Seminar. Dec 18th 2019 @ Nagoya Univ.

Clockwork origin of neutrino mixings

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Based on T.K., PRD100 (2019)

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Introduction

small number (large hierarchy) problems

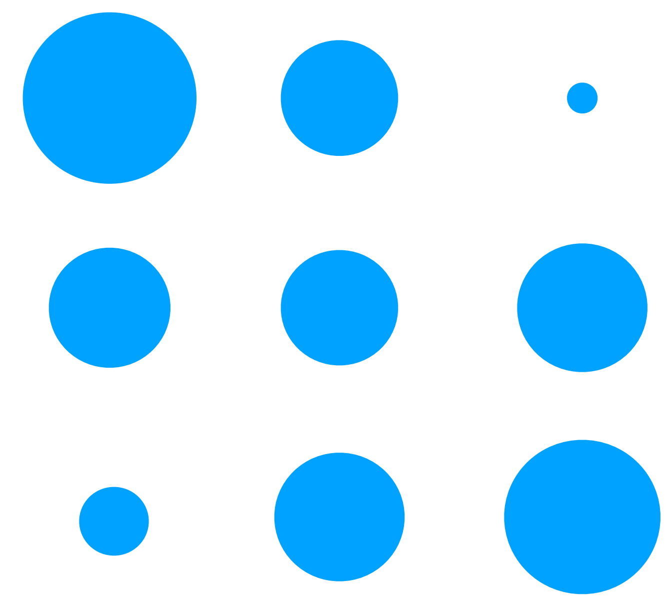
tiny neutrino mass

0.5MeV

< 0.1eV

https://higgstan.com

neutrino mixings



etc.

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new heavy particles?

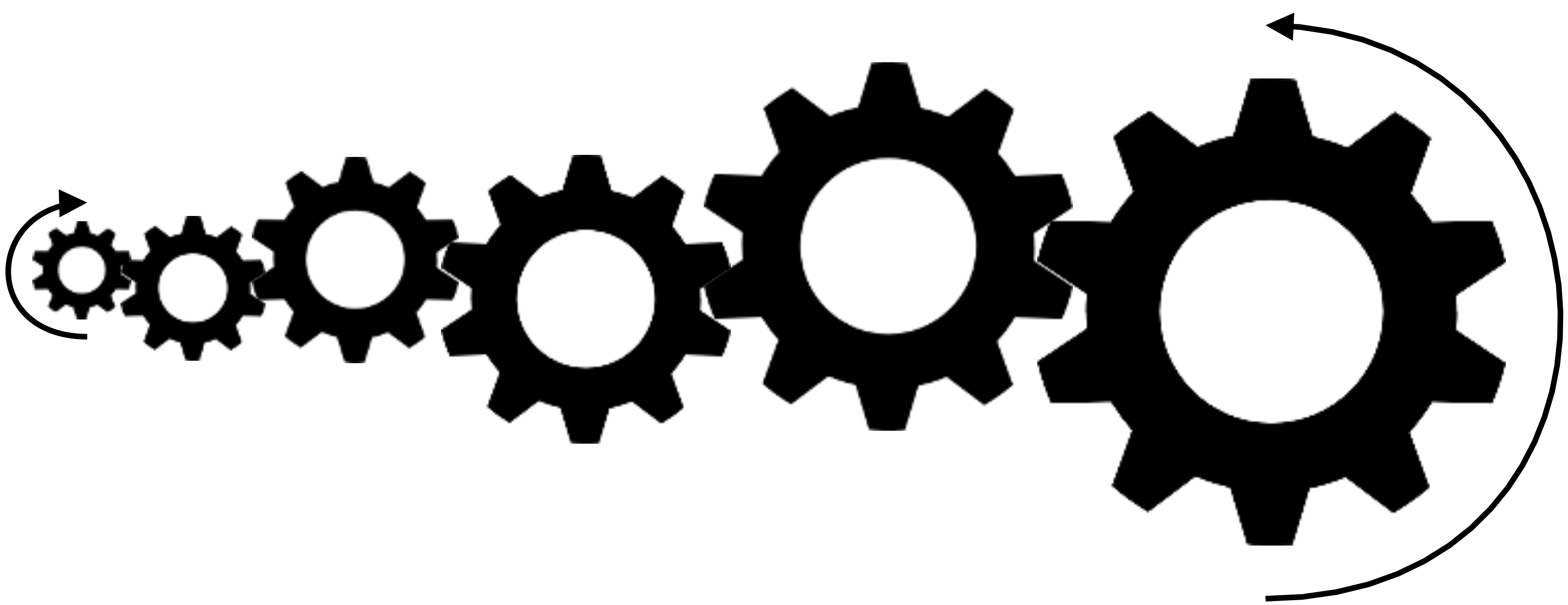
symmetries?

fine tuning?

Introduction

Clockwork mechanism

A natural way to obtain small number (or large hierarchies) with only O(1) couplings & N fields



Giudice and McCullough, JHEP (2017)

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- Introduction
- Review of Clockwork mechanism
  - basic concept
  - scalar CW, fermion CW
- Neutrinos & CW mechanism
  - review of masses and mixings
  - origin of tiny neutrino masses
  - neutrino mixings
    — CW-SM origin
    — CW origin
- Summary

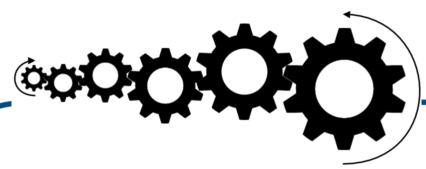
Basic concepts of CW mechanism

Teresi, arXiv:1705.09698

an origin of a small number

$$0.99 \times 0.99 \times \cdots \times 0.99 = \text{small number}$$

(1) N fields with O(1) couplings



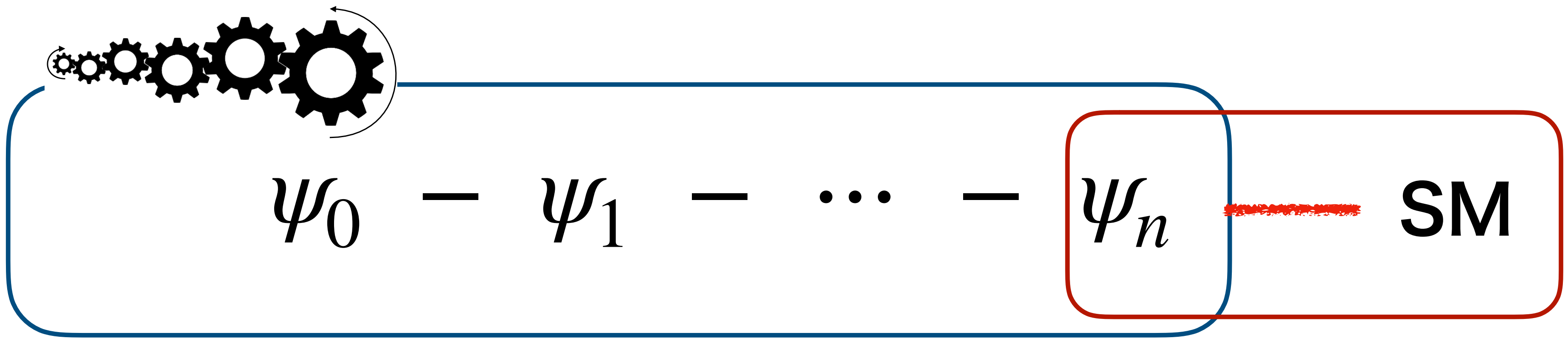
$$\psi_0 - \psi_1 - \cdots - \psi_n - \text{SM}$$

$$\frac{1}{q} \times \frac{1}{q} \times \cdots \times \frac{1}{q} \xrightarrow[\substack{q>1 \\ q=\mathcal{O}(1)}]{} \left(\frac{1}{q}\right)^n \ll 1$$

(2)SM is coupled to the last field

→ effective couplings will be small

Basic concept of CW mechanism



$$\mathscr{L} = \mathscr{L}^{\mathrm{CW}}_{\mathrm{kin}} + \mathscr{L}^{\mathrm{CW}}_{\mathrm{mass}} + \boxed{\mathscr{L}^{\mathrm{CW}}_{\mathrm{near}}} + \boxed{\mathscr{L}_{\mathrm{CW-SM}}} + \mathscr{L}_{\mathrm{SM}}$$

CW sector

- scalar CW fields
- fermion CW fields
- vector CW fields
- graviton CW fields

applications

- axion
- inflation
- dark matter
- neutrino masses & mixings

etc.

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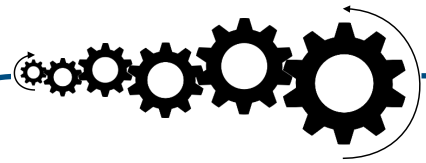
Scalar Clockwork



$$\phi_0 - \phi_1 - \phi_2 - \cdots - \phi_n$$

$$m^2 q \quad m^2 q \quad m^2 q \quad m^2 q$$

(1) N scalar fields $\phi_0, \phi_1, \phi_2, \cdots, \phi_n$

(2) $\phi_j - \phi_j$ coupling : m^2

(3) $\phi_j - \phi_{j+1}$ coupling : $m^2 q$

q : "gear ratio"

$$\mathscr{L}_{\text{near}} = \frac{1}{2} m^2 \sum_{j=0}^{n-1} (\phi_j - q\phi_{j+1})^2 + \mathcal{O}(\phi^4)$$

$$= \frac{1}{2} \sum_{i,j=0}^{n} \phi_i M^2_{ij} \phi_j + \mathcal{O}(\phi^4)$$

$$M^2 = m^2 \begin{pmatrix} 1 & -q & 0 & \cdots & 0 \\ -q & 1+q^2 & -q & \cdots & 0 \\ 0 & -q & 1+q^2 & \cdots & 0 \\ \vdots & \vdots & \vdots & & 0 \\ 0 & 0 & 0 & \cdots & q^2 \end{pmatrix}$$

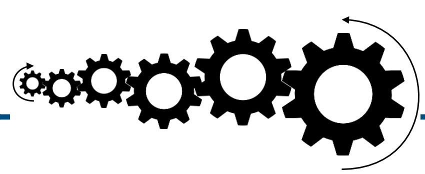
Scalar Clockwork

$$\begin{pmatrix} \phi_0 \\ \phi_1 \\ \vdots \\ \phi_n \end{pmatrix} = \begin{pmatrix} O_{00} & O_{01} & \cdots & O_{0n} \\ O_{10} & O_{11} & \cdots & O_{1n} \\ \vdots & \vdots & \ddots & \vdots \\ O_{n0} & O_{n1} & \cdots & O_{nn} \end{pmatrix} \begin{pmatrix} a_0 \\ a_1 \\ \vdots \\ a_n \end{pmatrix}$$

← massless

← massive

$$O^T M^2 O = \mathrm{diag}.\left(m_{a0}^2, m_{a1}^2, \cdots, m_{an}^2\right)$$



$$\phi_0 - \phi_1 - \cdots - \phi_n \text{ --- SM}$$

$$\phi_n = O_{n0}a_0 + O_{n1}a_1 + \cdots + O_{nn}a_n$$

Mass spectrum for large n

$m_{an} \sim m(q+1)$

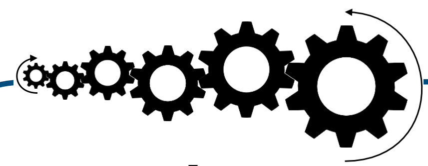
$\vdots$

$m_{a1} \sim m(q-1)$

massless state is hierarchically isolated

$m_{a0} = 0$

Scalar Clockwork



$$\phi_0 - \phi_1 - \phi_2 - \cdots - \phi_n \;-\; \text{SM}$$

$$m^2 q \quad m^2 q \quad m^2 q \quad m^2 q$$

$$\phi_n = O_{n0} a_0 + O_{n1} a_1 + \cdots + O_{nn} a_n \qquad \lambda_k = q^2 + 1 - 2q \cos \frac{k\pi}{n+1}$$

$$= \frac{1}{q^n} \sqrt{\frac{q^2 - 1}{q^2 - q^{-2n}}}\, a_0 + \sum_{k=1}^{n} \sqrt{\frac{2}{(n+1)\lambda_k}}\, q \sin \frac{nk\pi}{n+1} \cdot a_k$$

$$\mathscr{L}_{\text{CW}-\text{SM}} = f\, \phi_n\, SM$$

$$\to \quad \mathscr{L}^{\text{eff}}_{\text{CW}-\text{SM}} = f_0\, a_0\, SM$$

effective a0-SM coupling

$$f_0 = \frac{f}{q^n} \sqrt{\frac{q^2 - 1}{q^2 - q^{-2n}}} \sim \frac{f}{q^n} \ll 1$$

The effective a0-SM is highly suppressed by $1/q^n$. $q > 1$

Contents

Fermion Clockwork

chiral fields

$$\psi_{R0} \underbrace{-}_{m} \boxed{\psi_{L0} \underbrace{-}_{qm} \psi_{R1}} \underbrace{-}_{m} \boxed{\psi_{L1} \underbrace{-}_{qm} \psi_{R2}} - \cdots - \boxed{\psi_{Ln-1} \underbrace{-}_{qm} \psi_{Rn}}$$

(1) N+1 RH chiral fermions $\psi_{R0}, \psi_{R1}, \psi_{R2}, \cdots, \psi_{RN}$

(2) N LH chiral fermions $\psi_{L0}, \psi_{L1}, \psi_{L2}, \cdots, \psi_{LN-1}$

(3) $\psi_{Li} - \psi_{Ri}$ coupling : m

(4) $\psi_{Li} - \psi_{Ri+1}$ coupling : mq

$$\mathscr{L}_{\text{near}} = -\sum_{i=0}^{n-1} m(\bar{\psi}_{Li}\psi_{Ri} - q\bar{\psi}_{Li}\psi_{Ri+1}) = \bar{\psi}_L M \psi_R$$

we omit h.c.

Fermion Clockwork

$$\psi_{R0} - \psi_{L0} - \psi_{R1} - \cdots - \psi_{LN-1} - \psi_{Rn} - \text{SM}$$

mass eigenstates

$$\psi_R = U_R \chi_R$$

$$\psi_L = U_L \chi_L$$

$$\begin{pmatrix} \psi_{R0} \\ \psi_{R1} \\ \vdots \\ \psi_{Rn} \end{pmatrix} = \begin{pmatrix} O_{00} & O_{01} & \cdots & O_{0n} \\ O_{10} & O_{11} & \cdots & O_{1n} \\ \vdots & \vdots & \ddots & \vdots \\ O_{n0} & O_{n1} & \cdots & O_{nn} \end{pmatrix} \begin{pmatrix} \chi_{R0} \\ \chi_{R1} \\ \vdots \\ \chi_{Rn} \end{pmatrix}$$

$$\mathcal{L}^{\text{eff}}_{\text{CW}-\text{SM}} = \frac{f}{q^n}\sqrt{\frac{q^2-1}{q^2-q^{-2n}}}\ \chi_{R0}$$

$$= f_0\ \chi_{R0}\ SM$$

effective χ_{R0}-SM coupling

$$f_0 = \frac{f}{q^n}\sqrt{\frac{q^2-1}{q^2-q^{-2n}}} \sim \frac{f}{q^n} \ll 1$$

Contents

neutrino masses and mixings

problems in neutrino physics

(1) tiny masses (2) mass ordering (3) Dirac or Majorana

(4) mixing pattern and so on.

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(1)tiny masses

https://higgstan.com

0.5MeV

KATRIN, PRD (2019) $m \leq 1.1 \text{ eV}$

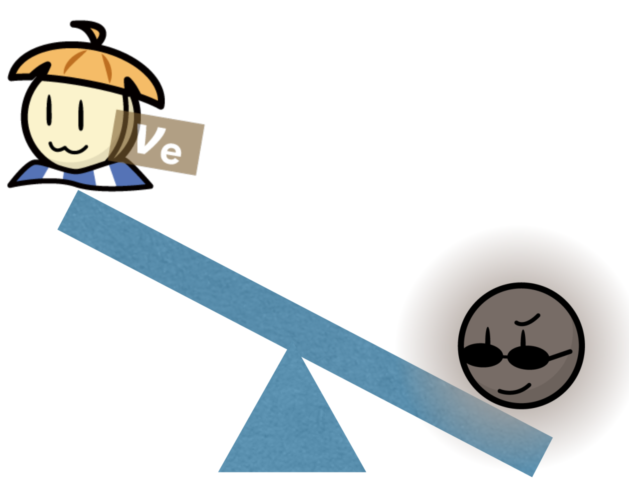
Planck (2018) $\sum m < 0.12 \text{ eV}$

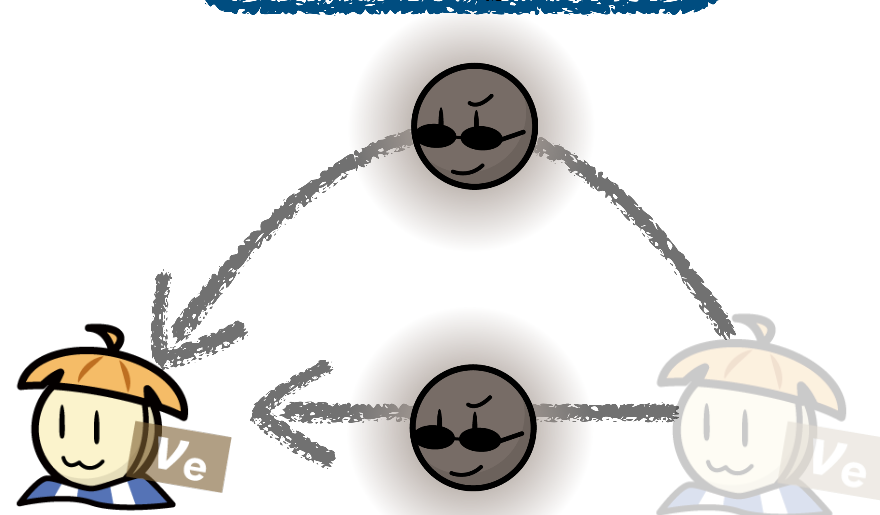
< 0.1eV

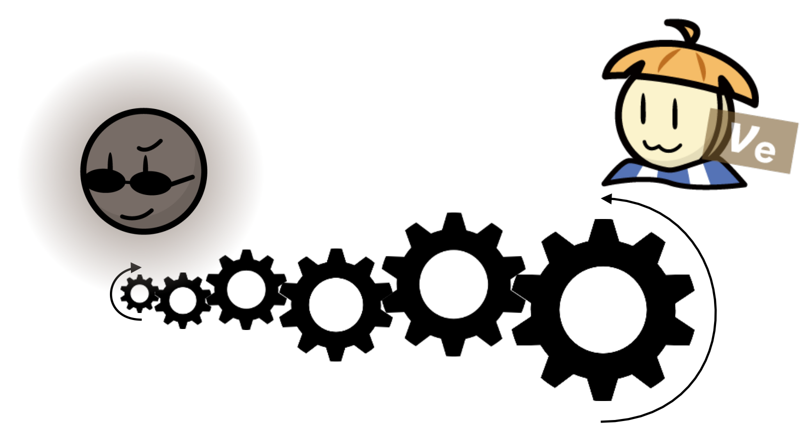
seesaw

scotogenic

clockwork





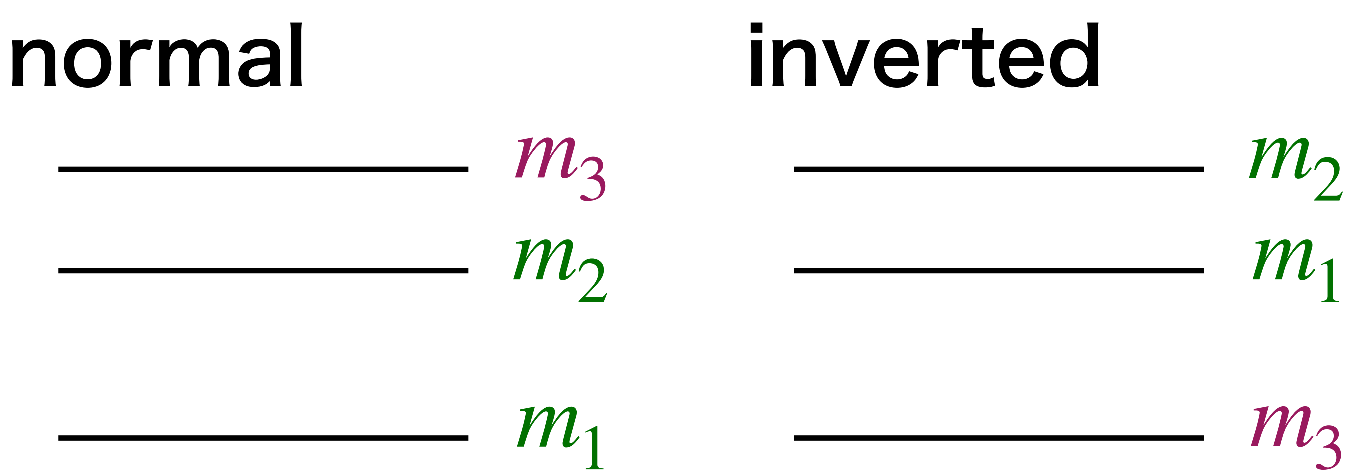


heavy neutrino(s)

loop ～ small

neutrino masses and mixings

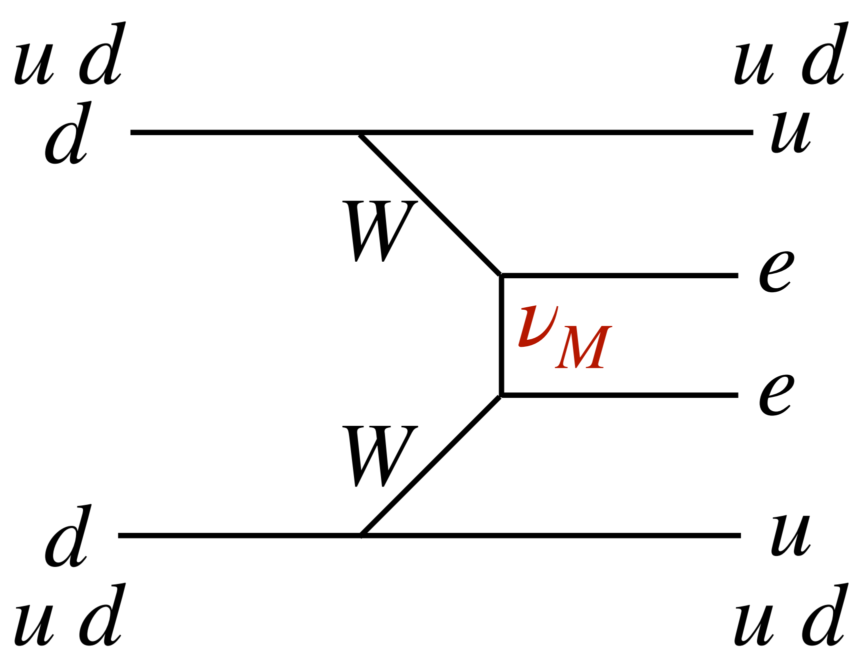
(2) mass ordering



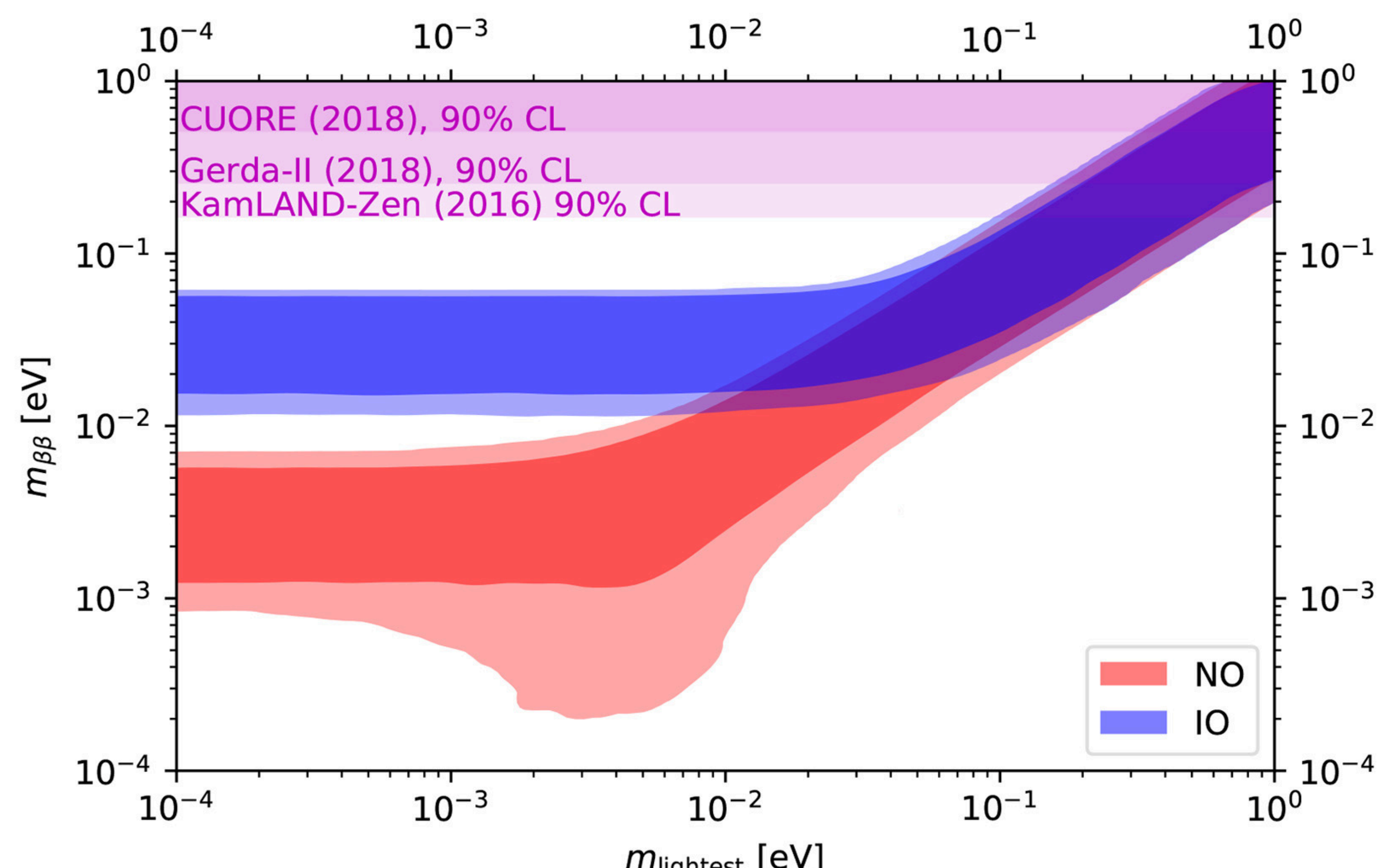
A global analysis shows a **preference for normal mass ordering.** Salas et al., PLB (2018)

(3) Dirac or Majorana

neutral ν → Majorana?



neutrinoless double beta decay



Salas et al.,Front.Astron.Space Sci (2018)

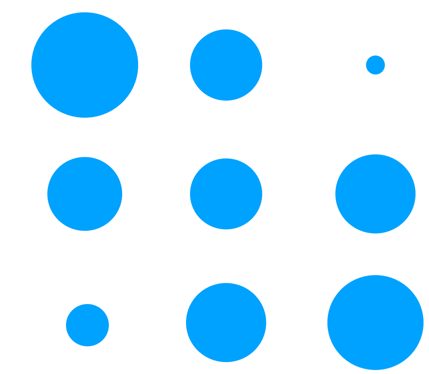
neutrino masses and mixings

(4) mixing pattern

flavor mass matrix m

$$m = \begin{pmatrix} m_{11} & m_{12} & m_{13} \\ m_{11} & m_{12} & m_{13} \\ m_{11} & m_{12} & m_{13} \end{pmatrix} = U^T \begin{pmatrix} m_1 & 0 & 0 \\ 0 & m_2 & 0 \\ 0 & 0 & m_3 \end{pmatrix} U$$

mixing matrix U



mass matrix textures

one zero

$$\begin{pmatrix} 0 & \times & \times \\ \times & \times & \times \\ \times & \times & \times \end{pmatrix}$$

Xing, PRD69 (2004)

μ-τ symmetry + perturbations

$$\begin{pmatrix} \times & B & B \\ B & C & D \\ B & D & C \end{pmatrix} + \begin{pmatrix} \epsilon_{11} & \epsilon_{12} & \epsilon_{13} \\ \epsilon_{21} & \epsilon_{22} & \epsilon_{23} \\ \epsilon_{31} & \epsilon_{32} & \epsilon_{33} \end{pmatrix}$$

Liao, et al, PRD87 (2013)

Magic + perturbations

$$\begin{pmatrix} A & B & C \\ B & D & A+C-D \\ C & A+C-D & B-C+D \end{pmatrix} + \epsilon$$

Gautam & Kumar, PRD94 (2016)

with symmetries

$Z_3, A_4, S_3, S_4, \cdots$

clockwork

and so on.

neutrino masses and mixings

In this seminar

(1) tiny masses → Clockwork

(2) mass ordering → normal ordering

(3) Dirac or Majorana → Dirac (easy for CW)

(4) mixing pattern → Clockwork

From neutrino oscillation experiments

$$m = \begin{pmatrix} 0.824 m_1 & 0.547 m_2 & 0.147 m_3 \\ -0.495 m_1 & 0.569 m_2 & 0.657 m_3 \\ 0.275 m_1 & -0.614 m_2 & 0.740 m_3 \end{pmatrix}$$

$$m_2 = \sqrt{7.50 \times 10^{-5} + m_1^2}\ \mathrm{eV}$$

$$m_3 = \sqrt{2.524 \times 10^{-3} + m_1^2}\ \mathrm{eV}$$

Our mission!
Obtain this mass matrix by CW mechanism!

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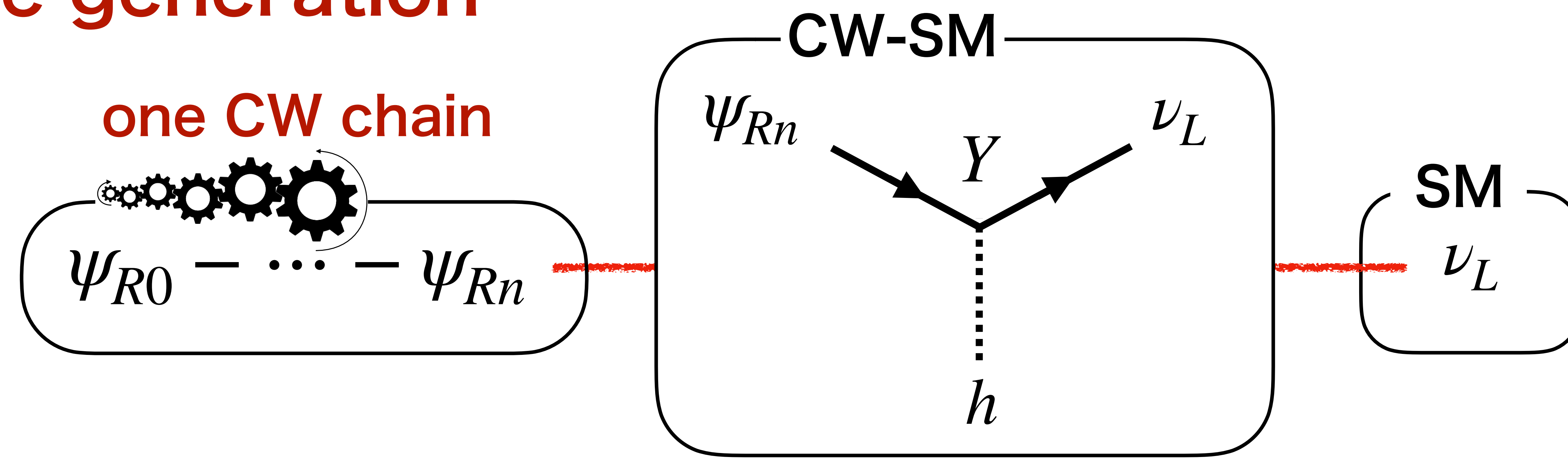
Tiny neutrino mass

one generation



$\mathscr{L}_{\mathrm{CW-SM}} = -YH\bar{L}_L\psi_{Rn}$ **Dirac neutrino**

$$\mathscr{L}^{\mathrm{eff}}_{\mathrm{CW-SM}} = -\frac{Y}{q^n}\sqrt{\frac{q^2-1}{q^2-q^{-2n}}}H\bar{L}_L\chi_{R0}$$

right-handed neutrino is identified with χ_{R0}

---

ex) $n = 25$, $q = 3$, $Y = 1$, $v = 246\mathrm{GeV}$

$$m_\nu \sim vy_0 \sim \frac{vy}{q^n} \sim 0.1\mathrm{eV}$$

Tiny neutrino masses

three generations $\boxed{\text{w/o mixings}}$

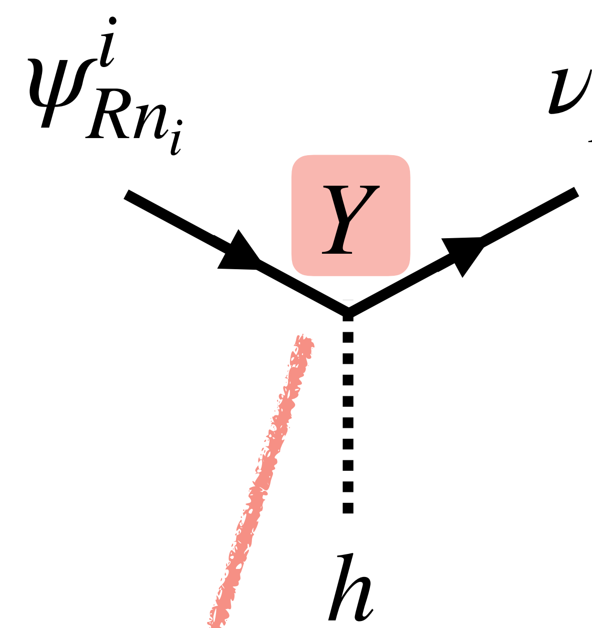
three CW chains

$$\psi^1_{R0} - \underset{q_1}{\cdots} - \psi^1_{Rn_1}$$

$$\psi^2_{R0} - \underset{q_2}{\cdots} - \psi^2_{Rn_2}$$

$$\psi^3_{R0} - \underset{q_3}{\cdots} - \psi^3_{Rn_3}$$

$$\mathscr{L}_{\mathrm{CW-SM}} = -YH\bar{L}^i_L\psi^i_{Rn_i}$$



$\mathscr{L}_{\mathrm{SM}}$

$$\nu^1_L$$

$$\nu^2_L$$

$$\nu^3_L$$

common Yukawa

$$m^i_\nu = v\,\boxed{\frac{1}{q_i^{n_i}}\sqrt{\frac{q_i^2-1}{q_i^2-q_i^{-2n_i}}}} \cdot Y \sim v\frac{1}{q_i^{n_i}} \cdot Y \ll 1$$

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neutrino mixings

Two ways

$\mathscr{L}_{\mathrm{CW-SM}}$ origin

Ibarra et al., PLB780 (2018)

$$m_{\alpha\beta} = v \boxed{\frac{1}{q_\beta^{n_\beta}} \sqrt{\frac{q_\beta^2 - 1}{q_\beta^2 - q_\beta^{-2n_\beta}}}} \cdot \boxed{Y^{\alpha\beta}}$$

pure CW
small masses

CW-SM
mixing pattern

pure CW origin

T.K., PRD100 (2019)
based on Ibarra's model

$$m_{\alpha\beta} = v \cdot \boxed{\frac{1}{q_{\alpha\beta}^{n_{\alpha\beta}}} \sqrt{\frac{q_{\alpha\beta}^2 - 1}{q_{\alpha\beta}^2 - q_{\alpha\beta}^{-2n_{\alpha\beta}}}}}$$

pure CW
masses & mixings

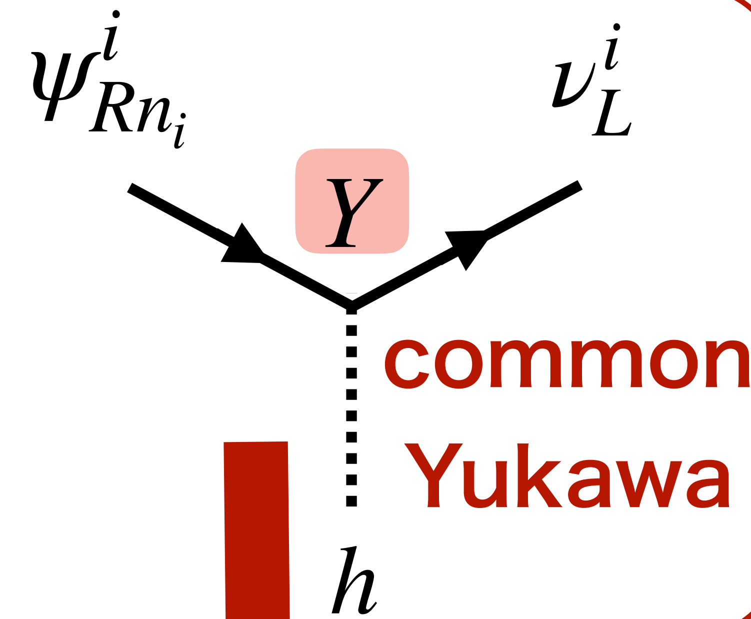
Contents

mixings:CW-SM origin

w/o mixings

$$\psi^1_{R0} - \cdots - \psi^1_{Rn_1}$$
$$\psi^2_{R0} - \cdots - \psi^2_{Rn_2}$$
$$\psi^3_{R0} - \cdots - \psi^3_{Rn_3}$$

CW-SM



$\mathscr{L}_{\rm SM}$

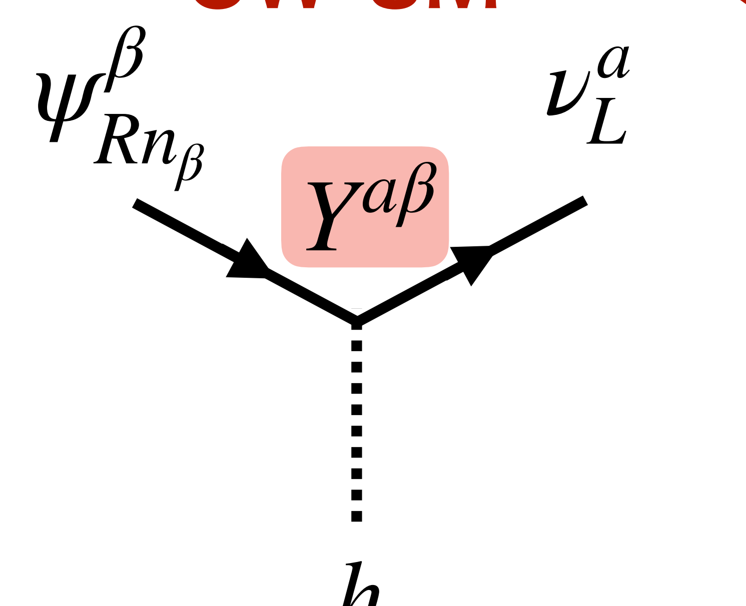
$$\nu^1_L$$
$$\nu^2_L$$
$$\nu^3_L$$

w/ mixings

Ibarra et al., PLB780 (2018)

$$\psi^1_{R0} - \cdots - \psi^1_{Rn_1}$$
$$\psi^2_{R0} - \cdots - \psi^2_{Rn_2}$$
$$\psi^3_{R0} - \cdots - \psi^3_{Rn_3}$$

CW-SM



$\mathscr{L}_{\rm SM}$

$$\nu^1_L$$
$$\nu^2_L$$
$$\nu^3_L$$

mixings:CW-SM origin

$$\mathscr{L}_{\mathrm{CW-SM}} = -Y^{a\beta} H \bar{L}_L^a \psi_{Rn_\beta}^a \;\rightarrow\; \mathscr{L}_{\mathrm{CW-SM}}^{\mathrm{eff}} = -Y^{a\beta} H \bar{L}_L^a \chi_{Rn0}^a$$

$$m_{a\beta} = \boxed{v \frac{1}{q_\beta^{n_\beta}} \sqrt{\frac{q_\beta^2 - 1}{q_\beta^2 - q_\beta^{-2n_\beta}}}} \cdot \boxed{Y^{a\beta}}$$

pure CW : small masses

CW-SM : mixing pattern

ex) two RH neutrinos

$$q_1 = q_2 = 1.79$$
$$n_1 = n_2 = 52$$

$$Y = \begin{pmatrix} 0.49 & 0.89 \\ 3.62 & 1.27 \\ 3.61 & 2.54 \end{pmatrix}$$

→ observed neutrino masses and mixings

mixings:CW-SM origin

$$m_{a\beta} = v\boxed{\frac{1}{q_\beta^{n_\beta}}\sqrt{\frac{q_\beta^2 - 1}{q_\beta^2 - q_\beta^{-2n_\beta}}}} \cdot \boxed{Y^{a\beta}}$$

pure CW : small masses

CW-SM : mixing pattern

$|Y_{a\beta}| = 1$ democratic Yukawa

$$\to \quad m = \begin{pmatrix} m_{11} & m_{12} & m_{13} \\ m_{11} & m_{12} & m_{13} \\ m_{11} & m_{12} & m_{13} \end{pmatrix} \neq \begin{pmatrix} 0.824m_1 & 0.547m_2 & 0.147m_3 \\ -0.495m_1 & 0.569m_2 & 0.657m_3 \\ 0.275m_1 & -0.614m_2 & 0.740m_3 \end{pmatrix}$$

We can not obtain correct mixing by only pure CW sector

Contents

mixings:pure CW origin

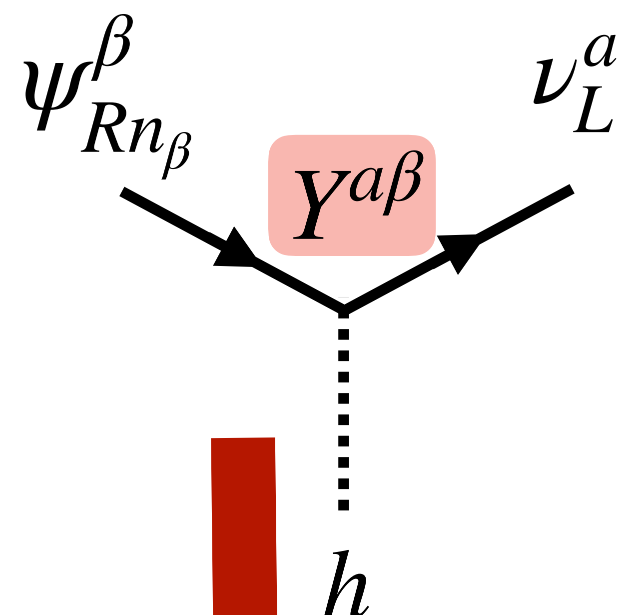
Ibarra et al.

$\psi^1_{R0} - \cdots - \psi^1_{Rn_1}$

$\psi^2_{R0} - \cdots - \psi^2_{Rn_2}$

$\psi^3_{R0} - \cdots - \psi^3_{Rn_3}$

CW-SM



$\mathscr{L}_{\rm SM}$

ν^1_L

ν^2_L

ν^3_L

T.K

T.K., PRD100 (2019)

nine CW chains

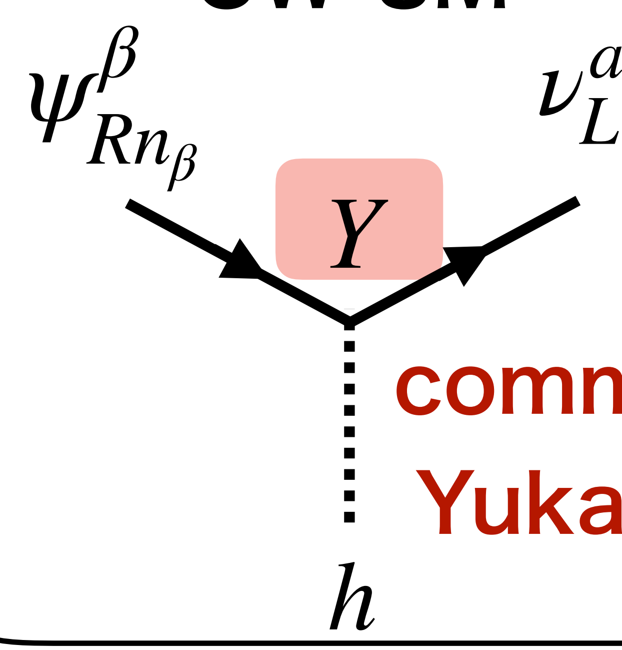
$\psi^1_{R0} - \cdots - \psi^1_{Rn_1}$

$\psi^2_{R0} - \cdots - \psi^2_{Rn_2}$

$\vdots$

$\psi^9_{R0} - \cdots - \psi^9_{Rn_9}$

CW-SM



common Yukawa

$\mathscr{L}_{\rm SM}$

ν^1_L

ν^2_L

ν^3_L

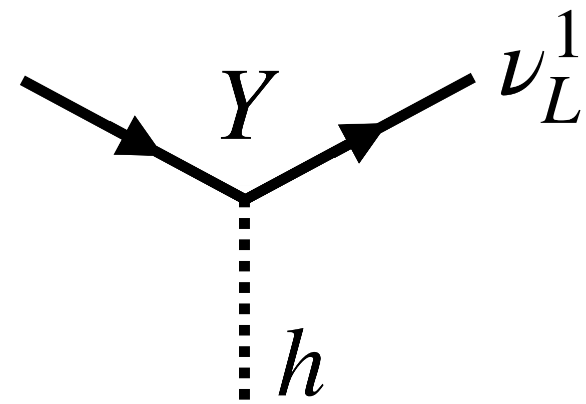
mixings : pure CW origin

$\psi^1_{R0} - \underset{q_{11}}{\cdots} - \psi^1_{Rn_{11}}$

$\psi^2_{R0} - \underset{q_{12}}{\cdots} - \psi^2_{Rn_{12}}$

$\psi^3_{R0} - \underset{q_{13}}{\cdots} - \psi^3_{Rn_{13}}$

$\psi^1_{Rn_{11}}, \psi^2_{Rn_{12}}, \psi^3_{Rn_{13}}$



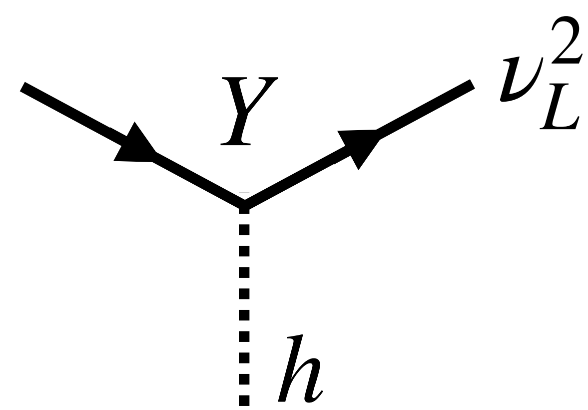
ν^1_L

$\psi^4_{R0} - \underset{q_{21}}{\cdots} - \psi^4_{Rn_{21}}$

$\psi^5_{R0} - \underset{q_{22}}{\cdots} - \psi^5_{Rn_{22}}$

$\psi^6_{R0} - \underset{q_{23}}{\cdots} - \psi^6_{Rn_{23}}$

$\psi^4_{Rn_{21}}, \psi^5_{Rn_{22}}, \psi^6_{Rn_{23}}$



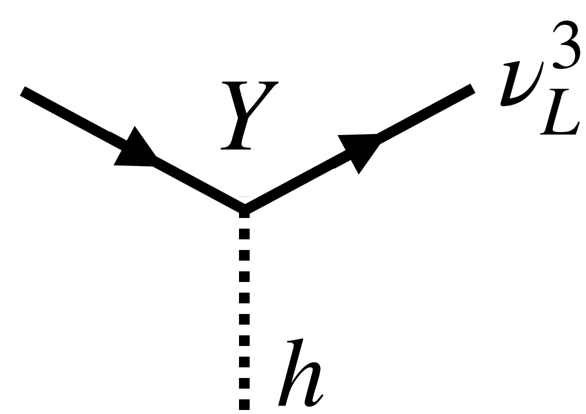
ν^2_L

$\psi^7_{R0} - \underset{q_{31}}{\cdots} - \psi^7_{Rn_{31}}$

$\psi^8_{R0} - \underset{q_{32}}{\cdots} - \psi^8_{Rn_{32}}$

$\psi^9_{R0} - \underset{q_{33}}{\cdots} - \psi^9_{Rn_{33}}$

$\psi^7_{Rn_{31}}, \psi^8_{Rn_{32}}, \psi^9_{Rn_{33}}$

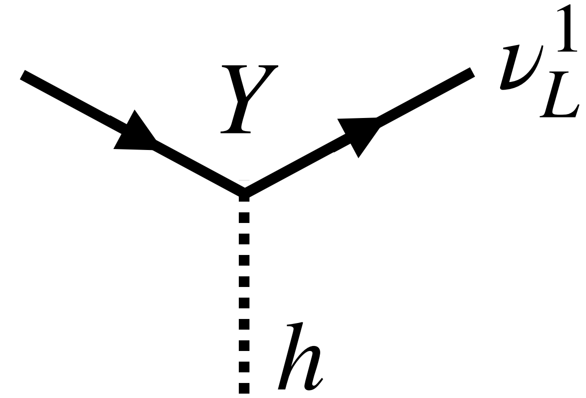


ν^3_L

mixings : pure CW origin

$$\psi^1_{R0} - \underset{q_{11}}{\cdots} - \psi^1_{Rn_{11}}$$
$$\psi^2_{R0} - \underset{q_{12}}{\cdots} - \psi^2_{Rn_{12}}$$
$$\psi^3_{R0} - \underset{q_{13}}{\cdots} - \psi^3_{Rn_{13}}$$

$\psi^1_{Rn_{11}}, \psi^2_{Rn_{12}}, \psi^3_{Rn_{13}}$



ν^1_L

$$m = \begin{pmatrix} m_{11} & m_{12} & m_{13} \\ m_{21} & m_{22} & m_{23} \\ m_{31} & m_{32} & m_{33} \end{pmatrix}$$

$$= \begin{pmatrix} f(n_{11}, q_{11}) & f(n_{12}, q_{12}) & f(n_{13}, q_{13}) \\ f(n_{21}, q_{21}) & f(n_{22}, q_{22}) & f(n_{23}, q_{23}) \\ f(n_{31}, q_{31}) & f(n_{32}, q_{32}) & f(n_{33}, q_{33}) \end{pmatrix}$$

$$m_{11} = v \cdot \frac{Y}{q_{11}^{n_{11}}} \sqrt{\frac{q_{11}^2 - 1}{q_{11}^2 - q_{11}^{-2n_{11}}}}$$

$$m_{12} = v \cdot \frac{Y}{q_{12}^{n_{12}}} \sqrt{\frac{q_{12}^2 - 1}{q_{12}^2 - q_{12}^{-2n_{12}}}}$$

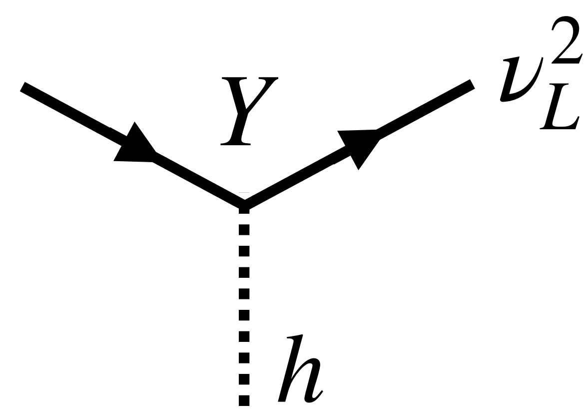
$$m_{13} = v \cdot \frac{Y}{q_{13}^{n_{13}}} \sqrt{\frac{q_{13}^2 - 1}{q_{13}^2 - q_{13}^{-2n_{13}}}}$$

mixings : pure CW origin

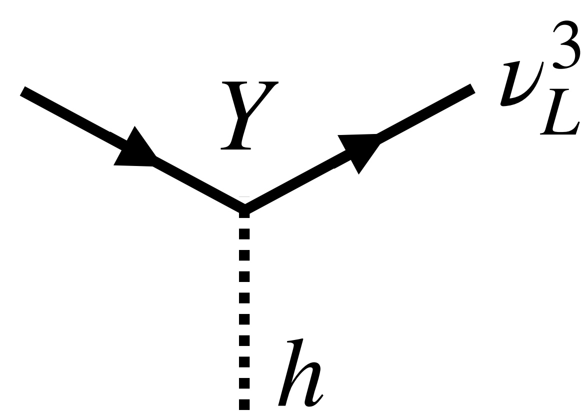
$$m = \begin{pmatrix} m_{11} & m_{12} & m_{13} \\ m_{21} & m_{22} & m_{23} \\ m_{31} & m_{32} & m_{33} \end{pmatrix} = \begin{pmatrix} f(n_{11}, q_{11}) & f(n_{12}, q_{12}) & f(n_{13}, q_{13}) \\ f(n_{21}, q_{21}) & f(n_{22}, q_{22}) & f(n_{23}, q_{23}) \\ f(n_{31}, q_{31}) & f(n_{32}, q_{32}) & f(n_{33}, q_{33}) \end{pmatrix}$$

$$\psi^4_{R0} - \underset{q_{21}}{\cdots} - \psi^4_{Rn_{21}}$$
$$\psi^5_{R0} - \underset{q_{22}}{\cdots} - \psi^5_{Rn_{22}}$$
$$\psi^6_{R0} - \underset{q_{23}}{\cdots} - \psi^6_{Rn_{23}}$$
$$\psi^7_{R0} - \underset{q_{31}}{\cdots} - \psi^7_{Rn_{31}}$$
$$\psi^8_{R0} - \underset{q_{32}}{\cdots} - \psi^8_{Rn_{32}}$$
$$\psi^9_{R0} - \underset{q_{33}}{\cdots} - \psi^9_{Rn_{33}}$$

$\psi^4_{Rn_{21}}, \psi^5_{Rn_{22}}, \psi^6_{Rn_{23}}$



$\psi^7_{Rn_{31}}, \psi^8_{Rn_{32}}, \psi^9_{Rn_{33}}$



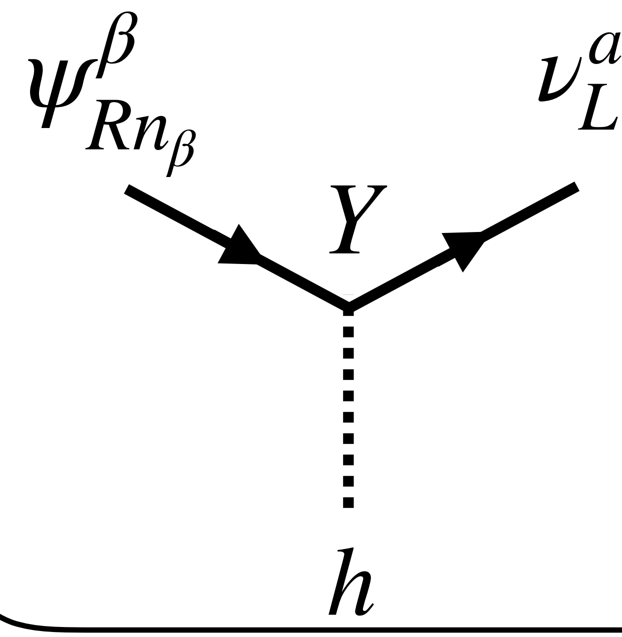
ν_L^2

ν_L^3

mixings : pure CW origin

$$\begin{matrix} \psi_{R0}^{1} - \cdots - \psi_{Rn_1}^{1} \\ \psi_{R0}^{2} - \cdots - \psi_{Rn_2}^{2} \\ \vdots \\ \psi_{R0}^{9} - \cdots - \psi_{Rn_9}^{9} \end{matrix}$$

CW-SM



$\mathscr{L}_{\mathrm{SM}}$

$$\begin{matrix} \nu_L^1 \\ \nu_L^2 \\ \nu_L^3 \end{matrix}$$

$$m_{a\beta} = v \cdot \frac{Y}{q_{a\beta}^{n_{a\beta}}} \sqrt{\frac{q_{a\beta}^2 - 1}{q_{a\beta}^2 - q_{a\beta}^{-2n_{a\beta}}}} \quad \xrightarrow[|Y_{a\beta}| = 1]{} \quad m_{a\beta} = v \cdot \frac{1}{q_{a\beta}^{n_{a\beta}}} \sqrt{\frac{q_{a\beta}^2 - 1}{q_{a\beta}^2 - q_{a\beta}^{-2n_{a\beta}}}}$$

pure CW origin

$$m = \begin{pmatrix} f(n_{11}, q_{11}) & f(n_{12}, q_{12}) & f(n_{13}, q_{13}) \\ f(n_{21}, q_{21}) & f(n_{22}, q_{22}) & f(n_{23}, q_{23}) \\ f(n_{31}, q_{31}) & f(n_{32}, q_{32}) & f(n_{33}, q_{33}) \end{pmatrix} = \begin{pmatrix} 0.824m_1 & 0.547m_2 & 0.147m_3 \\ -0.495m_1 & 0.569m_2 & 0.657m_3 \\ 0.275m_1 & -0.614m_2 & 0.740m_3 \end{pmatrix}$$

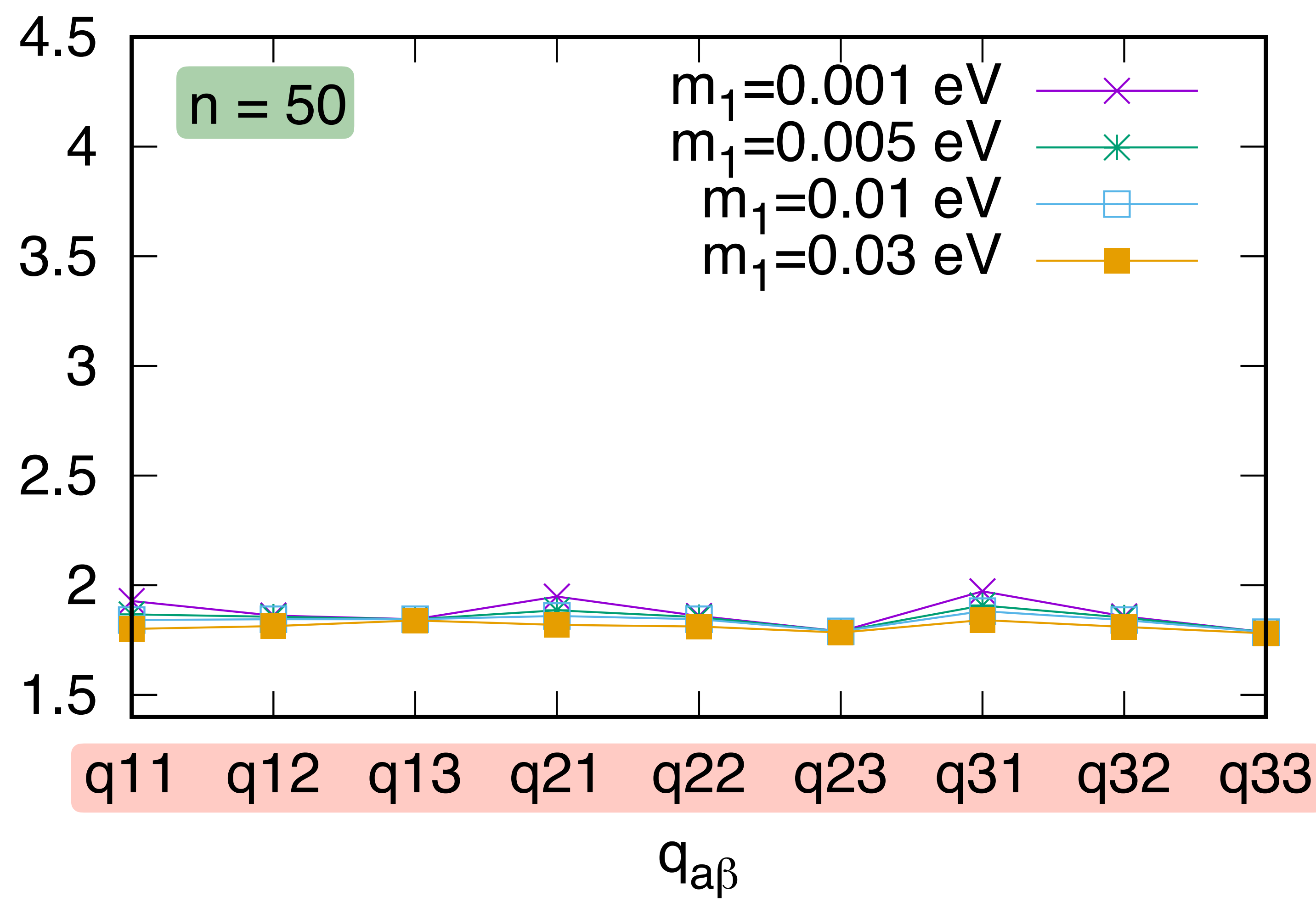
mixings : pure CW origin

universal n case

$$\psi^1_{R0} - \underset{q_{11}}{\cdots} - \psi^1_{Rn}$$
$$\psi^2_{R0} - \underset{q_{12}}{\cdots} - \psi^2_{Rn}$$
$$\psi^3_{R0} - \underset{q_{13}}{\cdots} - \psi^3_{Rn}$$
$$\psi^4_{R0} - \underset{q_{21}}{\cdots} - \psi^4_{Rn}$$
$$\psi^5_{R0} - \underset{q_{22}}{\cdots} - \psi^5_{Rn}$$
$$\psi^6_{R0} - \underset{q_{23}}{\cdots} - \psi^6_{Rn}$$
$$\psi^7_{R0} - \underset{q_{31}}{\cdots} - \psi^7_{Rn}$$
$$\psi^8_{R0} - \underset{q_{32}}{\cdots} - \psi^8_{Rn}$$
$$\psi^9_{R0} - \underset{q_{33}}{\cdots} - \psi^9_{Rn}$$

$$n = 50 \quad \begin{pmatrix} q_{11} & q_{12} & q_{13} \\ q_{21} & q_{22} & q_{23} \\ q_{31} & q_{32} & q_{33} \end{pmatrix} = \begin{pmatrix} 1.841 & 1.845 & 1.844 \\ 1.860 & 1.844 & 1.789 \\ 1.882 & 1.841 & 1.785 \end{pmatrix}$$

→ **observed masses and mixings**



mixings : pure CW origin

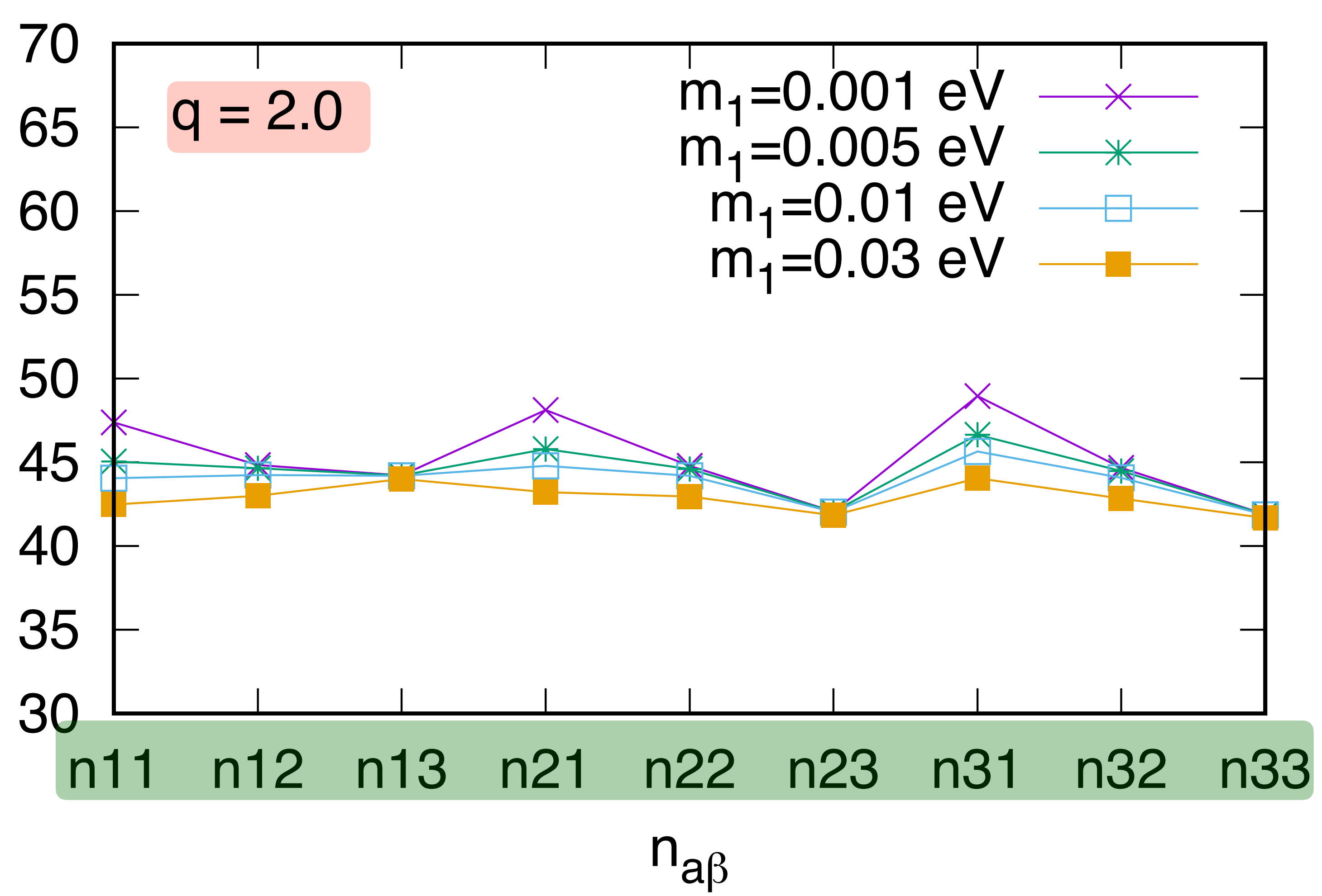
universal q case

$$\psi^1_{R0} - \underset{q}{\cdots} - \psi^1_{Rn_{11}}$$
$$\psi^2_{R0} - \underset{q}{\cdots} - \psi^2_{Rn_{12}}$$
$$\psi^3_{R0} - \underset{q}{\cdots} - \psi^3_{Rn_{13}}$$
$$\psi^4_{R0} - \underset{q}{\cdots} - \psi^4_{Rn_{21}}$$
$$\psi^5_{R0} - \underset{q}{\cdots} - \psi^5_{Rn_{22}}$$
$$\psi^6_{R0} - \underset{q}{\cdots} - \psi^6_{Rn_{23}}$$
$$\psi^7_{R0} - \underset{q}{\cdots} - \psi^7_{Rn_{31}}$$
$$\psi^8_{R0} - \underset{q}{\cdots} - \psi^8_{Rn_{32}}$$
$$\psi^9_{R0} - \underset{q}{\cdots} - \psi^9_{Rn_{33}}$$

$q = 2$

$$\begin{pmatrix} n_{11} & n_{12} & n_{13} \\ n_{21} & n_{22} & n_{23} \\ n_{31} & n_{32} & n_{33} \end{pmatrix} = \begin{pmatrix} 44.06 & 44.24 & 44.18 \\ 44.79 & 44.19 & 42.03 \\ 45.64 & 44.08 & 41.85 \end{pmatrix}$$

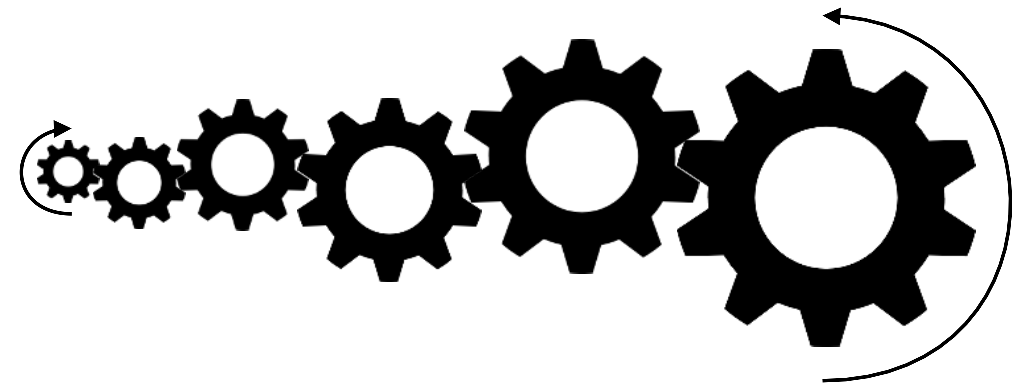
→ observed masses and mixings



Contents

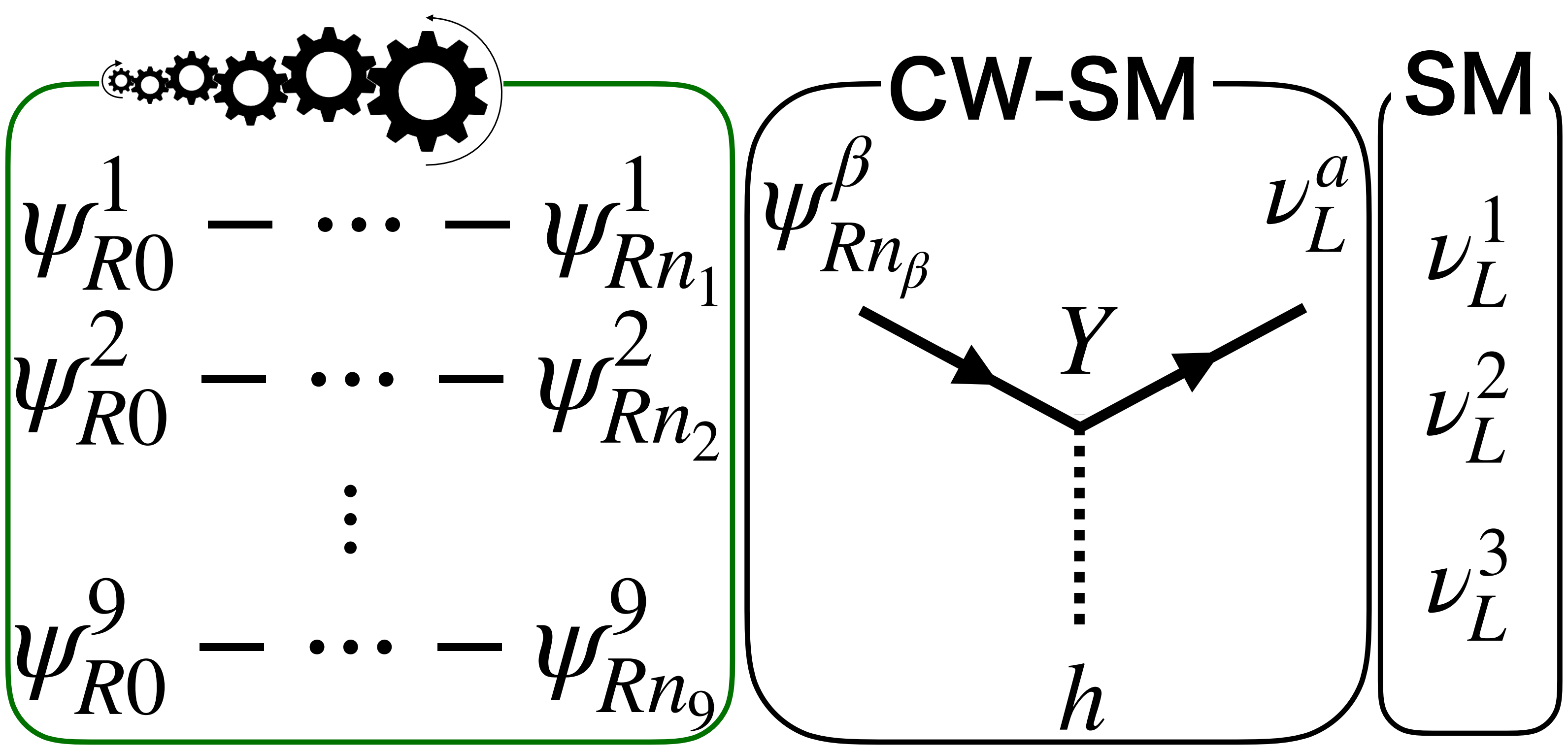
Summary

clockwork mechanism



A natural way to obtain small number (or large hierarchies) with only O(1) couplings & N fields

neutrino masses & mixings



$$|Y_{a\beta}| = 1$$

$$m_{a\beta} = v \cdot \frac{1}{q_{a\beta}^{n_{a\beta}}} \sqrt{\frac{q_{a\beta}^2 - 1}{q_{a\beta}^2 - q_{a\beta}^{-2n_{a\beta}}}}$$

→ observed masses and mixings

Thank you for your attention.