What is the CFT dual of BPS black hole microstates?

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IPhT, Saclay

Based on: BG and Samir Mathur, 1905.11923, 1912.05567, 2008.01274
BG, Marcel Hughes, Samir Mathur and Madhur Mehta 2208.07409

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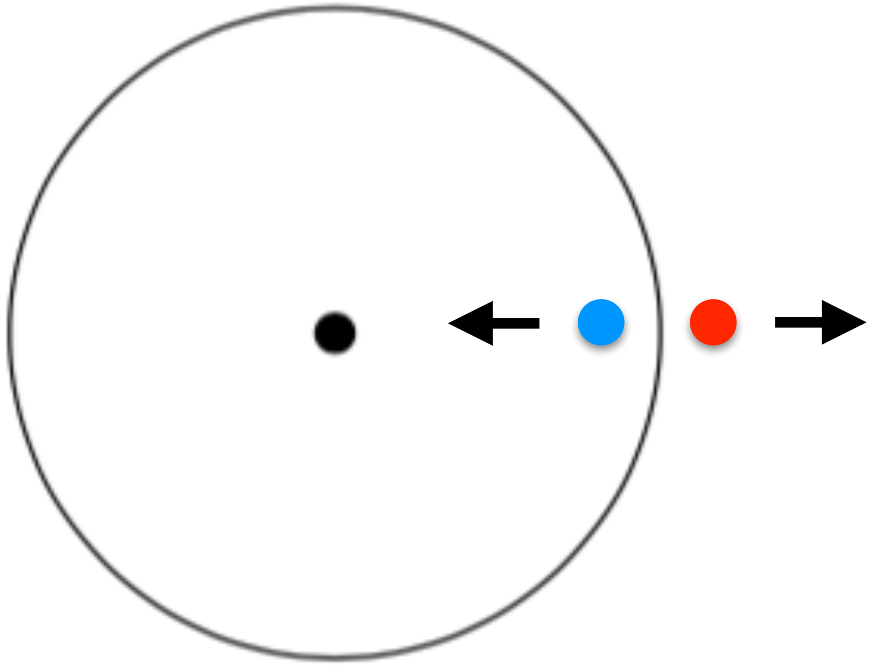
Outline

- Fuzzballs (black hole microstates)
- D1D5 system
- BPS states and lifting
- CFT dual of multi-supergraviton states

Fuzzballs (black hole microstates)

Black hole



Solutions with a smooth horizon.

Including quantum effects, black holes can radiate.

Hawking radiation: Hawking pairs are created near the horizon which is a local vacuum. The radiation does not contain information about the matter placed at the singularity. The black hole information paradox (Hawking 1974).

Quantum mechanics and gravity are not consistent at the horizon.

From the second law of thermodynamics, black holes have entropy

$$S = \frac{A}{4G}$$

No-hair theorems → no microstates

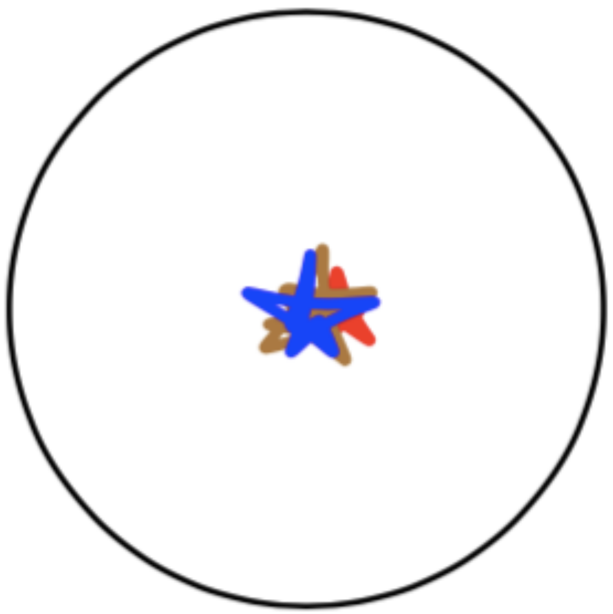
The entropy and the page curve can be reproduced in the Euclidean quantum gravity (the island formalism) without knowing the microstates.

Can we build these microstates?
What are their physical consequenses (observational effects)?

Making BH microstates in string theory

String theory contains higher dimensions and various extended objects to make a consistent quantum theory including gravity.

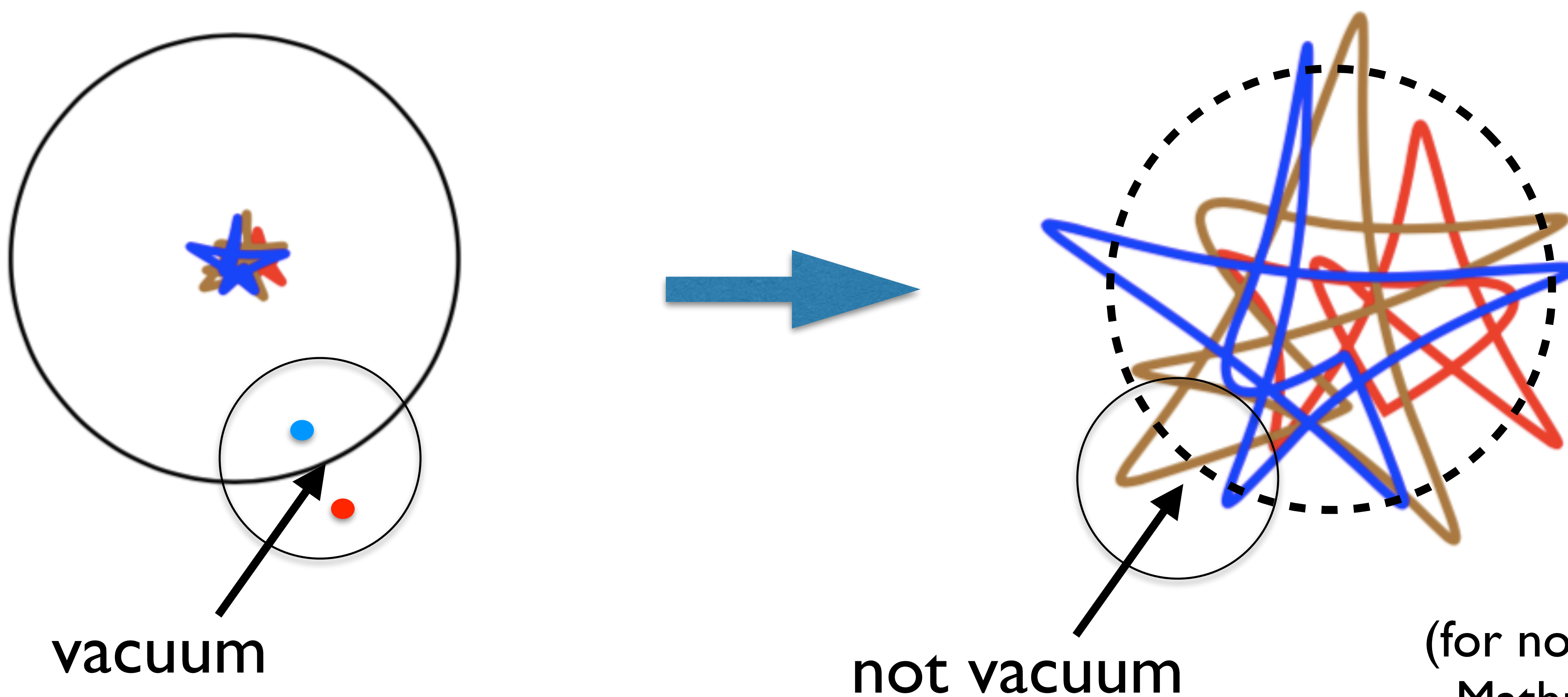
To create a point mass, put lots of vibrating strings at a 'point'.



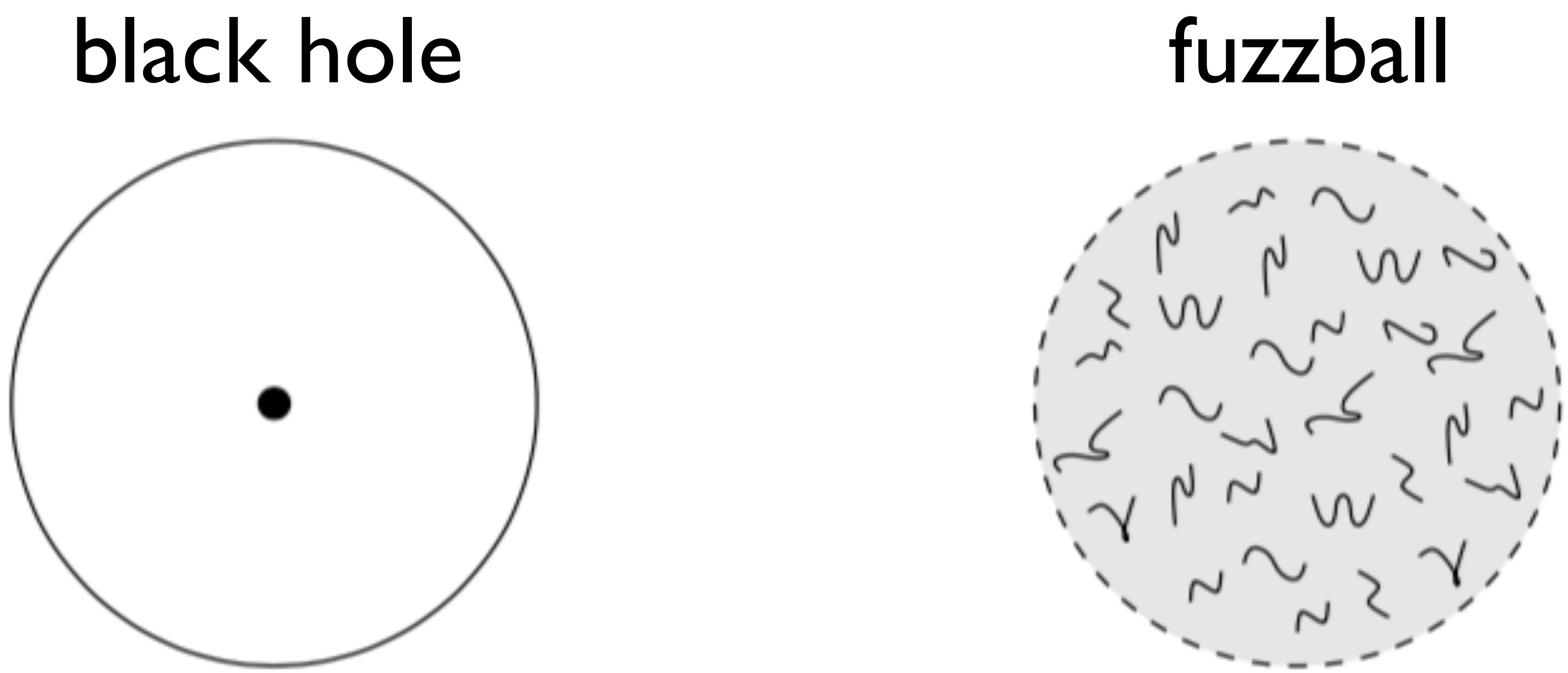
The growing of the size of strings is slower than the growing of the size of the horizon.

Cannot make microstates by using strings only.

Including D-branes and extra dimensions, the growing of the size is the same as the horizon.



(for non-BPS see Mathur, 2005)



Fuzzball

Fuzzballs are black hole microstates in string theory. They have no horizon or singularity. (Lunin and Mathur 2002, …)

Fuzzballs radiate like a burning piece of coal. The radiation contains the information about the fuzzballs.
The information paradox is naturally resolved.

Many fuzzballs have been made in supergravity.

D1D5 system

Setup in string theory: D1D5 system

String theory contains higher dimensions and various extended objects to make a consistent quantum theory including gravity.

String (1-d), D1-brane (1-d), D5-brane (5-d) …

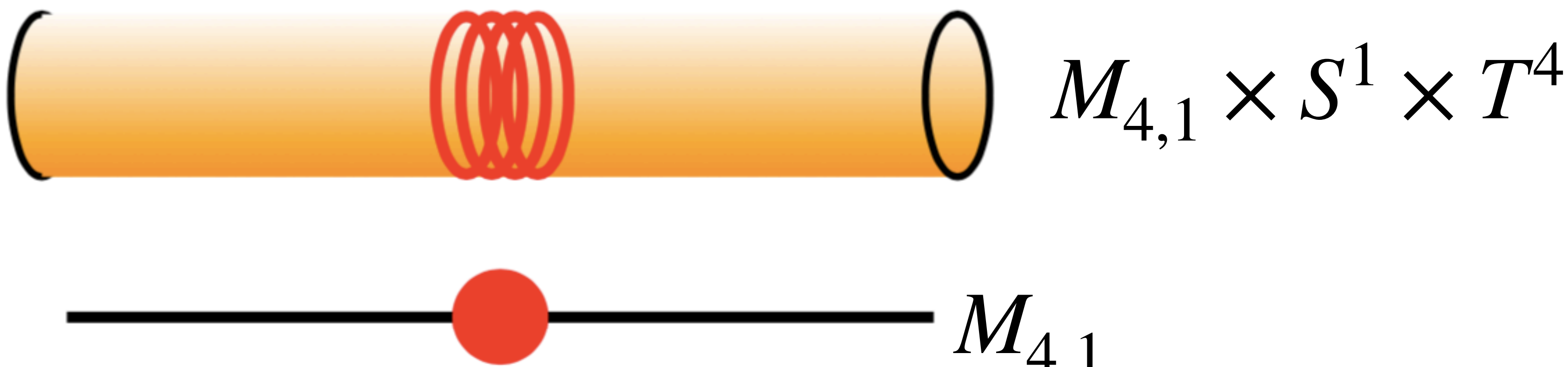
Build a 5-d black hole from the 10-d string theory

Compactification

$$M_{4,1} \times S^1 \times T^4$$

$M_{4,1}$: (4+1)-d Minkowski
S^1: a circle
T^4: a 4-d torus

D1D5 states: wrap N_1 D1-branes on S^1 and N_5 D5-branes on $S^1 \times T^4$



$V(T^4) \to 0$ ➡ lives only on S^1 effectively

Low energy effective theory: D1D5 CFT

1+1 dimensional theory with $\mathcal{N} = (4,4)$ superconformal symmetry

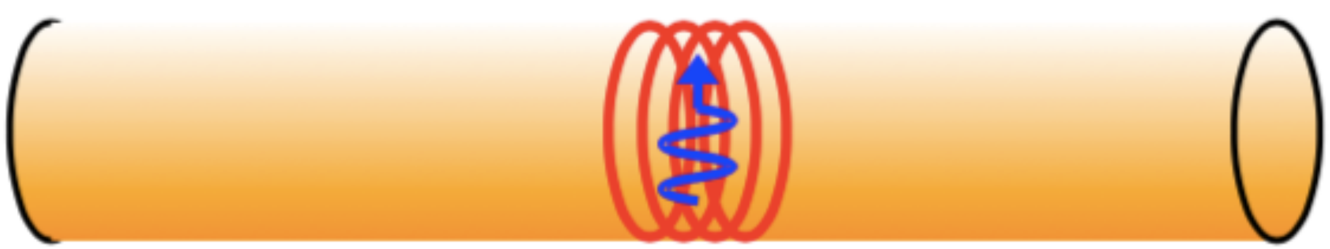
Symmetric orbifold

Fractional momentum, brane

A graviton moving in a circle

$$\Delta P \sim \frac{1}{L}$$

A graviton moving in a n_1-wound string



$$\Delta P \sim \frac{1}{n_1 L}$$

Total momentum is n_p/L

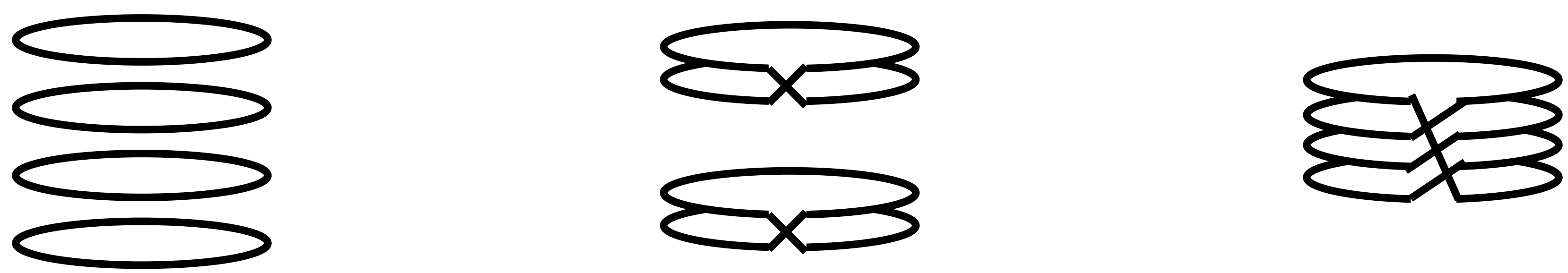
$$P \sim \frac{n_p}{L} \sim \frac{n_1 n_p}{n_1 L}$$

$n_1 n_p$ units of the fractional momentum

D1D5 states: wrap N_1 D1-branes on S^1 and N_5 D5-branes on $S^1 \times T^4$

→ $N_1 N_5$ fractional D1-branes (effective strings)

Example $N_1 N_5 = 4$

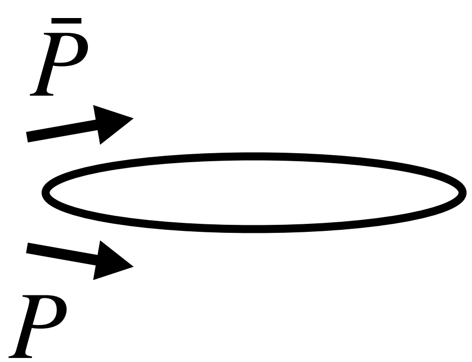


There are multi-wound strings.
Strings having the same length (and excitations) are identical.

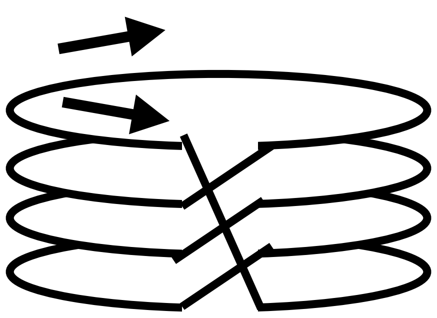
→ Symmetric orbifold $M^{N_1 N_5}/S_{N_1 N_5}$

Building blocks of states at the orbifold point

- Component string of winding k (a closed loop with length $2\pi k$)
- Left (P) and right ($\bar{P}$) moving excitations (4 bosons + 4 fermions)

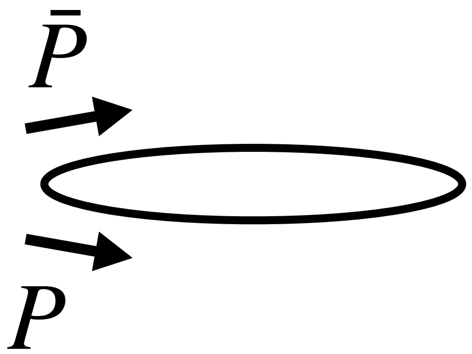


k=1



k=4

Left moving (P excitations): 4 free bosons + 4 free fermions

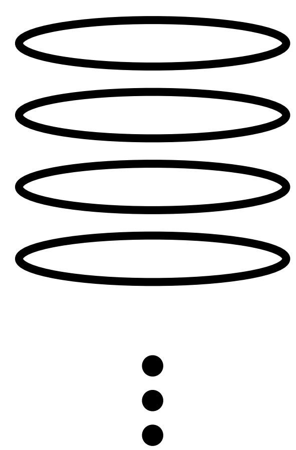


$\alpha_{A\dot{A},-n}$ $\quad d^{\alpha A}_{-n}$ ← momentum (left moving dimension h)

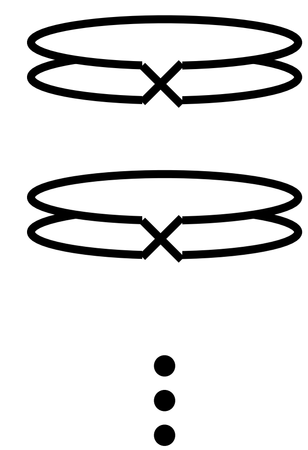
Right moving: $\bar{\alpha}_{A\dot{A},-n}$ $\quad \bar{d}^{\bar{\alpha}A}_{-n}$ ← right moving dimension $\bar{h}$

(4,4) supersymmetry: $G^{\alpha}_{\dot{A},-n}$ $\quad \bar{G}^{\bar{\alpha}}_{\dot{A},-n}$

A state is composed of component strings with total winding N.



N singly wound strings

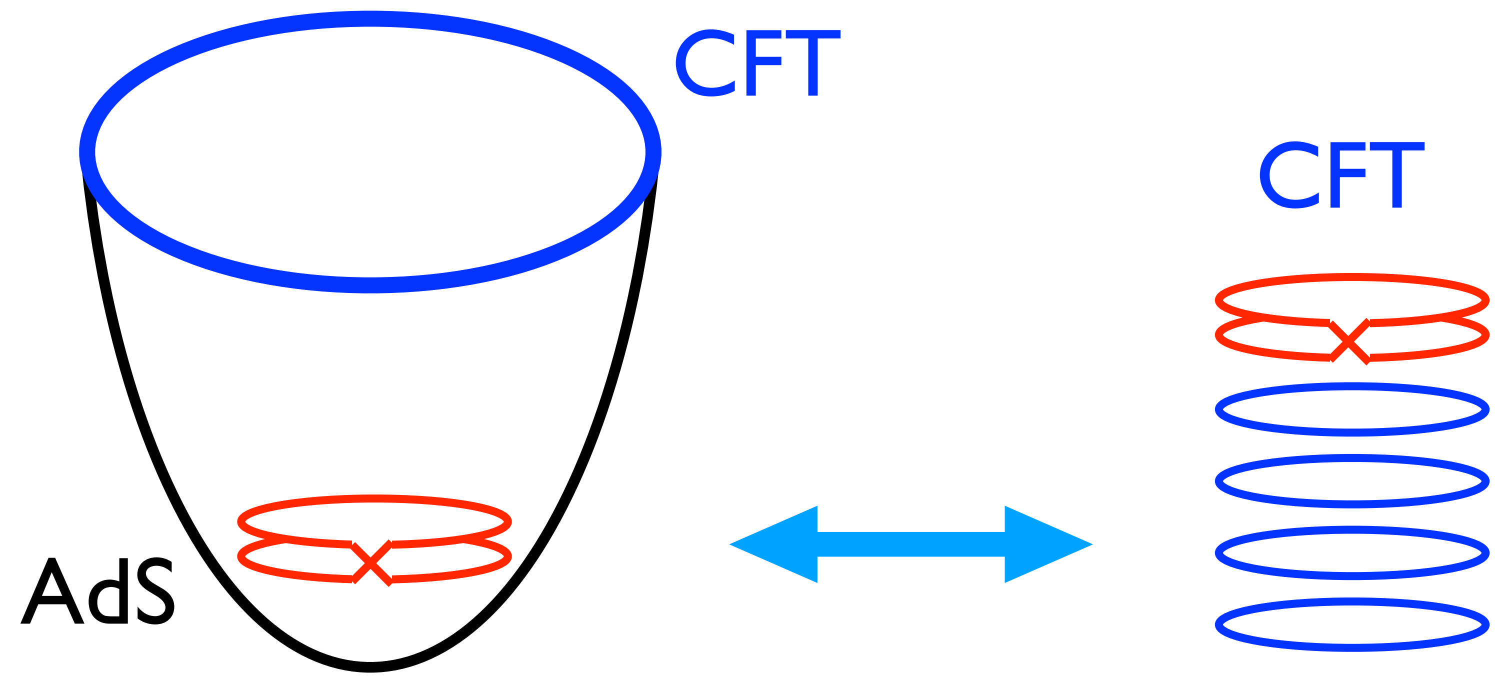


N/2 doubly wound strings

Moving away from the orbifold point (adding interactions)

AdS_3/CFT_2

Tensionless string on $AdS_3 \times S^3 \times T^4$ is dual to the symmetric orbifold $(T^4)^N/S_N$ (CFT_2) (Eberhardt, Gabriel and Gopakumar, 2019…)



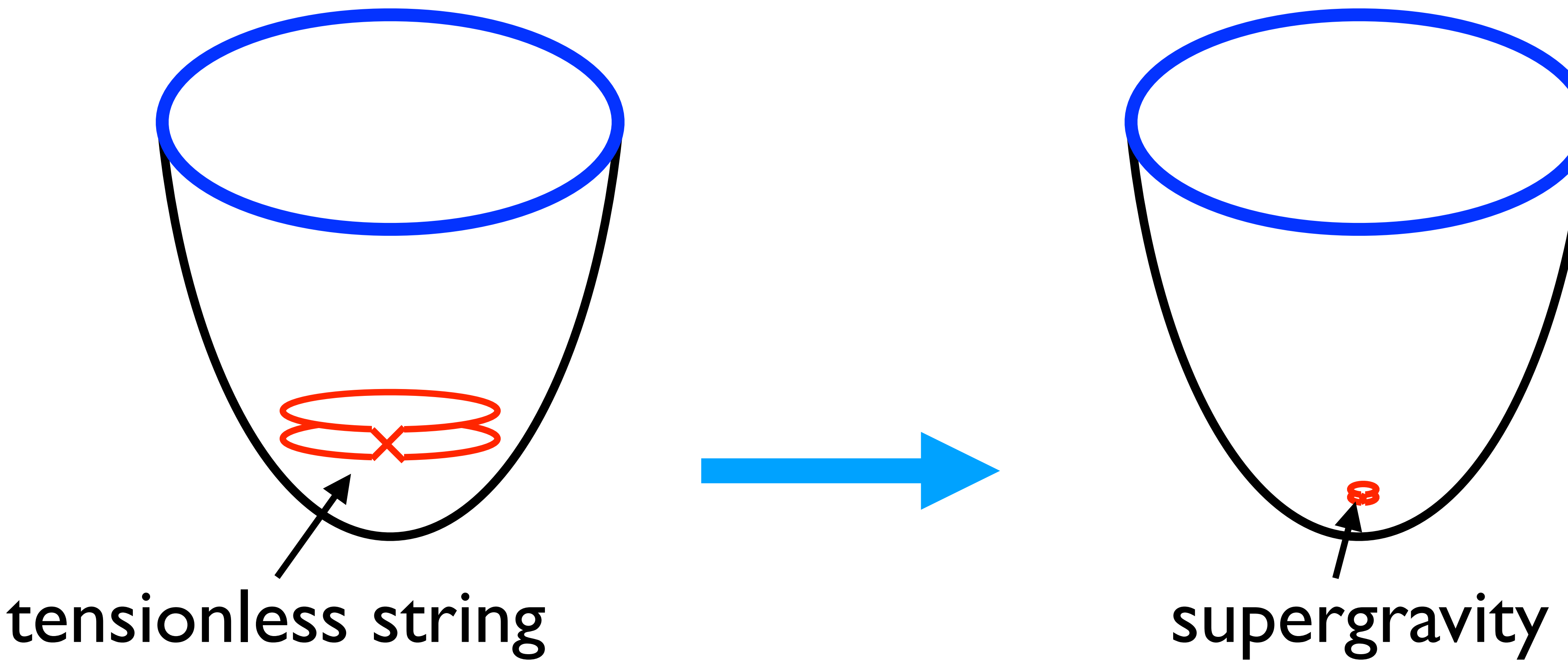
A single string with winding k in the bulk

A component string with winding k on the symmetric orbifold

$$g_s \sim \frac{1}{\sqrt{N}}$$

Adding interaction with coupling λ in the CFT

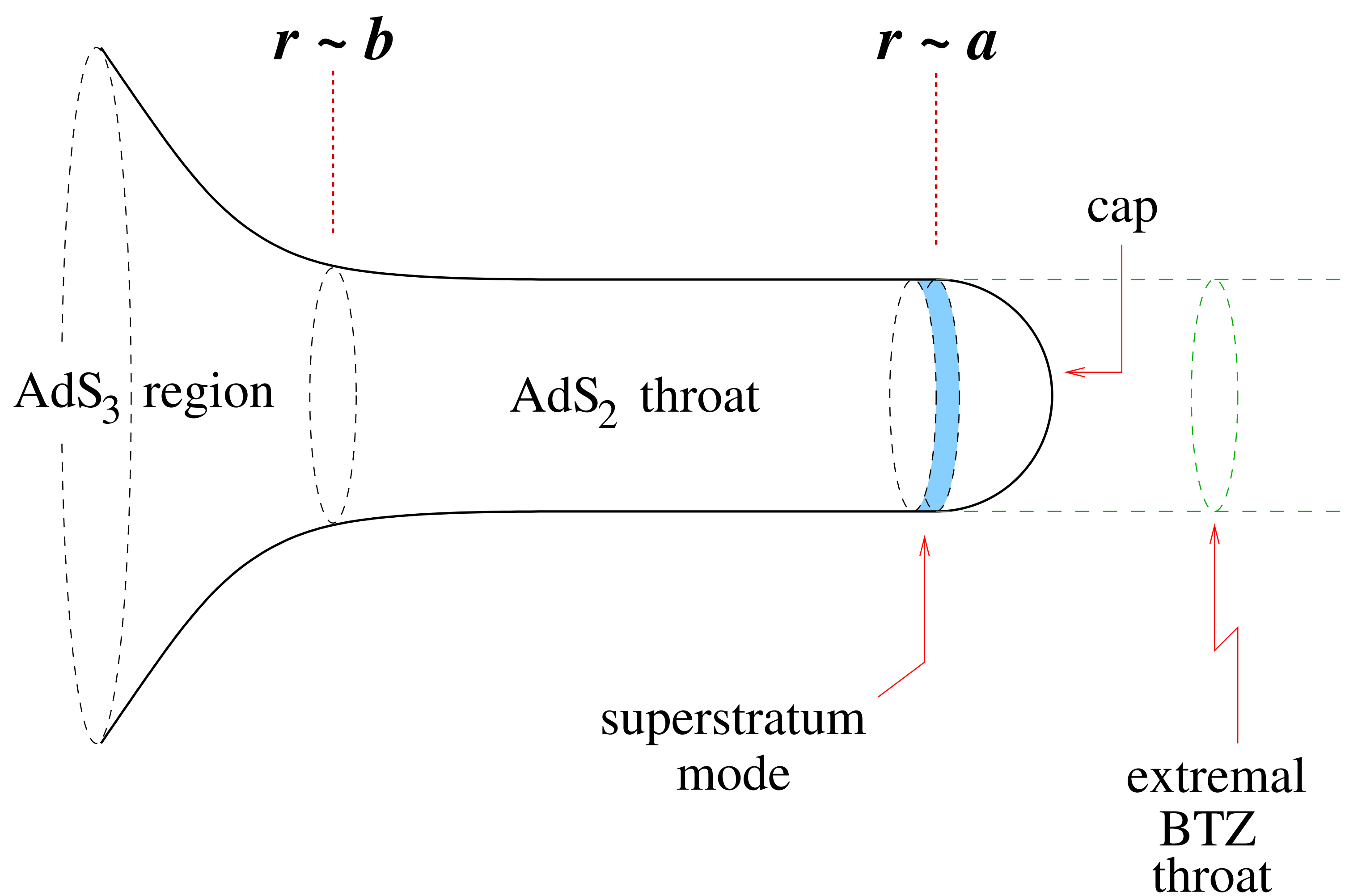
Adding tension, RR flux… in the AdS



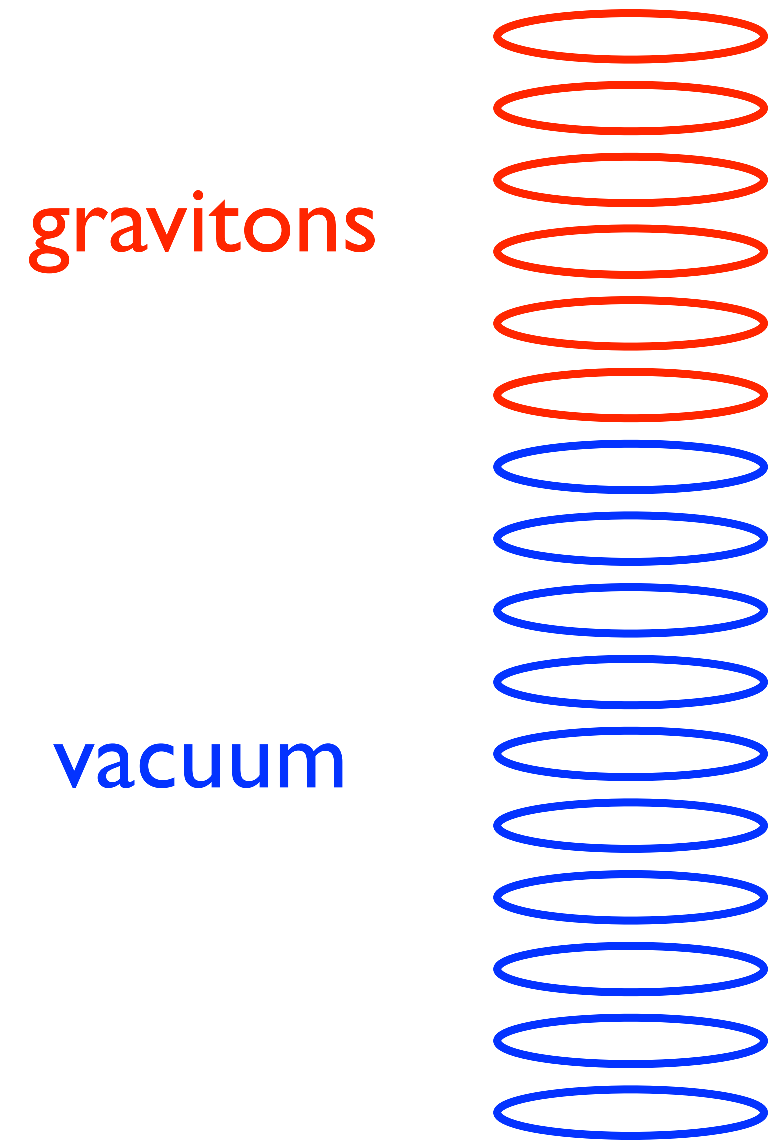
A graviton in the AdS corresponds to a component string in the CFT. (de Boer, 1998)

Superstrata (multi-graviton states)

(Bena, Giusto, Martinec, Russo, Shigemori, Turton and Warner, 2016…)



CFT state



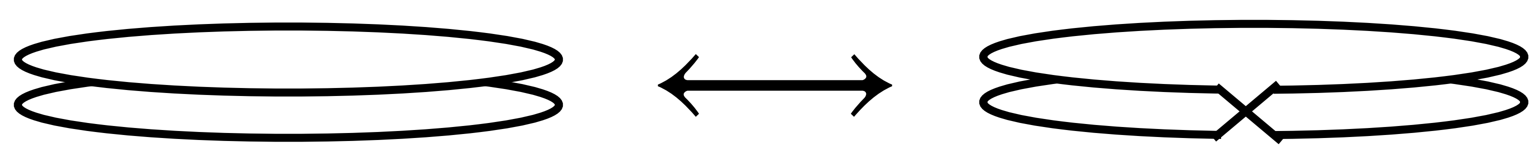
Many gravitons ⟷ (?) Many component strings with each corresponding to a graviton

Interaction

$$S \to S + \lambda \int d^2 z D(z, \bar{z}) \qquad D = \frac{1}{4}\epsilon^{\dot{A}\dot{B}}\epsilon_{\alpha\beta}\epsilon_{\bar{\alpha}\bar{\beta}} G^{\alpha}_{\dot{A},-\frac{1}{2}} \bar{G}^{\bar{\alpha}}_{\dot{B},-\frac{1}{2}} \sigma^{\beta\bar{\beta}}$$

Twist operator: join two strings or break a string

$$\sigma : \quad (k_i, k_j) \leftrightarrow (k_i + k_j)$$



(1) Excitations live on strings cannot interact directly but through the joining or breaking of strings.

(2) Change length of strings important in this talk

BPS states and lifting

BPS states

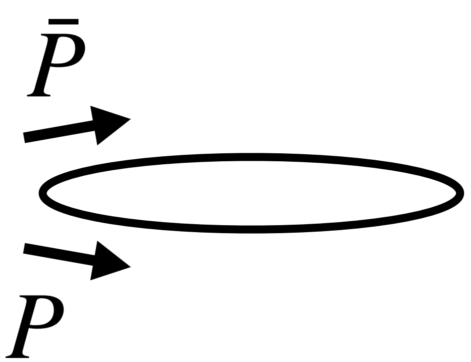
(4,4) supersymmetry: $G^{\alpha}_{\dot{A},0}$ $\bar{G}^{\bar{\alpha}}_{\dot{A},0}$ R sector

$$\{G^{\alpha}_{\dot{A},0}, G^{\beta}_{\dot{B},0}\} = \epsilon_{\dot{A}\dot{B}}\epsilon^{\alpha\beta}\, h \qquad \{\bar{G}^{\alpha}_{\dot{A},0}, \bar{G}^{\beta}_{\dot{B},0}\} = \epsilon_{\dot{A}\dot{B}}\epsilon^{\bar{\alpha}\bar{\beta}}\, \bar{h}$$

Raising operators: $G^{+}_{+,0}$ $G^{+}_{-,0}$ $\bar{G}^{+}_{+,0}$ $\bar{G}^{+}_{-,0}$

At orbifold point

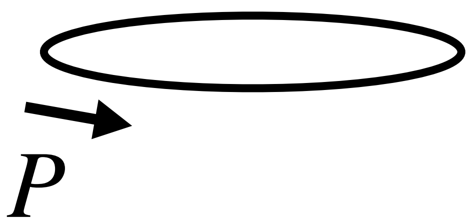
$$\phi: \;\; h \neq 0, \;\; \bar{h} \neq 0$$



$G^{+}_{+,0}$ $G^{+}_{-,0}$ $\bar{G}^{+}_{+,0}$ $\bar{G}^{+}_{-,0}$

A multiplet containing 16 states (a long multiplet, non-BPS)

$$\phi : \ h \neq 0, \ \ \bar{h} = 0$$

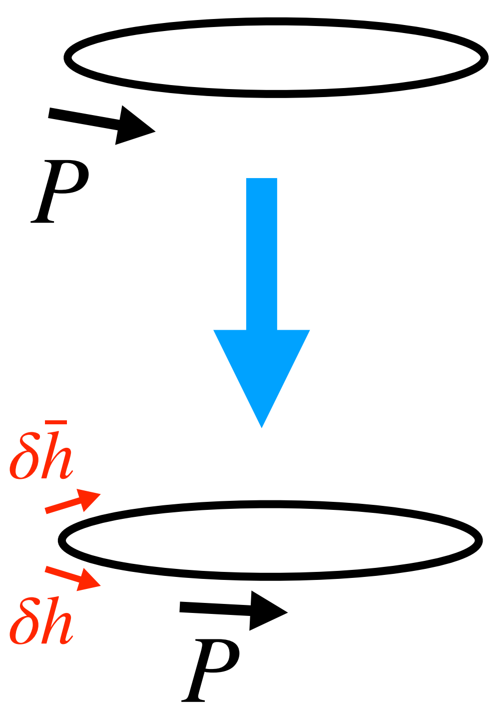


$$G^+_{+,0} \quad G^+_{-,0} \quad \bar{G}^+_{+,0} \quad \bar{G}^+_{-,0}$$

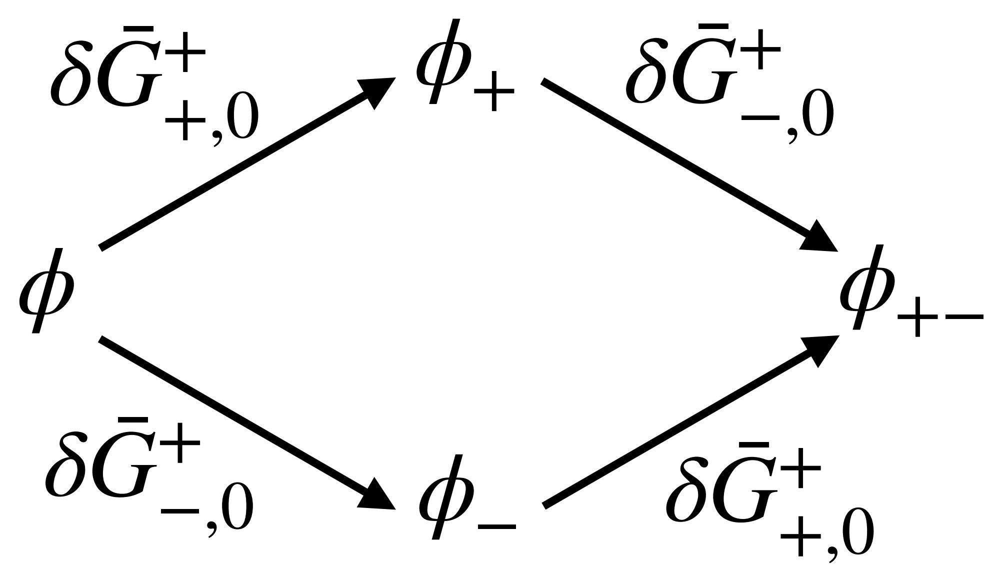
A multiplet containing 4 states (a short multiplet, BPS)

Away from the orbifold point (adding interaction)

$$\phi : \ h \neq 0, \ \ \bar{h} = 0$$



4 short multiplets must join into a long multiplet



$$h \neq 0, \ \ \bar{h} \neq 0 \qquad\qquad h - \bar{h} \in Z$$

Lifting problem

What is $\delta\bar{G}^{\alpha}_{\dot{A},0}$?

Which 4 states join into a long multiplet and lift?

What is the value of the lift δh?

Which states are unlifted?

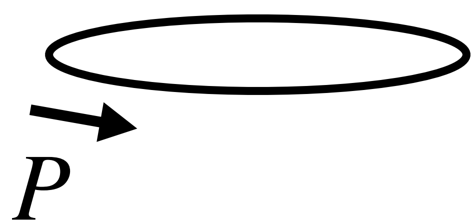
Superstrata are BPS states in supergravity. Their CFT dual should be BPS (unlifted) in the limit $\lambda \to 0$.

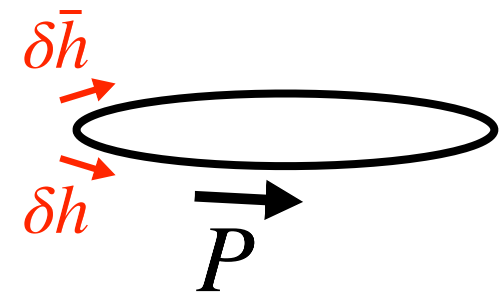
The Gava-Narain method

The Gava-Narain Method

(Gava and Narain, 2002, BG and Mathur, 2019)

'level-n' states $(h,\bar h)=(n,0)$ $\xrightarrow{\text{interaction}}$ $(h,\bar h)=(n+\epsilon,\epsilon)$





$\epsilon\geq 0$ $\longrightarrow$ lifting starts at $O(\lambda^2)$

4 short multiplets join into a long multiplet by

$$\delta\bar G^{\alpha}_{\dot A,0}=\bar G^{\bar\alpha(P)}_{\dot A,0}=\pi\mathcal{P}G^{+}_{\dot A,-\frac{1}{2}}\sigma^{-\bar\alpha}\mathcal{P}$$

$\mathcal{P}$ projects states into level (n,0)

twist operator

The energy is lifted by $E^{(2)} = \delta h + \delta \bar{h}$

Only level (n,0) contribute

$$\epsilon_{\dot{A}\dot{B}}\epsilon^{\bar{\alpha}\bar{\beta}}E^{(2)}_{ba} = 2\lambda^2\left\langle O^{(0)}_b\right|\left\{\bar{G}^{\bar{\alpha}(P)}_{\dot{A},0}, \bar{G}^{\bar{\beta}(P)}_{\dot{B},0}\right\}\left|O^{(0)}_a\right\rangle$$

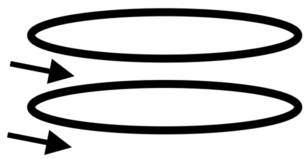
Diagonalize the matrix $E^{(2)}$. The eigenvalues are the values of the lift. The corresponding eigenstates are the lifted states.

(1) The 4 short multiplets have the same lift.

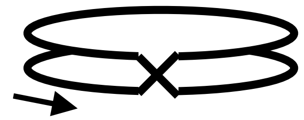
(2) Descendants have the same lift as their primary.

Lifting for total winding two

Total winding two



two singly
wound strings

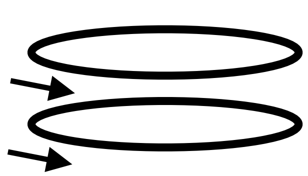


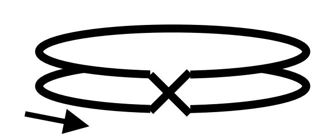
one doubly
wound string

level-1: $h = 1$
2688 states
12 primaries

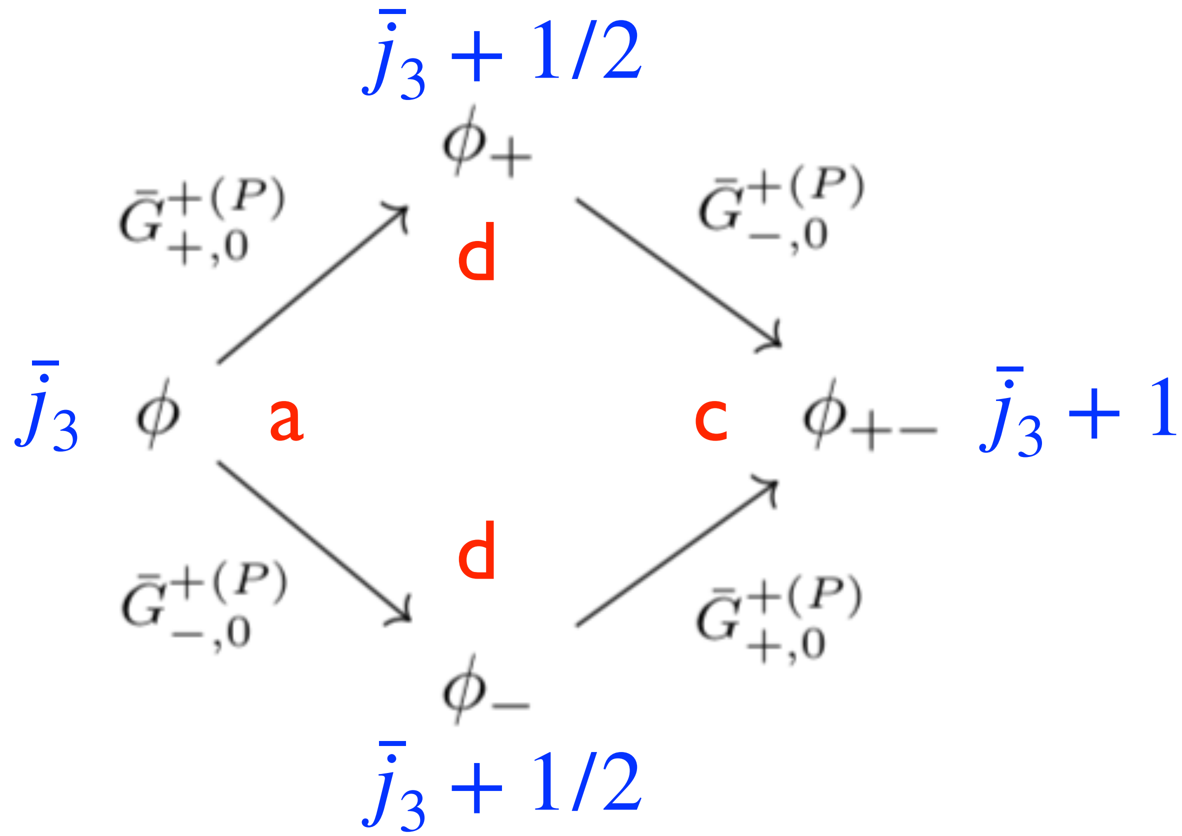
Number of primaries

level	sector	$\chi^l_{j=1,h}\bar{\chi}_{\bar{j}_3=-1}$	$\chi^l_{j=1,h}\bar{\chi}_{\bar{j}_3=-1/2}$	$\chi^l_{j=1,h}\bar{\chi}_{\bar{j}_3=0}$	sector	$\chi^l_{j=1,h}\bar{\chi}_{\bar{j}_3=-1/2}$
		ϕ	unlifted	ϕ_{+-}		ϕ_+,ϕ_-,unlifted
$h=1$	(1,1)	3	0	3	(2)	6
$h=2$	(1,1)	1	16	1	(2)	28
$h=3$	(1,1)	18	8	18	(2)	98
$h=4$	(1,1)	15	72	15	(2)	282
$h=5$	(1,1)	68	80	68	(2)	728
$h=6$	(1,1)	89	264	89	(2)	1734





Index



States allowed to be lifted by index and the twist nature of $\delta\bar{G}^{\alpha}_{\dot{A},0}$?

level	sector	$\chi^l_{j=1,h}\bar{\chi}_{\bar{j}_3=-1}$	$\chi^l_{j=1,h}\bar{\chi}_{\bar{j}_3=-1/2}$	$\chi^l_{j=1,h}\bar{\chi}_{\bar{j}_3=0}$	sector	$\chi^l_{j=1,h}\bar{\chi}_{\bar{j}_3=-1/2}$
		ϕ	unlifted	ϕ_{+-}		ϕ_+,ϕ_-,unlifted
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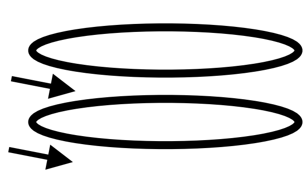
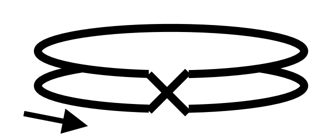


a

b

c



d

$$a = c \leq b/2$$

All states in a and c are allowed to be lifted.

We have checked that up to level-4 they are indeed lifted by $\delta \bar{G}^{\alpha}_{\dot{A},0}$.

If it is true for any level,

(1) The deformation is complicated enough to saturate the index.
(2) No extra symmetry to protect states.

Lifted states come from both the untwisted and twisted sectors.
There are unlifted states in the untwisted sector (colum b).

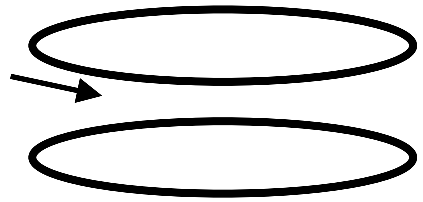
↑

corresponding BPS supergravity solutions?

Universality of lifting

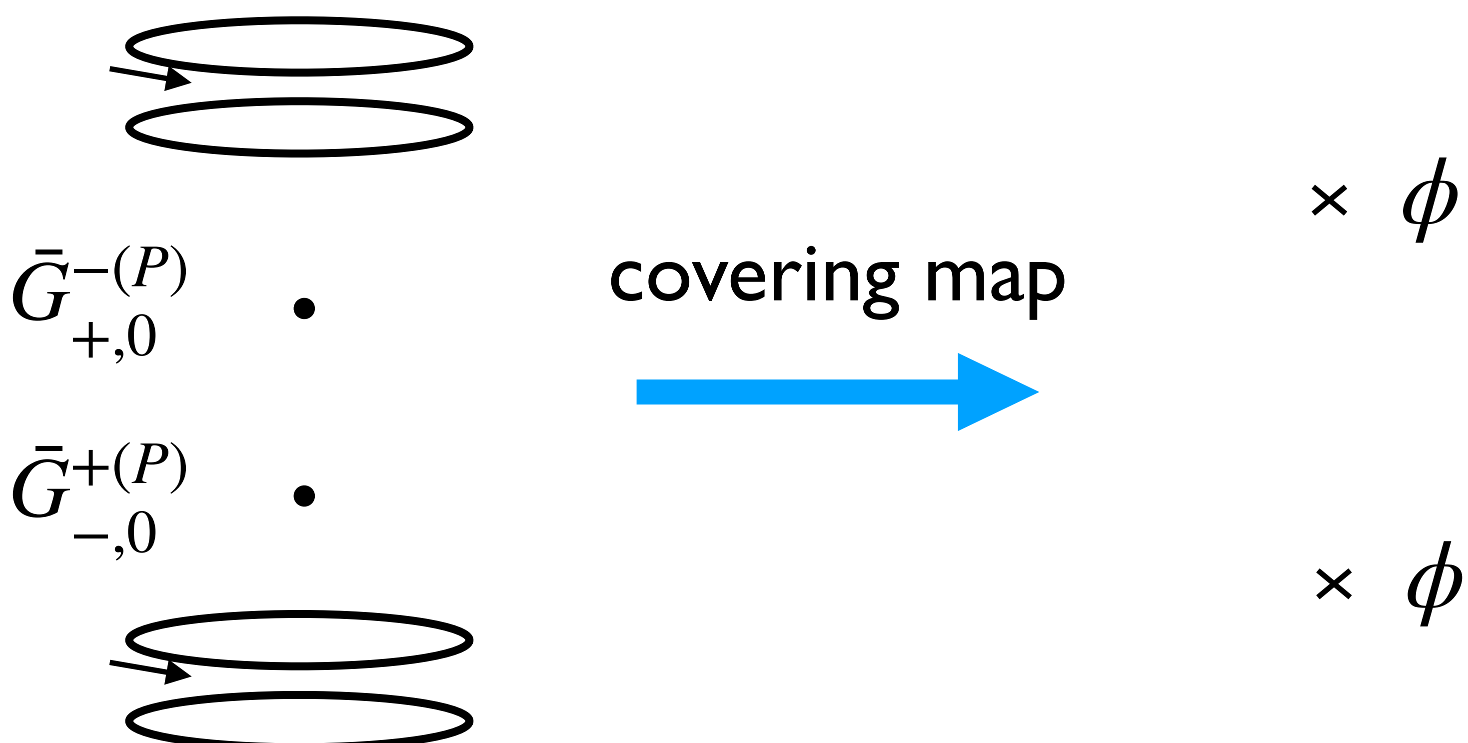
Universality

Excite only one string.



The expectation value of the lift

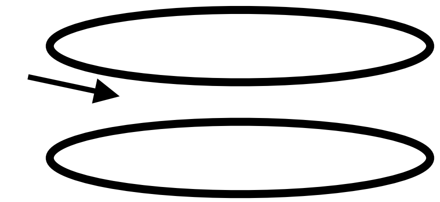
$$\langle E^{(2)} \rangle = 2\lambda^2 \langle \phi \,|\, \{ \bar{G}^{-(P)}_{+,0}, \bar{G}^{+(P)}_{-,0} \} \,|\, \phi \rangle$$



Universality

two-point function

Excite only one string with a primary.
A special class in column a.

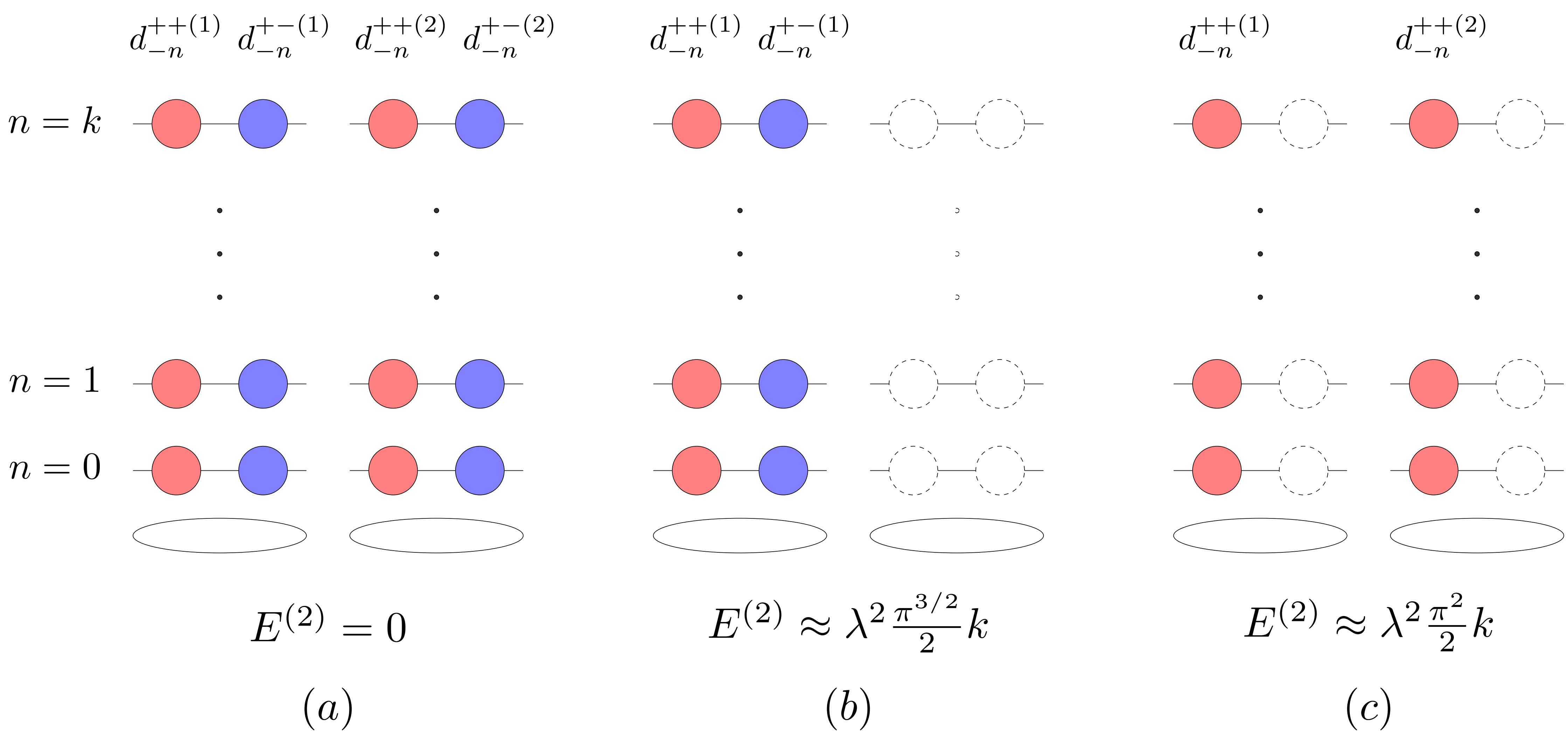


A primary of a singly wound string has $j = 0$.

$$\langle E^{(2)} \rangle = \frac{\lambda^2 \pi^2}{2^{2h-1}} \frac{\Gamma(2h)}{\Gamma[h]^2} \approx \lambda^2 \pi^{\frac{3}{2}} \sqrt{h} + O(h^{-\frac{1}{2}})$$

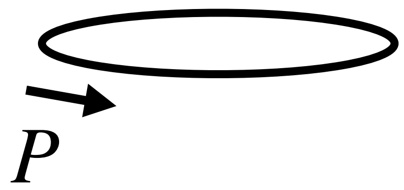
It depends only on the dimension h but not the details of the primary (universality).
They are nonzero, which supports the lifting conjecture for total winding two.

$\sqrt{h}$ behavior also found in other cases



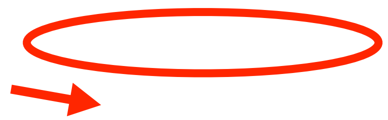
Multi-supergraviton states

generic states



$\alpha_{A\dot{A},-n}$ $d^{\alpha A}_{-n}$

supergraviton states

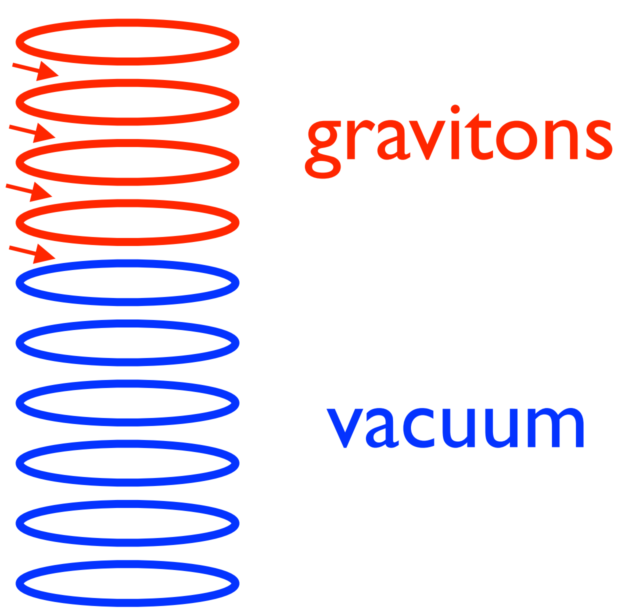


(for a review see Shigemori, superstrata)

rigid generators of the superconformal symmetry

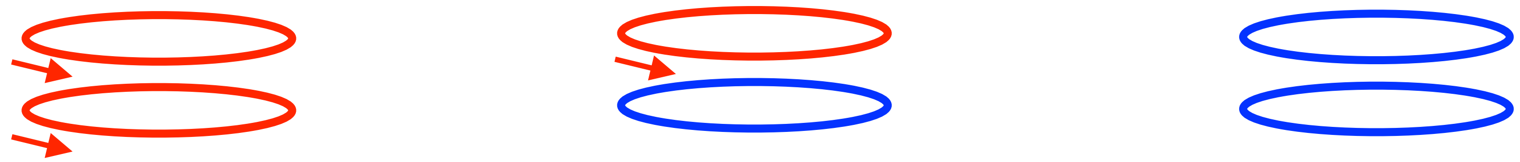
L_{-1} ~~$L_{-(n>1)}$~~ $\cdots$

Enough gravitons ⟶ black hole, fuzzball (superstratum)



Lifted or unlifted?

$\delta\bar{G}^{\alpha}_{\dot{A},0}$ can join any two component strings



Lift of multi-supergraviton states in the large N limit

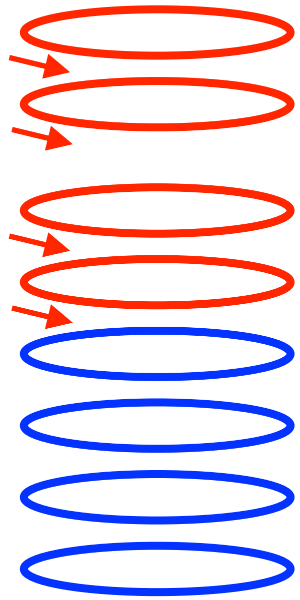
N_1 gravitons $\qquad E^{(2)} \sim \lambda^2 N_1^2 \sim \frac{g^2}{N} N_1^2 \qquad g \equiv \lambda\sqrt{N}$

$N_1 \sim O(1)$ (low lying states) $\qquad E^{(2)} \sim g^2 O(N^{-1})$ unlifted

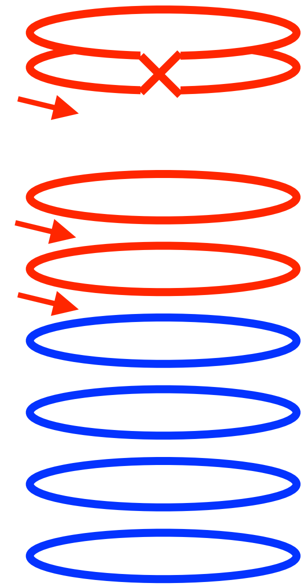
$N_1 \sim O(N)$ (superstrata) $\qquad E^{(2)} \sim g^2 O(N)$ lifted

Can we modify it to be unlifted?

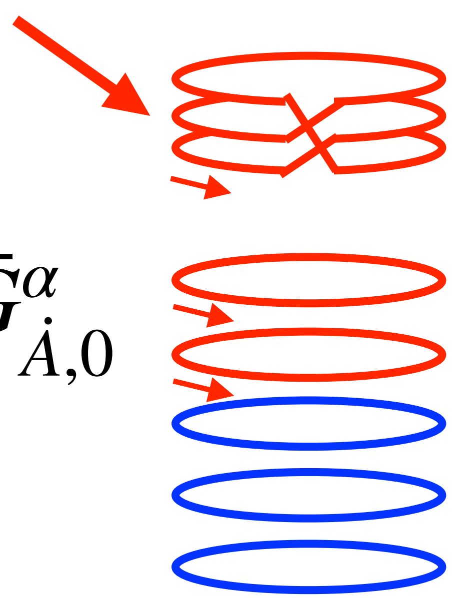
Mixing with long string states



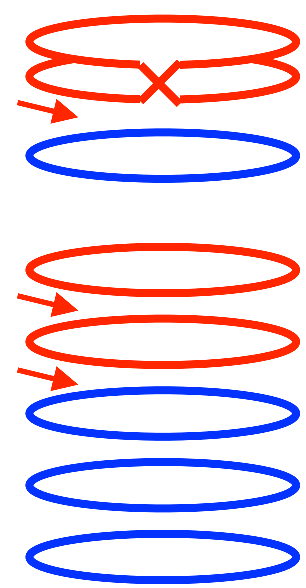
$$\delta \bar{G}^{\alpha}_{\dot{A},0} \sim N_1$$



a triply wound string

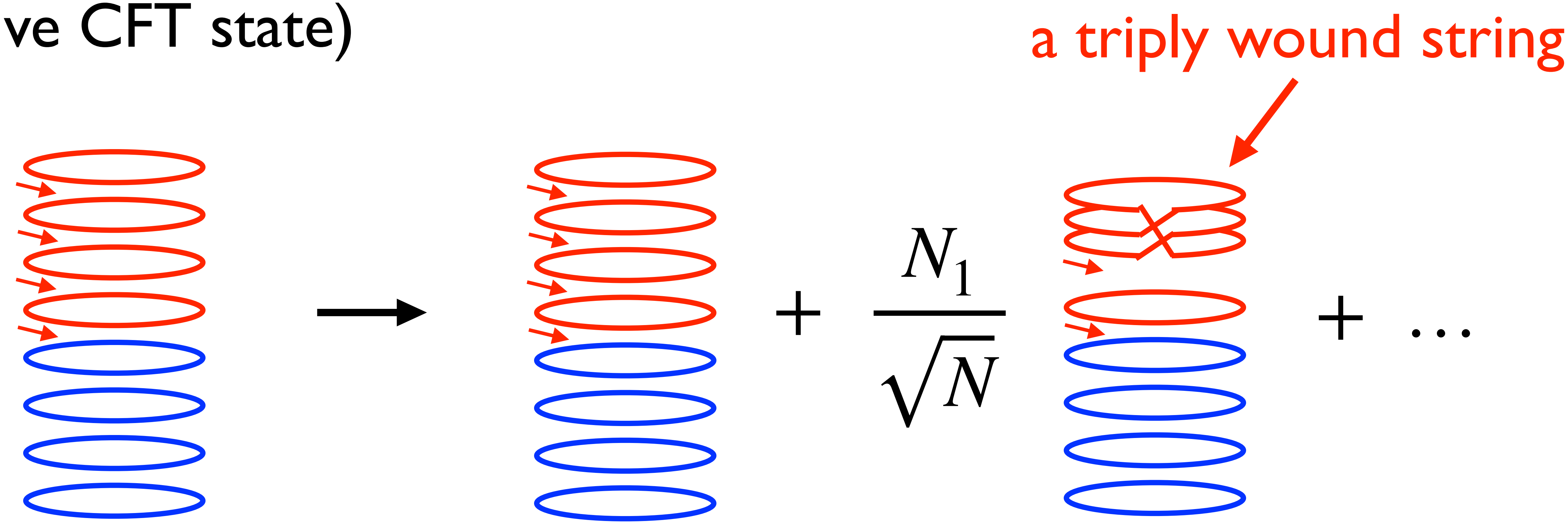


$$\delta \bar{G}^{\alpha}_{\dot{A},0} \sim \sqrt{N}$$



lifted state
(naive CFT state)

unlifted state



$N_1 \sim O(1)$ (low lying states) $O(N^{-1/2})$ correction

(Benjamin, Keller and Zadeh, 2021)

$N_1 \sim O(N)$ (superstrata) $O(N^{1/2})$ correction

The corrections do not depend on the coupling.

Taking the coupling to zero, the unlifted states should be the CFT dual of BPS black hole microstates.

Long string states are important when the number of super gravitons is large $\sim N$.
The naive CFT states are completely changed.

Thank you