson loop, vortex surface,...

Conditions for the topologically ordered phase:

1. Non-zero VEV of non-local order parameters
2. Fractional linking phases between non-local operators

- New classification e.g. SC (charge 2) $\neq$ charge 1 Abelian Higgs

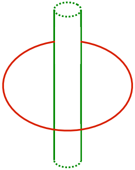
Topological order in Abelian Higgs model

Review based on

Hansson, et al. '04; Banks & Seiberg '10;

Seiberg & Kapustin '14; Gaiotto, et al. '14

Message



$$= \exp\frac{2\pi i}{k} \in \mathbb{Z}_k$$

Abelian Higgs models can be classified by topological order.

- Order parameters are non-local: Wilson loop & Vortex surface.
- Fractional linking phase of extended objects
- E.g. (s-wave) superconductor (charge 2) $\neq$ charge 1 Higgs model
- Topological order $\leftrightarrow$ $\mathbb{Z}_k$ 1- & 2-form symmetry breaking

Low energy limit of Abelian Higgs model

Topology often becomes dominant in low energy limit.

Stückelberg action

$$\frac{v^2}{2}\int |d\chi - kA|^2 \quad \text{or} \quad \frac{v^2}{2}\int d^4x|\partial_m\chi - kA_m|^2$$

- χ: NG boson of Higgs $\Phi = ve^{i\chi}$ (v: vev)
- $A\sim A_m$: $U(1)$ 1-form gauge field
- k: charge of Higgs field e.g. $k = 2$ for superconductor

How to see topological properties? $\rightarrow$ dual action

Original Lagrangian: $\frac{1}{4e^2}F^{mn}F_{mn} + |\partial_m\Phi - ikA_m\Phi|^2 + \frac{\lambda}{2}(|\Phi|^2 - v^2)^2$

Dual BF-action [Horowitz '89; Blau & Tompson '89]

Stückelberg action $\overset{\text{dual}}{\longleftrightarrow}$ BF-action

$$\frac{v^2}{2}\int |d\chi - kA|^2 \overset{\text{dual}}{\longleftrightarrow} \frac{k}{2\pi}\int B\wedge dA \text{ or } \frac{ik}{2\pi}\int d^4x\,\epsilon^{mnpq}B_{mn}\partial_p A_q$$

$B\sim B_{mn}$: 2-form gauge field

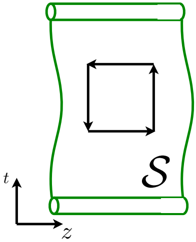
Derivation: $\chi \overset{\text{dual}}{\longleftrightarrow}$ 2-form B

1. Rewrite Stückelberg action by adding 3-form $H\sim H_{mnp}$
   $\frac{1}{8\pi^2 v^2}\int |H|^2 + \frac{1}{2\pi}\int H\wedge(d\chi - kA)$
2. Eliminating χ by EOM: $dH = 0 \to H = dB$
3. Take low-energy limit $(v\to\infty)$

Why does 2-form gauge field B arise?

2-form gauge field B can be coupled to vortices electrically

[Sugamoto '78; Lee '93]



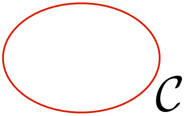
$$\int_{\mathcal{S}} B$$

- Charge: topological charge of vortices
- Gauge symmetry of 2-form $\leftrightarrow$ conservation of topological charge
- 2nd order tensor $\leftrightarrow$ spacetime dimensions of vortices
- Anti-symmetric tensor $\leftrightarrow$ orientation (Jacobian) of vortices

Observables in BF-theory [Horowitz '89]

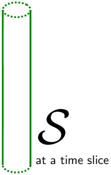
Wilson loop

$$W(\mathcal{C}) = e^{i \int_{\mathcal{C}} A}$$



Surface of vortex

$$V(\mathcal{S}) = e^{i \int_{\mathcal{S}} B}$$

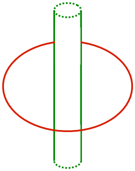


- Observables are non-local
- Observables depend on only topology
- No local DOF: $dA = 0, dB = 0$ (by EOM)

Topological order? $\to$ calculate $\langle W(\mathcal{C}) V(\mathcal{S}) \rangle$

Notes: the observables are invariant under gauge transf. $A \to A + d\lambda^{(0)}$, $B \to B + d\lambda^{(1)}$

Abelian Higgs model is topologically ordered [Hansson et al. '04]



$$= \exp \frac{2\pi i}{k} \in \mathbb{Z}_k$$

1. **Fractional** linking phase: $\langle W(\mathcal{C}) V(\mathcal{S}) \rangle = \exp\left(\frac{2\pi i}{k} \operatorname{link}(\mathcal{C}, \mathcal{S})\right)$ [Derivation]

   $\operatorname{link}(\mathcal{C}, \mathcal{S})$: linking number **(Topological invariant!)**

2. Non-zero VEVs of order parameters $W(\mathcal{C})$ & $V(\mathcal{S})$:

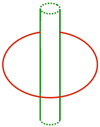
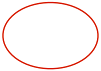
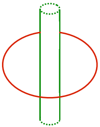
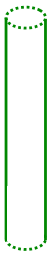
   $\langle W(\mathcal{C}) \rangle = \langle V(\mathcal{S}) \rangle = 1$

Classification: e.g. superconductor ($\mathbb{Z}_2$) $\neq$ charge 1 Higgs system ($\mathbb{Z}_1 = 1$)

Ordered phase $\leftrightarrow$ symmetry breaking?

1- & 2-form symmetries [Kapustin & Seiberg '14; Gaiotto, et al. '14]

Symmetries under transf. of 1- & 2-dim. objects

 $= e^{\frac{2\pi i}{k}}$     $= e^{\frac{2\pi i}{k}}$ 

	Group	Object	Generator	Transf.
1-form	$\mathbb{Z}_k$	$W(\mathcal{C})$	$V(\mathcal{S})$	$\langle V(\mathcal{S})W(\mathcal{C})\rangle = e^{\frac{2\pi i}{k}} \langle W(\mathcal{C})\rangle$
2-form	$\mathbb{Z}_k$	$V(\mathcal{S})$	$W(\mathcal{C})$	$\langle W(\mathcal{C})V(\mathcal{S})\rangle = e^{\frac{2\pi i}{k}} \langle V(\mathcal{S})\rangle$

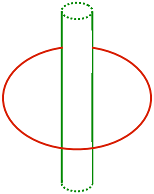
Topologically ordered phase $\leftrightarrow$ spontaneously broken $\mathbb{Z}_k$ 1- & 2-form syms.

- Symmetry breaking: $\langle W(\mathcal{C})\rangle \neq 0$, $\langle V(\mathcal{S})\rangle \neq 0$ (at large distance limit)

Notes

- ordinary symmetry is 0-form symmetry for particles
- BF-action is invariant under 1- & 2-form transf. (up to $2\pi\mathbb{Z}$).

Summary of Abelian Higgs model



$$= \exp \frac{2\pi i}{k} \in \mathbb{Z}_k$$

Abelian Higgs models can be classified by topological order.

- Order parameters are Wilson loop & Vortex surface.
- Fractional linking phase of extended objects
- Symmetry breaking: 1- & 2-form symmetries

Topological order in CFL phase of $U(N)$ gauge-Higgs system

Y. Hidaka, Y Hirono, M. Nitta, Y. Tanizaki, RY,

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Topological order in non-Abelian Higgs models?

- Many symmetry breaking patterns
  - Some of gauge fields are massive, others are not.
  - Some NG boson may not be eaten.

    (cf. no topological order in CFL of QCD [Hirono & Tanizaki '18])
- Hints from Abelian case:
  - No local excitations in low-energy limit
  - Existence of extended objects

The hints suggest:

there may be a topologically ordered phase

in a $U(N)$ gauge theory with N-flavor Higgs fields

$U(N)$ gauge theory with N-flavor Higgs

[Hanany & Tong '03; Auzzi et al. '03; Gorsky et al. 04]

Action (after Wick rotation)

$$\int \frac{1}{2g_1^2}\,\mathrm{tr}\,|F|^2 + \frac{1}{2g_2^2}|\,\mathrm{tr}\,F|^2 + |d\Phi - iA\Phi - ik\,\mathrm{tr}\,(A)\Phi|^2 + V(\Phi)$$

- Higgs fields: N(color) $\times$ N(flavor) matrix $\Phi = (\Phi_{cf})$
- Transf. law $\Phi \to (\det U_{\mathrm{col}})^k\, U_{\mathrm{col}}\, \Phi\, U_{\mathrm{flav}}^T$

  $U_{\mathrm{col}} \in U(N)_{\mathrm{col}}$, $U_{\mathrm{flav}} \in SU(N)_{\mathrm{flav}}/(\mathbb{Z}_N)_{\mathrm{flav}}$
- A: $U(N)$ 1-form gauge field

How about Higgs phase?

Higgs phase with non-Abelian vortex

[Hanany & Tong '03; Auzzi et al. '03; Gorsky et al. 04]

1. There is a Higgs phase with no massless excitation

   (# of gauge fields = # of Higgs fields)

   - VEV of Higgs can be diagonized $\langle\Phi\rangle = v\mathbf{1}_{N\times N}$
   - Color-flavor locked (CFL) phase:

     simultaneous color-flavor transf. remains

     (cf. QCD case [Alford et al. '98])

2. CFL phase Admits non-Abelian vortices $\Phi \sim \mathrm{diag}\,(e^{i\theta}, 1, ..., 1)$

   - Fractional $(1/N)$ magnetic flux

We will show that this CFL phase can be a topologically ordered phase.

$$\mathrm{diag}\,(e^{i\theta}, 1, ..., 1) = e^{i\theta/N}\,\mathrm{diag}\,(e^{i(N-1)\theta/N}, e^{-i\theta/N}, ..., e^{-i\theta/N}) \sim e^{i\theta/N}\mathbf{1}_{N\times N}$$

How to see topological order?

Conditions for the topological ordered phase

1. Existence of fractional linking phase,
2. Spontaneously broken 1- and 2-form symmetries

Procedure: similar to the case of Abelian Higgs model

1. Dual theory: $\int |d\Phi - iA\Phi - ik\,\mathrm{tr}\,(A)\Phi|^2 \quad \xleftrightarrow{\text{dual}} \quad \frac{i}{2\pi} K_{iA} \int b_i \wedge da_A$
2. Fractional linking phase
3. Non-zero VEVs of order parameters

In this talk, we consider $N = 3$ case for simplicity.

Low energy limit in CFL phase

Action is simplified in Abelian gauge: (cf. ['t Hooft '81])

Stückelberg action

$$\frac{v^2}{2}\int |d\phi_i - K_{iA}a_A|^2, \quad K_{iA} = \begin{pmatrix} k+1 & k & k \\ k & k+1 & k \\ k & k & k+1 \end{pmatrix}$$

- Abelian gauge: $\Phi = \frac{1}{\sqrt{2}} v \, \mathrm{diag}\left(e^{i\phi_1}, e^{i\phi_2}, e^{i\phi_3}\right) \in U(3)$
- a_A: Cartan of $U(3)$ gauge field
- K_{iA}: matrix of charges $(\det(K_{iA}) \neq 0)$

$$|d\phi_i - K_{iA}a_A|^2 = |d\phi_1 - (k+1)a_1 - ka_2 - ka_3|^2 + |d\phi_2 - ka_1 - (k+1)a_2 - ka_3|^2 + |d\phi_3 - ka_1 - ka_2 - (k+1)a_3|^2$$

Dual BF-type theory

The action

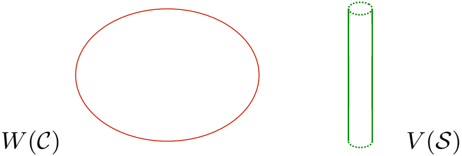
$$\frac{i}{2\pi} K_{iA} \int b_i \wedge da_A$$

NG boson $\phi_i \xleftrightarrow{\text{dual}}$ 2-form b_i

Deriavtion

1. Original action $\frac{v^2}{2} \int |d\phi_i - K_{iA} a_A|^2$
2. First order action by adding 3-form H_{3i}
   $\frac{1}{8\pi^2 v^2} \int |H_{3i}|^2 + \frac{i}{2\pi} \int H_{3i} \wedge (d\phi_i - K_{iA} a_A)$
3. Eliminating ϕ_i by EOM: $dH_{3i} = 0 \to H_{3i} = db_i$
4. Take low-energy limit ($v \to \infty$)

Observables in $\frac{i}{2\pi} K_{iA} \int b_i \wedge da_A$

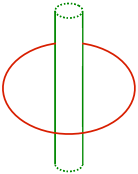


- Wilson loop: $W(\mathcal{C}) = \frac{1}{N} \operatorname{tr} \mathcal{P} e^{i \int_{\mathcal{C}} A} = \frac{1}{N} \sum_{A=1}^{N} e^{i \int_{\mathcal{C}} a_A}$
- Vortex surface operator: $V_i(\mathcal{S}) = e^{i \int_{\mathcal{S}} b_i}$

Observables depend on only topology

(No local DOF: $da_A = 0$, $db_i = 0$ by EOM)

CFL phase is topologically ordered!



$$= \exp \frac{2\pi i k}{3k+1} \in \mathbb{Z}_{3k+1}$$

1. $\mathbb{Z}_{3k+1}$ fractional phase in correlation function

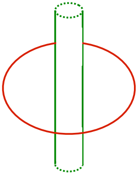
$$\langle W(\mathcal{C}) V_i(\mathcal{S}) \rangle = \exp\left(\frac{2\pi i k}{3k+1} \operatorname{link}(\mathcal{C}, \mathcal{S})\right)$$

2. Non-zero VEVs $\langle W(\mathcal{C}) \rangle = \langle V_i(\mathcal{S}) \rangle = 1$

Corresponding symmetries: $\mathbb{Z}_{3k+1}$ 1- & 2-form symmetries

- $\langle W(\mathcal{C}) V_i(\mathcal{S}) \rangle = \exp\left(\frac{2\pi i k}{3k+1} \operatorname{link}(\mathcal{C}, \mathcal{S})\right) \langle W(\mathcal{C}) \rangle$
- $\langle W(\mathcal{C}) V_i(\mathcal{S}) \rangle = \exp\left(\frac{2\pi i k}{3k+1} \operatorname{link}(\mathcal{C}, \mathcal{S})\right) \langle V(\mathcal{S}) \rangle$

Some comments



$$= \exp \frac{2\pi i k}{3k+1} \in \mathbb{Z}_{3k+1}$$

- $U(1)$ charge k in $\Phi \to (\det U_{\mathrm{col}})^k \, U_{\mathrm{col}} \, \Phi \, U_{\mathrm{flav}}^T$ is important for the existence of topological order.
- Non-Abelian vortices in the previous research [Gorsky, et al. 04]: $k = 0$ case ($\mathbb{Z}_1$). No nontrivial topological order.
- Possible fractional phase is restricted by color N:
  e.g. $\mathbb{Z}_4, \mathbb{Z}_7, \mathbb{Z}_{10}, \ldots$ are allowed for $N = 3$ case.

Summary

CFL phase of a $U(N)$ gauge theory with N-Higgs fields is topologically ordered if the Higgs fields have non-trivial $U(1)$ charge k.

- Non-Abelian vortex and Wilson loop has $\mathbb{Z}_{Nk+1}$ fractional linking phase.
- There are $\mathbb{Z}_{Nk+1}$ 1- and 2-form symmetries, and both of them are spontaneously broken.
- Adding θ-term: self linking number of $\mathcal{S}$ also contributes to the correlation function.

Future work: more general gauge group, numerical solutions of charge k non-Abelian vortex, ...

Appendix

Derivation of correlation function in BF-theory - I

[cf. Chen, Tiwari, Ryu '15 [1509.04266]]

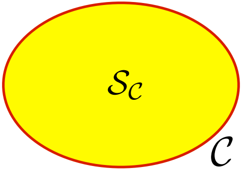
Let us evaluate the following correlation function

$$\langle W(\mathcal{C})V(\mathcal{S})\rangle = \mathcal{N}\int \mathcal{D}A\mathcal{D}B e^{\frac{ik}{2\pi}\int B\wedge dA + i\int_{\mathcal{C}} A + i\int_{\mathcal{S}} B}$$

By the Stokes' theorem, we can rewrite the integration $\int_{\mathcal{C}} A$ as

$$\int_{\mathcal{C}} A = \int_{\partial \mathcal{S}_{\mathcal{C}}} A = \int_{\mathcal{S}_{\mathcal{C}}} dA.$$

$\mathcal{S}_{\mathcal{C}}$ is a 2D surface whose boundary is $\mathcal{C}$



Derivation of correlation function in BF-theory - II

[cf. Chen, Tiwari, Ryu '15 [1509.04266]]

Surface integral $\rightarrow$ spacetime integral

$$\int_{\mathcal{S}_\mathcal{C}} dA = \int dA \wedge \delta_2(\mathcal{S}_\mathcal{C}) = \frac{1}{2!} \int F_{mn} J^{mn}(\mathcal{S}_\mathcal{C}).$$

$\delta_2(\mathcal{S}_\mathcal{C})$: delta function 2-form for $\mathcal{S}_\mathcal{C}$:

$$\delta_2(\mathcal{S}_\mathcal{C}) = \frac{\epsilon_{mnpq}}{2!2!} \left(\int_{\mathcal{S}_\mathcal{C}} dy^m \wedge dy^n \delta^4(x-y) \right) dx^p \wedge dx^q.$$

$$J^{mn}(\mathcal{S}_\mathcal{C}) = \int_{\mathcal{S}_\mathcal{C}} dy^m \wedge dy^n \delta^4(x-y)$$

Derivation

$$\begin{aligned}
\int_{\mathcal{S}_\mathcal{C}} dA &= \frac{1}{2!} \int_{\mathcal{S}_\mathcal{C}} dy^m \wedge dy^n F_{mn}(y) \\
&= \frac{1}{2!} \int d^4x \int_{\mathcal{S}_\mathcal{C}} dy^m \wedge dy^n F_{mn}(x) \delta^4(x-y) \\
&= \frac{1}{2!} \int d^4x F_{mn}(x) \int_{\mathcal{S}_\mathcal{C}} dy^m \wedge dy^n \delta^4(x-y)
\end{aligned}$$

Derivation of correlation function in BF-theory - III

[cf. Chen, Tiwari, Ryu '15 [1509.04266]]

The correlation function can be rewritten as

$$\langle V(\mathcal{S})W(\mathcal{C})\rangle = \mathcal{N}\int \mathcal{D}A\mathcal{D}B\, e^{\frac{ik}{2\pi}\int (B+\frac{2\pi}{k}\delta_2(\mathcal{S}_{\mathcal{C}}))\wedge dA + i\int_{\mathcal{S}} B}.$$

By the redefinition $B + \frac{2\pi}{k}\delta_2(\mathcal{S}_{\mathcal{C}}) \to B$, we have

Integrating $W(\mathcal{C})$

$$\begin{aligned}\langle V(\mathcal{S})W(\mathcal{C})\rangle &= e^{-\frac{2\pi i}{k}\int_{\mathcal{S}}\delta_2(\mathcal{S}_{\mathcal{C}})}\mathcal{N}\int \mathcal{D}A\mathcal{D}B\, e^{\frac{ik}{2\pi}\int B\wedge dA + i\int_{\mathcal{S}} B} \\ &= e^{-\frac{2\pi i}{k}\int_{\mathcal{S}}\delta_2(\mathcal{S}_{\mathcal{C}})}\langle V(\mathcal{S})\rangle\,.\end{aligned}$$

One can integrate out $\int_{\mathcal{S}} B$ similarly.

Derivation of correlation function in BF-theory - IV

[cf. Chen, Tiwari, Ryu '15 [1509.04266]]

$\int_{\mathcal{S}} B$ can be rewritten as

$$\int_{\mathcal{S}} B = \int_{\partial \mathcal{V}_{\mathcal{S}}} B = \int_{\mathcal{V}_{\mathcal{S}}} dB_2 = \int dB_2 \wedge \delta_1(\mathcal{V}_{\mathcal{S}}) = -\int B_2 \wedge d\delta_1(\mathcal{V}_{\mathcal{S}}),$$

where $\partial \mathcal{V}_{\mathcal{S}} = \mathcal{S}$.

The correlation function can be written as

$$\langle V(\mathcal{S}) W(\mathcal{C}) \rangle = e^{-\frac{2\pi i}{k} \int_{\mathcal{S}} \delta_2(\mathcal{S}_{\mathcal{C}})} \mathcal{N} \int \mathcal{D}A\mathcal{D}B \, e^{\frac{ik}{2\pi} \int B \wedge d(A - \frac{2\pi}{k} \delta_1(\mathcal{V}_{\mathcal{S}}))}$$

By the redefinition $A \to A + \frac{2\pi}{k} \delta_1(\mathcal{V}_{\mathcal{S}})$, we have

Correlation function

$$\langle V(\mathcal{S}) W(\mathcal{C}) \rangle = e^{-\frac{2\pi i}{k} \int_{\mathcal{S}} \delta_2(\mathcal{S}_{\mathcal{C}})}$$

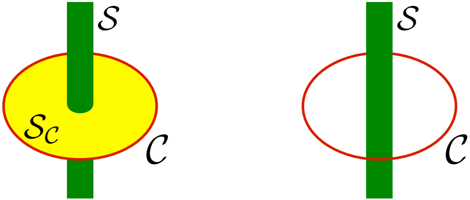
What is $\int_{\mathcal{S}} \delta_2(\mathcal{S}_{\mathcal{C}})$?

Derivation of correlation function in BF-theory - V

[cf. Chen, Tiwari, Ryu '15 [1509.04266]]

Linking number

$$\int_{\mathcal{S}} \delta_2(\mathcal{S}_{\mathcal{C}}) = \mathrm{link}\,(\mathcal{C}, \mathcal{S})$$



- $\int_{\mathcal{S}} \delta_2(\mathcal{S}_{\mathcal{C}})$: intersection number between $\mathcal{S}_{\mathcal{C}}$ and $\mathcal{S}$.

  This is equal to the **linking number** between $\mathcal{C}$ and $\mathcal{S}$.

Derivation of correlation function in BF-theory - VI

[cf. Chen, Tiwari, Ryu '15 [1509.04266]]

Therefore, we obtain

Correlation function

$$\langle V(\mathcal{S})W(\mathcal{C})\rangle = e^{-\frac{2\pi i}{k}\,\mathrm{link}\,(\mathcal{C},\mathcal{S})}.$$

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Bibliography

Bibliography - I

[1] K. A. Intriligator and N. Seiberg, “Lectures on supersymmetric gauge theories and electric-magnetic duality,” Nucl. Phys. Proc. Suppl. **45BC** (1996) 1–28, [arXiv:hep-th/9509066 [hep-th]]. [Subnucl. Ser. **34** (1997) 237].
(page 4).

[2] A. A. Abrikosov, “On the Magnetic properties of superconductors of the second group,” Sov. Phys. JETP **5** (1957) 1174–1182. [Zh. Eksp. Teor. Fiz. **32** (1957) 1442].
(page 6).

[3] H. B. Nielsen and P. Olesen, “Vortex Line Models for Dual Strings,” Nucl. Phys. **B61** (1973) 45–61.
(page 6).

[4] X. G. Wen, “Vacuum Degeneracy of Chiral Spin States in Compactified Space,” Phys. Rev. **B40** (1989) 7387–7390.
(page 7).

[5] X.-G. Wen, “Topological orders and Chern-Simons theory in strongly correlated quantum liquid,” Int. J. Mod. Phys. **B5** (1991) 1641–1648.
(page 7).

[6] T. H. Hansson, V. Oganesyan, and S. L. Sondhi, “Superconductors are topologically ordered,” Annals Phys. **313** (2004) no. 2, 497–538, [arXiv:cond-mat/0404327 [cond-mat.supr-con]].
(pages 8, 14).

Bibliography - II

[7] T. Banks and N. Seiberg, “Symmetries and Strings in Field Theory and Gravity,” Phys. Rev. **D83** (2011) 084019, [arXiv:1011.5120 [hep-th]].
(page 8).

[8] A. Kapustin and N. Seiberg, “Coupling a QFT to a TQFT and Duality,” JHEP **04** (2014) 001, [arXiv:1401.0740 [hep-th]].
(pages 8, 15).

[9] D. Gaiotto, A. Kapustin, N. Seiberg, and B. Willett, “Generalized Global Symmetries,” JHEP **02** (2015) 172, [arXiv:1412.5148 [hep-th]].
(pages 8, 15).

[10] G. T. Horowitz, “Exactly Soluble Diffeomorphism Invariant Theories,” Commun. Math. Phys. **125** (1989) 417.
(pages 11, 13).

[11] M. Blau and G. Thompson, “A New Class of Topological Field Theories and the Ray-singer Torsion,” Phys. Lett. **B228** (1989) 64–68.
(page 11).

[12] A. Sugamoto, “Dual Transformation in Abelian Gauge Theories,” Phys. Rev. **D19** (1979) 1820.
(page 12).

Bibliography - III

[13] K.-M. Lee, "The Dual formulation of cosmic strings and vortices," Phys. Rev. **D48** (1993) 2493–2498, [arXiv:hep-th/9301102 [hep-th]].
(page 12).

[14] Y. Hirono and Y. Tanizaki, "Quark-Hadron Continuity beyond the Ginzburg-Landau Paradigm," Phys. Rev. Lett. **122** (2019) no. 21, 212001, [arXiv:1811.10608 [hep-th]].
(page 18).

[15] A. Hanany and D. Tong, "Vortices, instantons and branes," JHEP **07** (2003) 037, [arXiv:hep-th/0306150 [hep-th]].
(pages 19, 20).

[16] R. Auzzi, S. Bolognesi, J. Evslin, K. Konishi, and A. Yung, "NonAbelian superconductors: Vortices and confinement in N=2 SQCD," Nucl. Phys. **B673** (2003) 187–216, [arXiv:hep-th/0307287 [hep-th]].
(pages 19, 20).

[17] A. Gorsky, M. Shifman, and A. Yung, "Non-Abelian meissner effect in Yang-Mills theories at weak coupling," Phys. Rev. **D71** (2005) 045010, [arXiv:hep-th/0412082 [hep-th]].
(pages 19, 20, 26).

[18] M. G. Alford, K. Rajagopal, and F. Wilczek, "Color flavor locking and chiral symmetry breaking in high density QCD," Nucl. Phys. **B537** (1999) 443–458, [arXiv:hep-ph/9804403 [hep-ph]].
(page 20).

Bibliography - IV

[19] G. 't Hooft, "Topology of the Gauge Condition and New Confinement Phases in Nonabelian Gauge Theories," Nucl. Phys. **B190** (1981) 455–478.
(page 22).

[20] X. Chen, A. Tiwari, and S. Ryu, "Bulk-boundary correspondence in (3+1)-dimensional topological phases," Phys. Rev. **B94** (2016) no. 4, 045113, [arXiv:1509.04266 [cond-mat.str-el]]. [Addendum: Phys. Rev. **B94** (2016) no. 7, 079903].
(pages 29, 30, 31, 32, 33, 34).