and S-duality

Peculiar index relation for Argyres-Douglas theories

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arXiv: 1907.01579

w/ Matt Buican,
Linfeng Li

S-duality in QFT

In classic QFTs, "S-duality" means the invariance of the theory under

$$S: \quad \tau \to -\frac{1}{\tau}$$

where τ : complexified gauge coupling

For this to make sense, τ must be a free parameter.

(The β-function must vanish.)

$\boxed{\mathcal{N}=4 \text{ SYM}}$ $\quad \tau \equiv \frac{\theta}{2\pi} + \frac{4\pi i}{g^2}$

$$S: \quad \text{W-boson} \xleftrightarrow{\tau \to -\frac{1}{\tau}} \text{monopole}$$

$$M_W \qquad\qquad \frac{4\pi}{g^2} M_W$$

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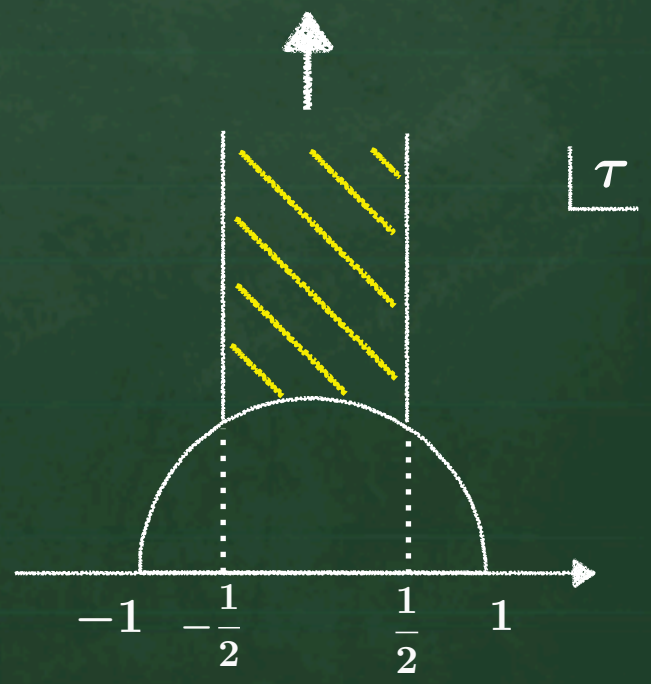
$$S: \ \tau \to -\frac{1}{\tau} \qquad T: \ \tau \to \tau + 1$$

$$\Longrightarrow \text{PSL(2,Z)}$$

weak coupling cusp
$(\tau = i\infty)$



- The space of τ has a single cusp at $\tau = i\infty$

$\mathcal{N} = 2$ SU(N) QCD
w/ 2N flavors

SU(N) — 2N

$$\tau \equiv \frac{\theta}{\pi} + \frac{8\pi i}{g^2}$$

$$\tau \to -\frac{1}{\tau}$$

S : fund. quark $\longleftrightarrow$ monopole

M_{quark} $\qquad$ $\frac{8\pi}{g^2} M_{\text{quark}}$

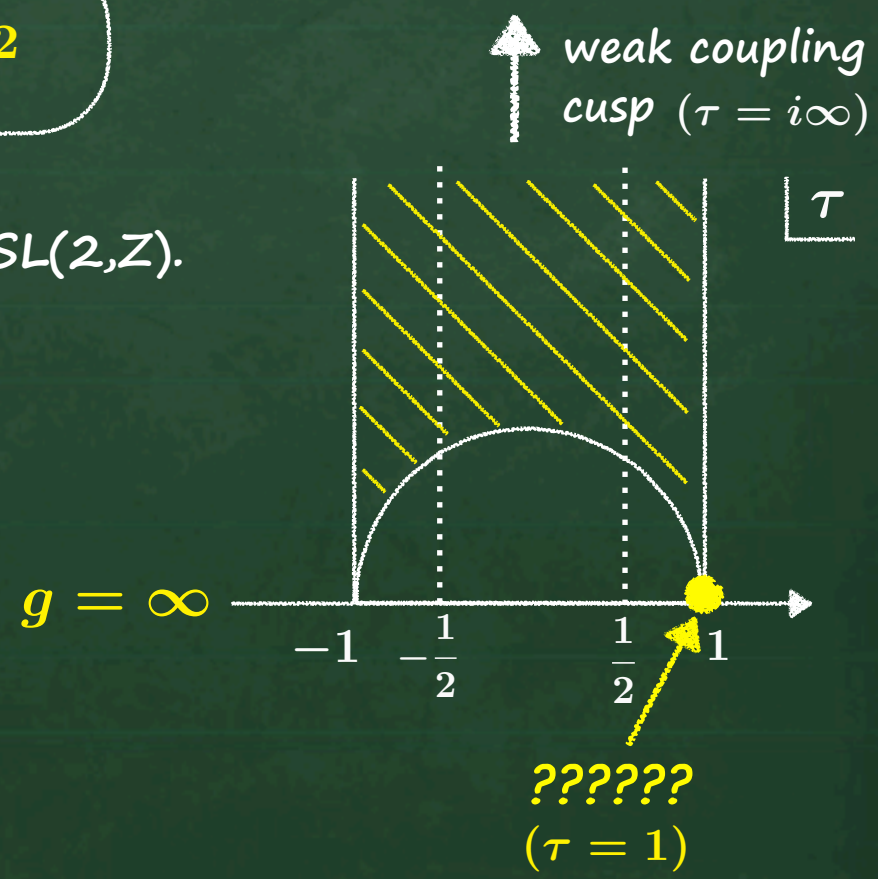
$$S: \ \tau \to -\frac{1}{\tau} \qquad T: \ \tau \to \tau + 2$$

- The S-duality group is smaller than PSL(2,Z).

  (except in the N=2 case)

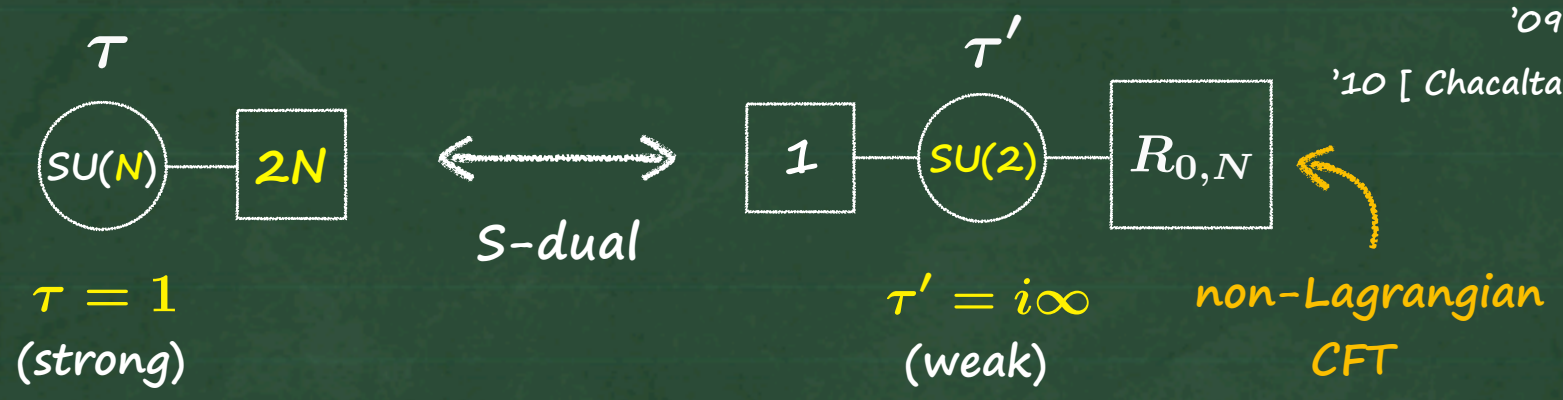
- The space of τ has two cusps!

  $\Longrightarrow$ physics at $\tau = 1$???

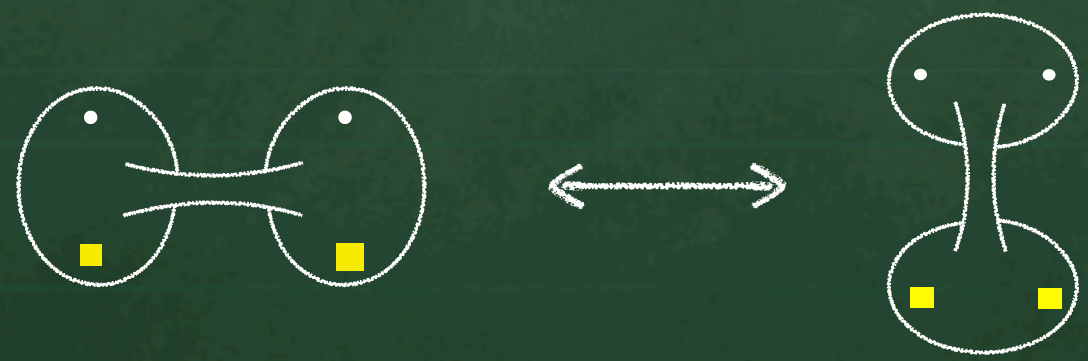


- The physics at $\tau = 1$ is understood via a new "S-duality."

'07 [Argyres-Seiberg]
'09 [Gaiotto]
'10 [Chacaltana-Distler]



- This duality is interpreted as N M5-branes wrapped on

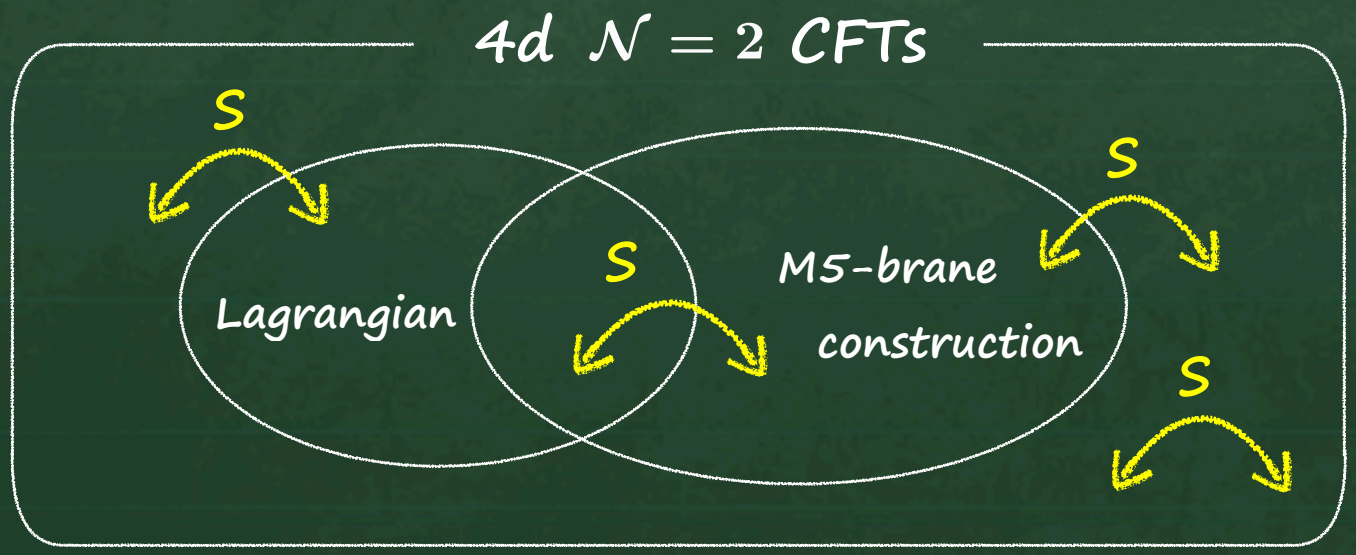


This interpretation led to the discovery of
- class S dualities
- AGT relation
- etc.

The moral

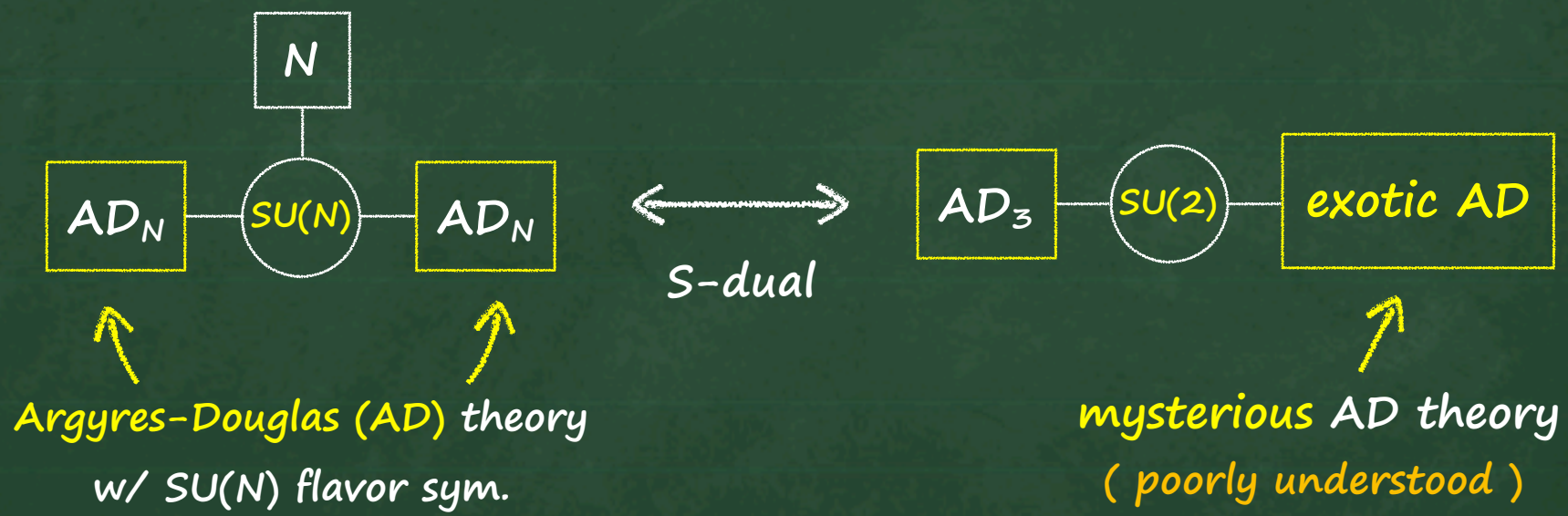
- Understanding S-duality reveals non-trivial relations among N=2 CFTs.

- S-duality sometimes predicts the existence of new N=2 CFTs, which would be useful in classifying 4d N=2 CFTs.



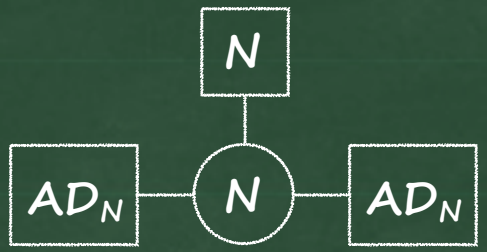
My talk today

- Introduce yet another class of $\mathcal{N} = 2$ S-dualities.



- Its Schur index is obtained via a change of variables from that of

Outline

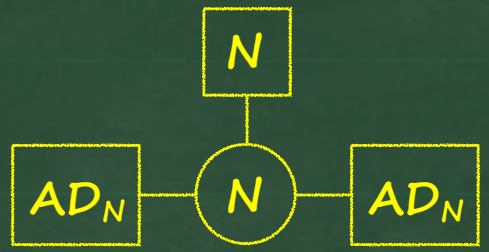
1. S-duality for 

2. Schur index relation

3. Interpretation in the S-dual side

Outline

1. S-duality for



2. Schur index relation

3. Interpretation in the S-dual side

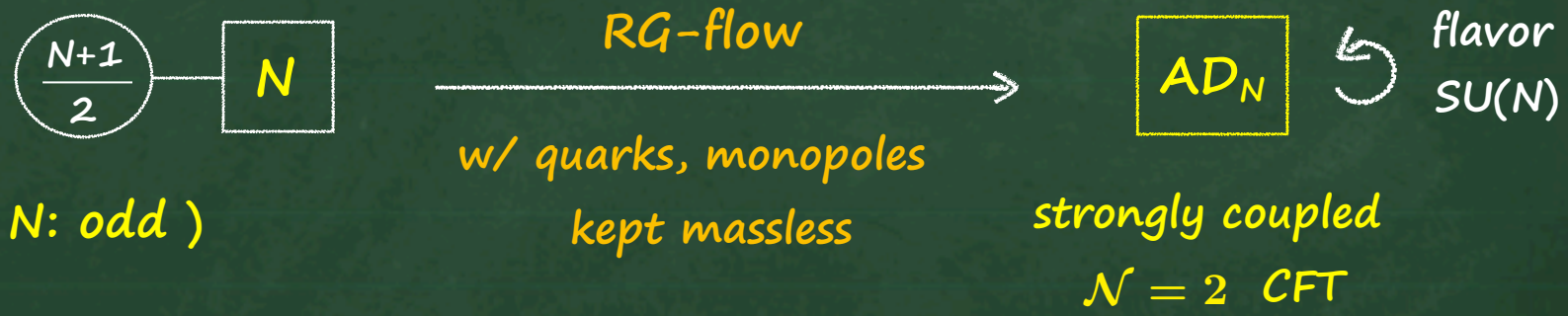
What is $\boxed{AD_N}$?

RG-flow construction

[usually refereed to as $D_2(SU(N))$]

[Cecotti - Del Zotto]

$SU(\frac{N+1}{2})$ QCD w/ N flavors



(N: odd)

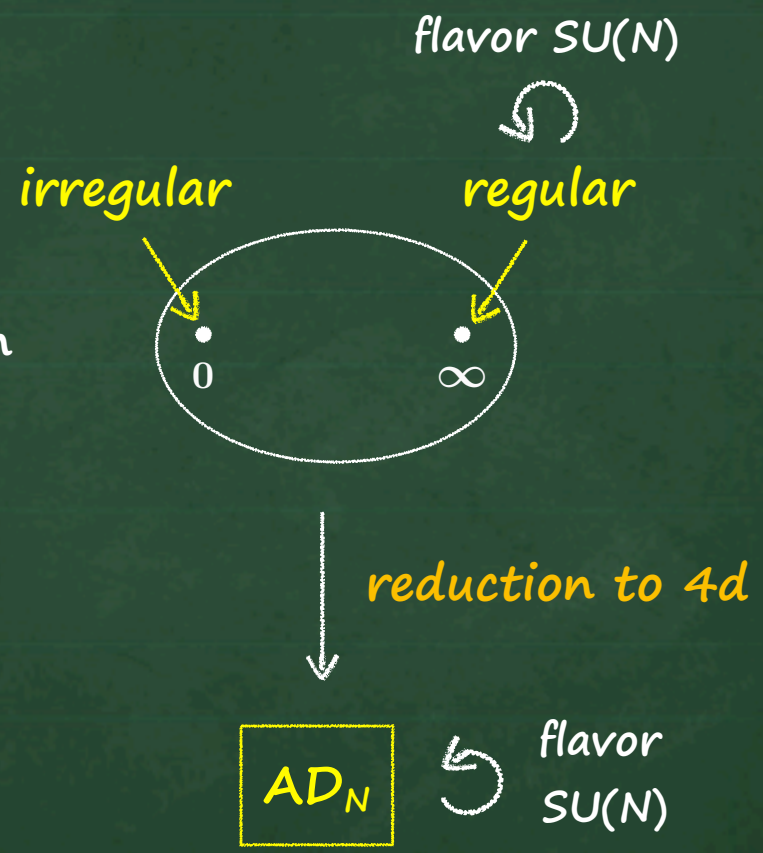
- Flavor SU(N) symmetry
- Coulomb branch operators of dimension $\frac{3}{2}, \frac{5}{2}, \frac{7}{2}, \cdots, \frac{N}{2}$

What is $\boxed{AD_N}$?

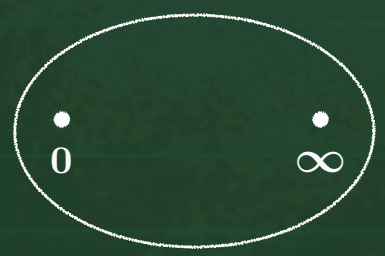
M5 construction

N M5-branes compactified on

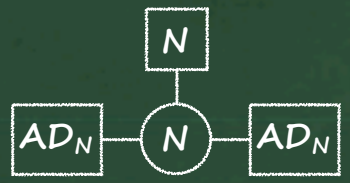
(N: odd)



- Some physical quantities of $\boxed{AD_N}$ can be computed via 2d QFT on

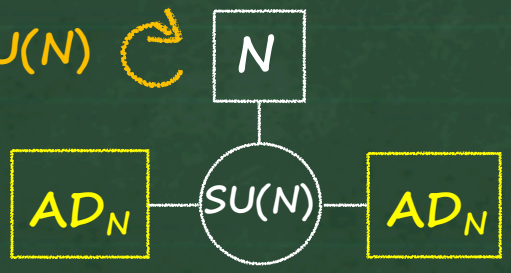


S-duality for



- The following gauging is exactly marginal:

flavor U(N)

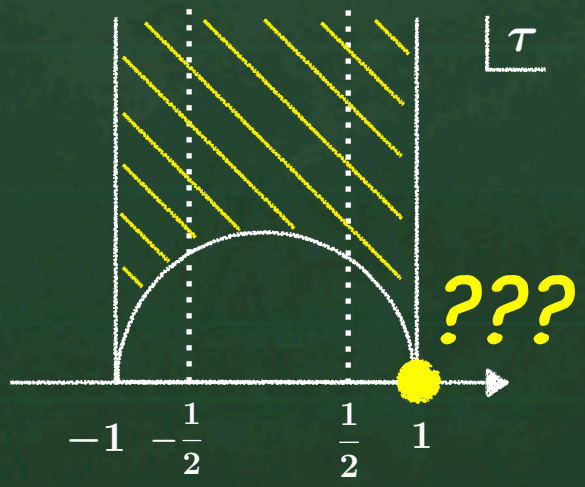


$$\beta = 2N + N + N - 4N = 0$$

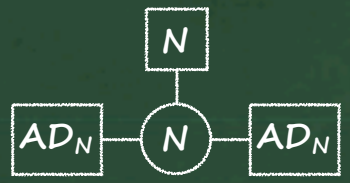
- Exactly marginal coupling

$$\tau = \frac{\theta}{\pi} + \frac{8\pi i}{g^2}$$

$\Longrightarrow$ We expect two cusps.



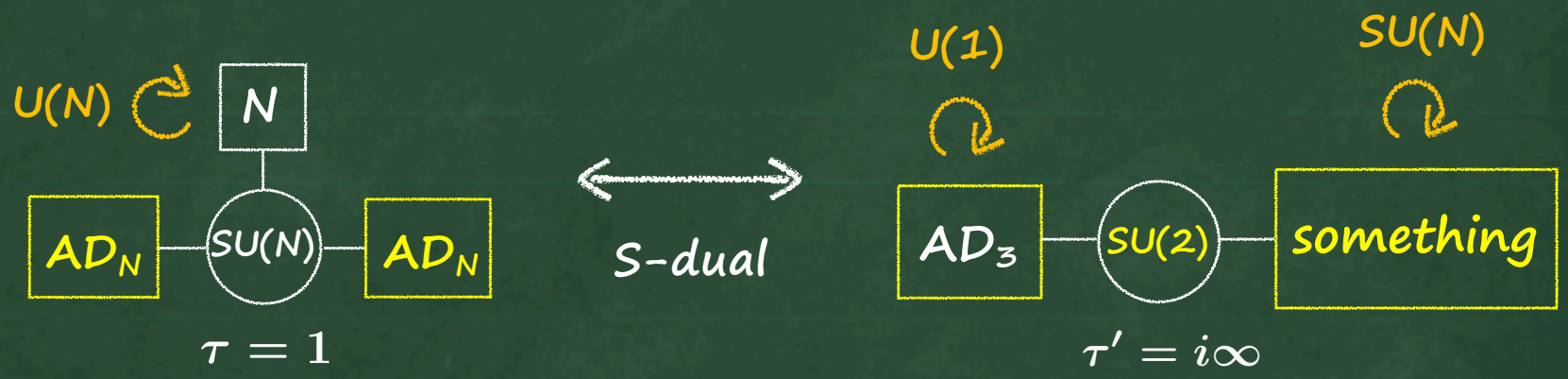
S-duality for



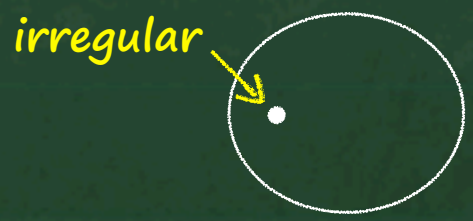
'14 [Buican-Giacomelli-Nishinaka-Papageorgakis]

'19 [Buican-Li-Nishinaka]

Our conjecture c.f.) '16 [Xie]



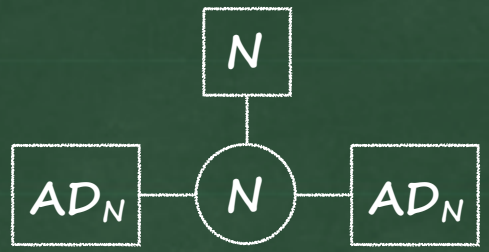
- **something** is identified with an exotic AD theory obtained by compactifying 2N M5-branes on



(The flavor sym, the spectrum of Coulomb branch ops. match.)

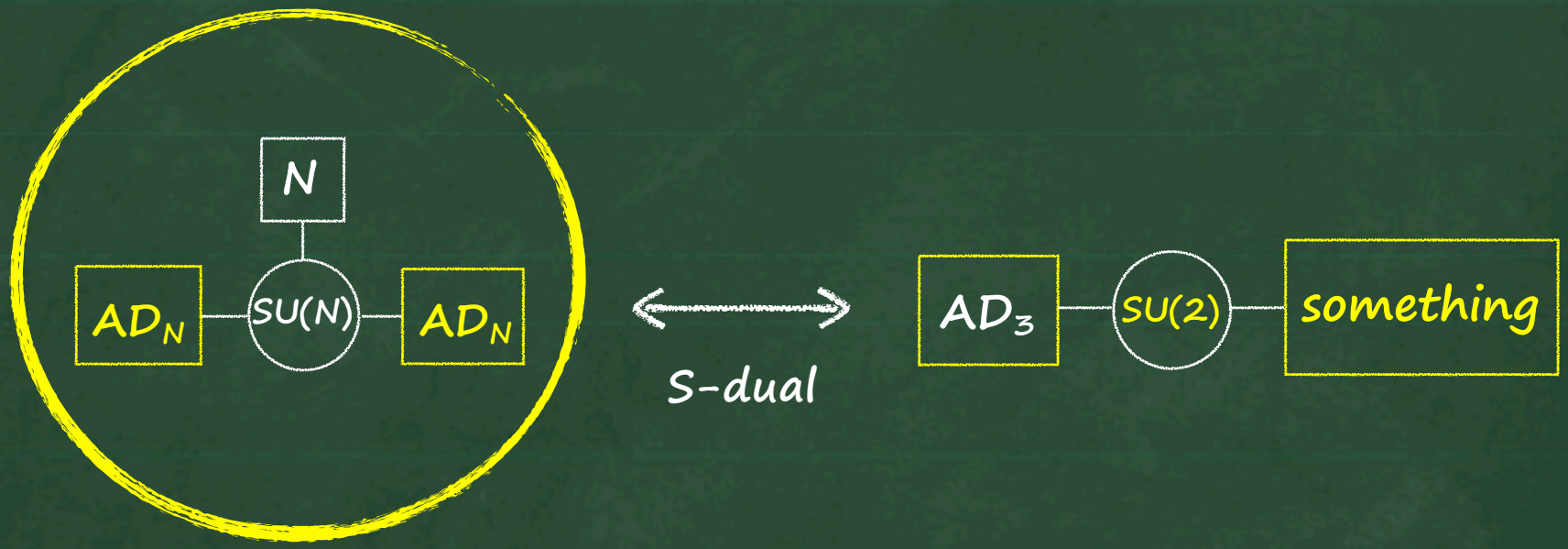
- The SW curve argument.

Outline

1. S-duality for 

2. Schur index relation

3. Interpretation in the S-dual side



We now compute the Schur index of this guy.

Schur index relation

Schur index

SU(2)$_R$ charge

scaling dim.

flavor sym.

space of local operators

flavor charge

$$\mathcal{I}(q;\vec{z}) \equiv \mathrm{Tr}_{\mathcal{H}}\,(-1)^F q^{E-R} \prod_{k=1}^{\mathrm{rank}\,G_F} (z_k)^{A_k} \qquad (\,q,\ z_k \in \mathbb{C}\,)$$

- $E-R$ and A_k commute with 1/4 of $\mathcal{N}=2$ supercharges.

$$\Longrightarrow \quad \mathcal{I}(q;\vec{z}) \text{ counts 1/4 BPS operators.}$$

- $\mathcal{I}(q;\vec{z})$ is particularly useful for studying strongly coupled SCFT.

Schur index relation

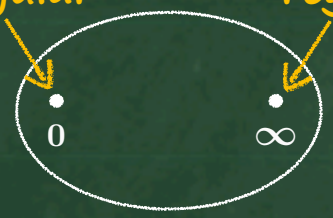
$\boxed{AD_N}$ ↺ flavor SU(N)

Conjecture for AD_N

flavor SU(N) fugacities

$$\mathcal{I}_{\mathrm{AD}_N}(q;\vec{z}) \;=\; \mathrm{Tr}_{\mathcal{H}}\,(-1)^F q^{E-R} \prod_{k=1}^{\mathrm{rank}\,G_F} (z_k)^{A_k}$$

on



correlator in q-deformed YM

$$\langle \mathcal{O}_{\mathrm{irreg}}\, \mathcal{O}_{\mathrm{reg}} \rangle_{q\mathrm{YM}}$$

= '11 [Gadde et al]

×

R^4 $\boxed{AD_N \text{ theory}}$

Schur index

$$\mathcal{I}(q;\vec{z})$$

Schur index relation

$\boxed{\mathrm{AD}_N}$ ↺ flavor SU(N)

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$$\mathcal{I}_{\mathrm{AD}_N}(q;\vec{z}) = \mathrm{Tr}_{\mathcal{H}}\,(-1)^F q^{E-R} \prod_{k=1}^{\mathrm{rank}\,G_F} (z_k)^{A_k}$$

'15 [Song]

'16 [Xie-Yan-Yau]

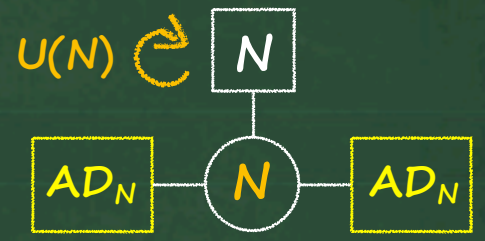
$$= \langle \mathcal{O}_{\mathrm{irreg}}\, \mathcal{O}_{\mathrm{reg}} \rangle_{q\mathrm{YM}}$$

character of adj. rep. of SU(N)

$$= \exp\left[\sum_{n=1}^{\infty} \frac{1}{n} \frac{q^n}{1-q^{2n}} \chi_{\mathrm{adj}}^{SU(N)}({z_k}^n)\right]$$

$$= \prod_{k=0}^{\infty}\left[\frac{1}{(1-q^{2k+1})^{N-1}} \prod_{1\le i\neq j\le N} \frac{1}{1-q^{2k+1} z_i z_j^{-1}}\right]$$

Schur index relation



Index of the gauge theory

gauge invariance

Haar measure

N

AD$_N$ x 2

N $\vec{y}$

$$\mathcal{I}(q;\vec{y}) = \oint_{|z_k|=1} \left(\prod_{k=1}^{N-1} \frac{dz_k}{2\pi i z_k} \right) \Delta(\vec{z})\, \underbrace{\mathcal{I}_{\text{vec}}(q;\vec{z}) \times \left[\mathcal{I}_{\text{AD}_N}(q;\vec{z}) \right]^2}_{\text{Our NEW identity}} \times \mathcal{I}_{\text{hyp}}(q;\vec{z},\vec{y})$$

$\vec{y}$: flavor U(N) fugacity

$\vec{z}$: color SU(N) fugacity

$$= \mathcal{I}_{\text{vec}}(q^2;\vec{z})\, \mathcal{I}_{\text{hyp}}(q^2;\vec{z},q^{\frac{1}{2}}\vec{y})\, \mathcal{I}_{\text{hyp}}(q^2;\vec{z},q^{-\frac{1}{2}}\vec{y})$$

N

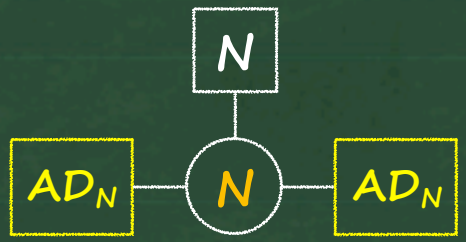
N $q^{\frac{1}{2}}\vec{y}$

N $q^{-\frac{1}{2}}\vec{y}$

= Schur index of N — 2N with $q \to q^2$

(+ a change of flavor fugacity)

Schur index of



$\parallel$ **WOW!**

SU(N) QCD

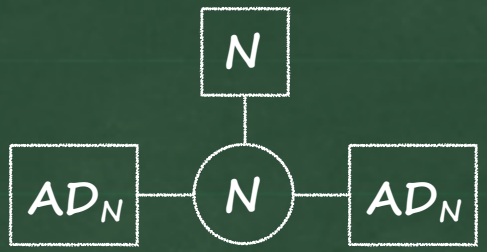
Schur index of

w/ $q \to q^2$

$$\mathcal{I} = \mathrm{Tr}\left[(-1)^F q^{E-R} \times (\cdots)\right]$$

E : scaling dim $\quad$ R : SU(2)$_R$ charge

Outline

1. S-duality for 

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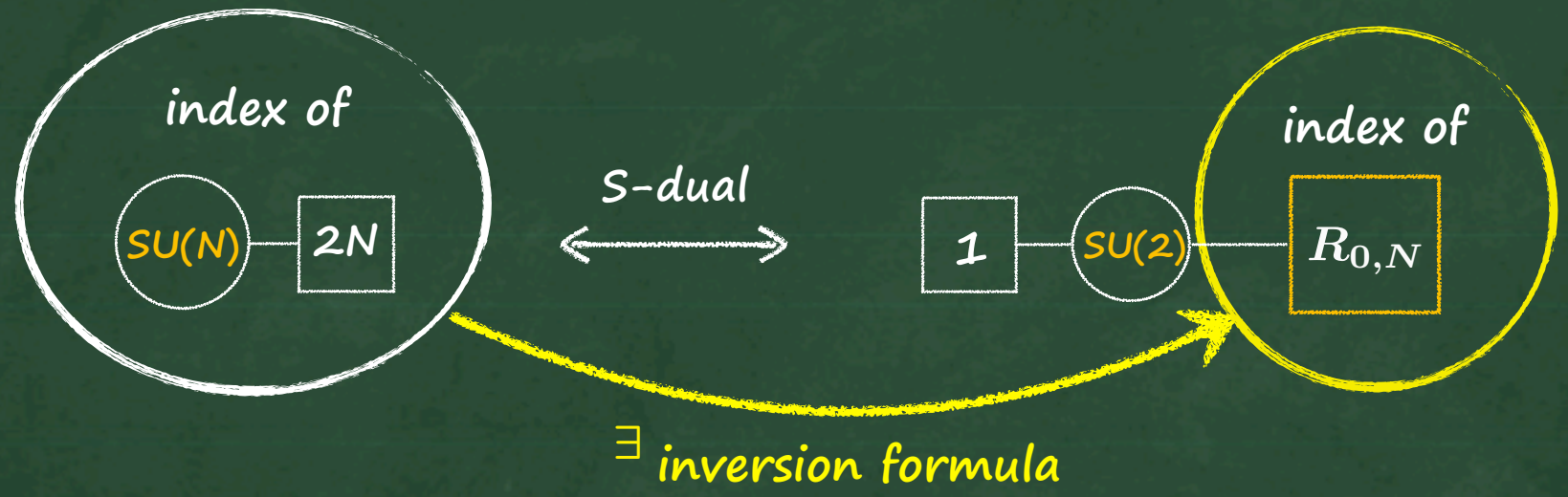
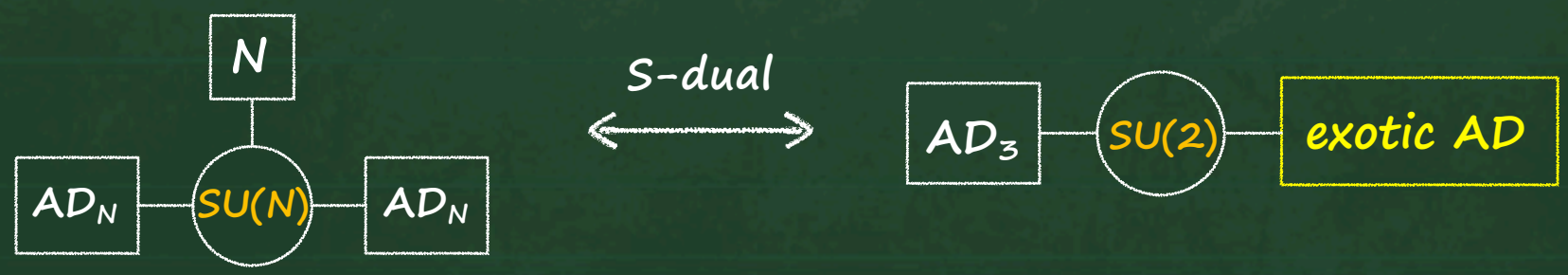
Interpretation in the dual side

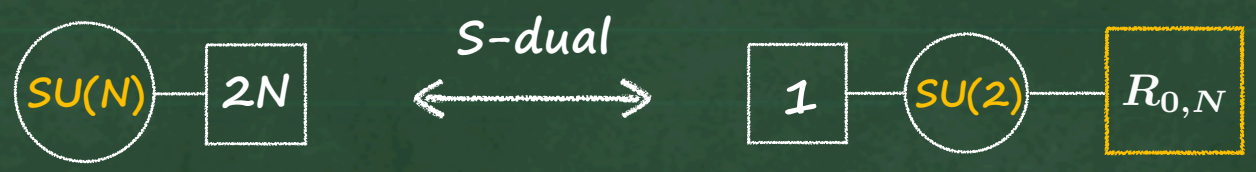




'10 [Gadde - Rastelli - Razamat - Yan]

Inversion formula

- The Schur index is invariant under the duality.



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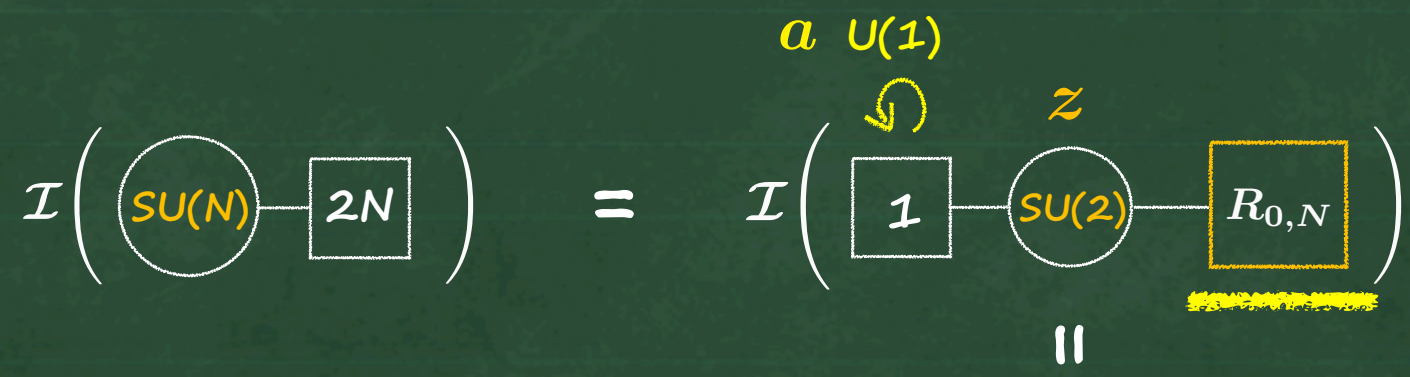
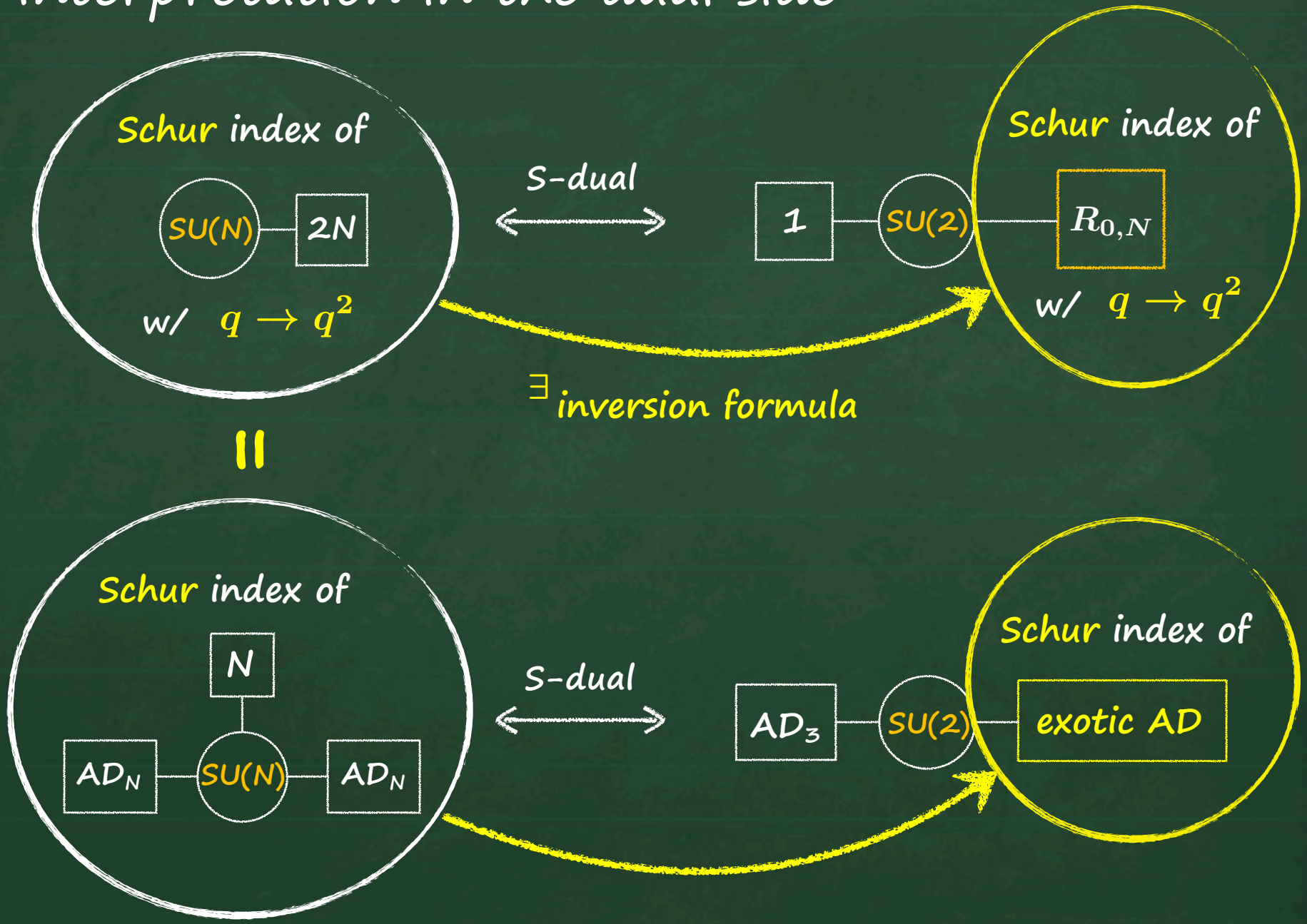


$$\oint \frac{dz}{2\pi i z}\Delta(z)\,\mathcal{I}_{\text{hyp}}(q;z,a)\times\mathcal{I}_{\text{vec}}(q;z)\times\mathcal{I}_{R_{0,N}}(q;z,\vec{y})$$

z : color SU(2)	"Fourier transform" of a function of z
a : flavor U(1)	into a function of a

$\exists$ "inverse Fourier trasform" : $\mathcal{I}_{R_{0,N}}(q;z,\vec{y})$ from $\mathcal{I}$ (SU(N) — 2N)

Interpretation in the dual side



$\exists$ closed form expression in terms of 2D q-deformed YM

Schur index of

$R_{0,N}$

w/ $q \to q^2$

well-understood

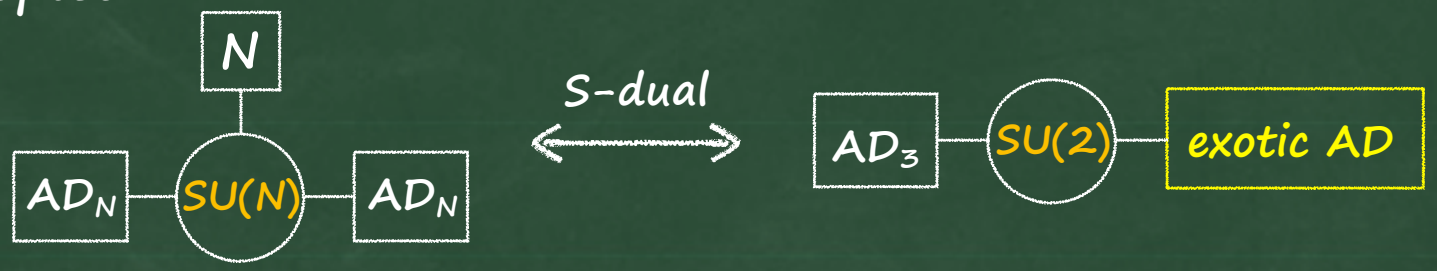
Schur index of

exotic AD

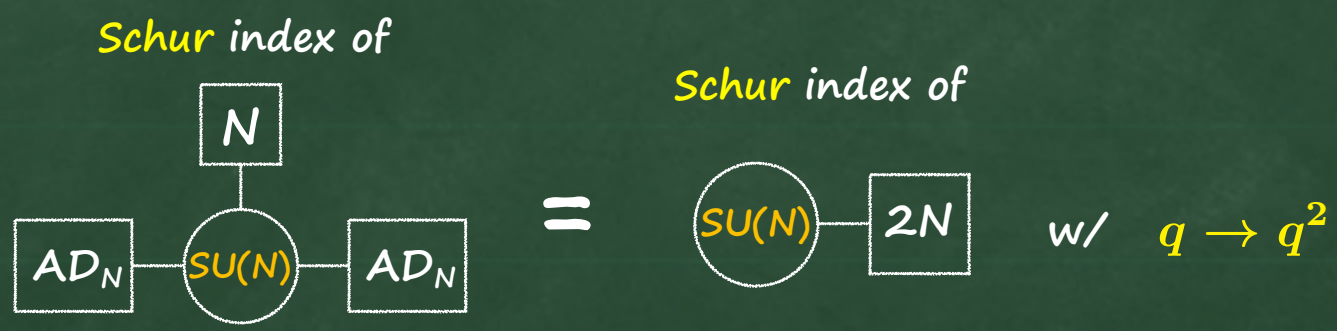
We obtain a closed-form formula for the Schur index of exotic AD !!

Summary

- We propose



- We found



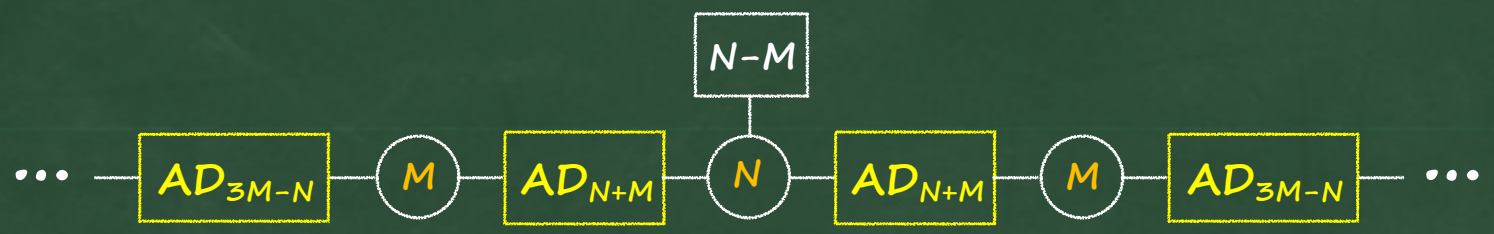
from which you can read off the index of $\boxed{\text{exotic AD}}$.

- Infinite generalizations $\Longrightarrow$ various NEW S-dualities

- Open problem: Reason for the index relation???

  Other quantities consistent w/ the relation???

Schur index of



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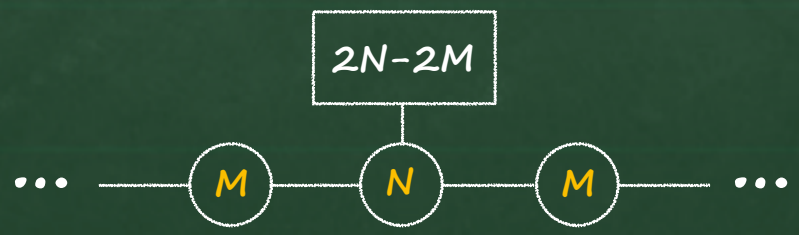
Schur index of



w/ $\begin{cases} q \to q^2 \\ \text{replacement of flavor fugacities} \end{cases}$