Norway grants

NATIONAL SCIENCE CENTRE POLAND

Quantum information and CP measurement in $H \to \tau^+\tau^-$ at future lepton colliders

Kazuki Sakurai

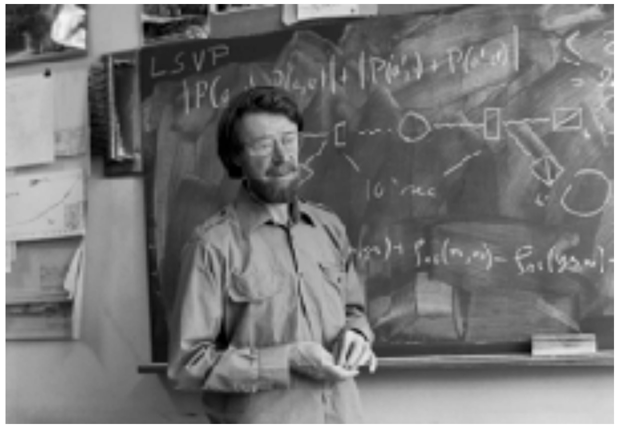
(University of Warsaw)

In collaboration with:

Mohammad Altakach, Fabio Maltoni, Kentarou Mawatari, Priyanka Lamba

2023/2/1, Seminar @ Nagoya University

Bell inequalities



- Bell inequalities have been formulated in 1964 by John Bell.
- Bell inequalities are very powerful!: derived only by assuming **locality** and **reality** of physical observables.
- Bell inequalities must be satisfied for any local-real hidden variable theories.
- QM is neither local nor real. Indeed Bell inequalities can be violated in QM.
- In 1970's-80's, the violation of Bell inequalities have been experimentally confirmed. The laws of physics cannot be both local and real. Local-real hidden variable theories were falsified.

Crauser, Horne, Shimony, Holt (1969),
Freedman and Clauser (1972),
A. Aspect et. al. (1981, 1982),
Y. H. Shih, C. O. Alley (1988),
L. K. Shalm et al. (2015) [5σ]

- In 1970's-80's, the violation of Bell inequalities have been experimentally confirmed. The laws of physics cannot be both local and real. Local-real hidden variable theories were falsified.

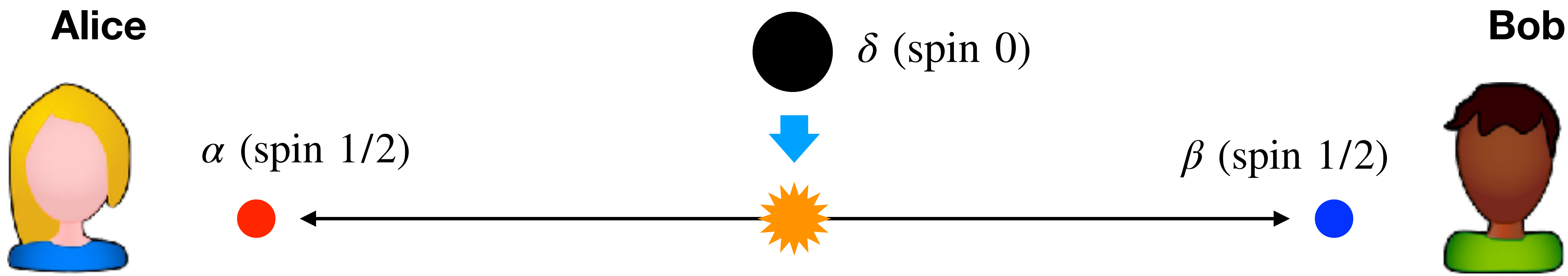
Crauser, Horne, Shimony, Holt (1969),
Freedman and Clauser (1972),
A. Aspect et. al. (1981, 1982),
Y. H. Shih, C. O. Alley (1988),
L. K. Shalm et al. (2015) [5σ]

[Einstein, Podolsky, Rosen 1935]

Reality: Physical observables (positions, momentum, etc.) have certain values regardless of the measurements (even when nobody looks).

Locality: The effect of an event at point-A cannot propagate faster than the speed of light to another point-B.

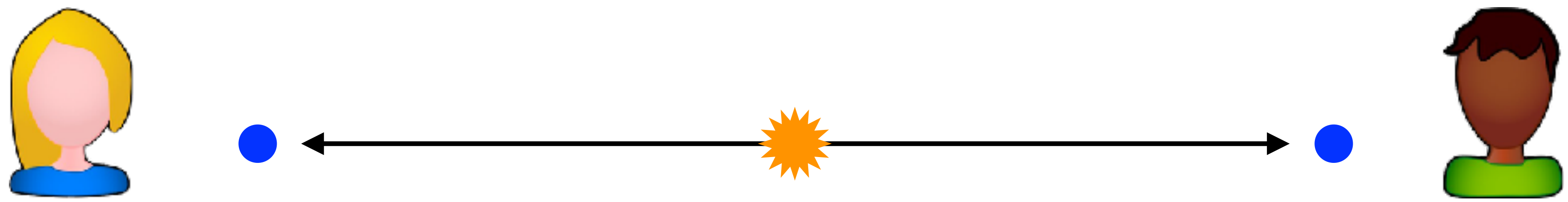
(In special relativity, the causality is broken if information travels faster than the speed of light.)



- Alice and Bob measure the spin Z-component of their particles.
- Their results look random, but 100% anti-correlated.

Alice	+	+	-	+	-	-	+	+	+	-	+	-
Bob	-	-	+	-	+	+	-	-	-	+	-	+
Alice x Bob	-	-	-	-	-	-	-	-	-	-	-	-

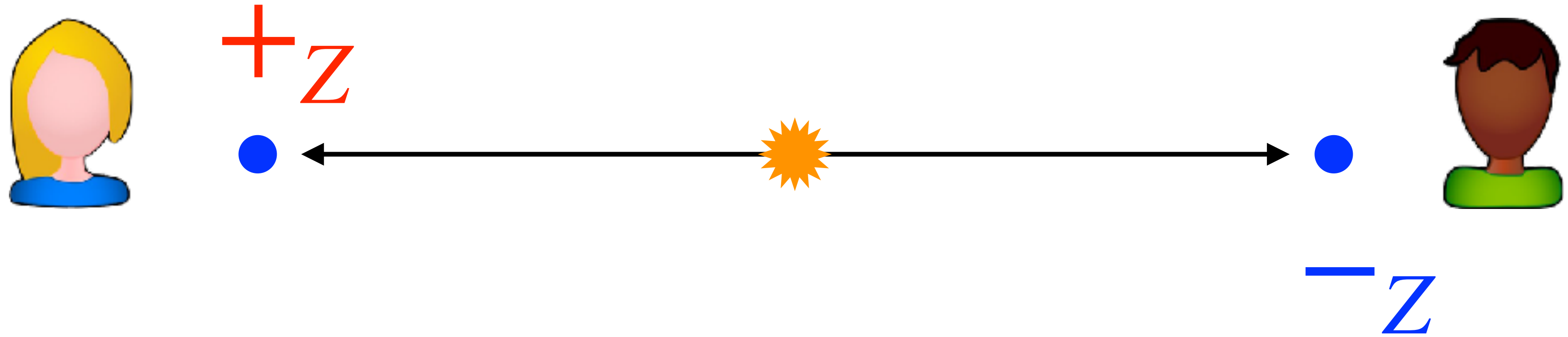
$$\langle S_z^{\alpha} \cdot S_z^{\beta} \rangle = -1$$



In QM, the state is:

$$|\Psi^{(0,0)}\rangle \doteq \frac{|+_z -_z\rangle - |-_z +_z\rangle}{\sqrt{2}}$$

S_Z of Alice's particle is in a superposition of +1 and -1. **Not real**



In QM, the state is:

$$|\Psi^{(0,0)}\rangle \doteq \frac{\boxed{|+_z -_z\rangle} - |-_z +_z\rangle}{\sqrt{2}} \Longrightarrow |+_z -_z\rangle$$

the state collapses by
the Alice's measurement

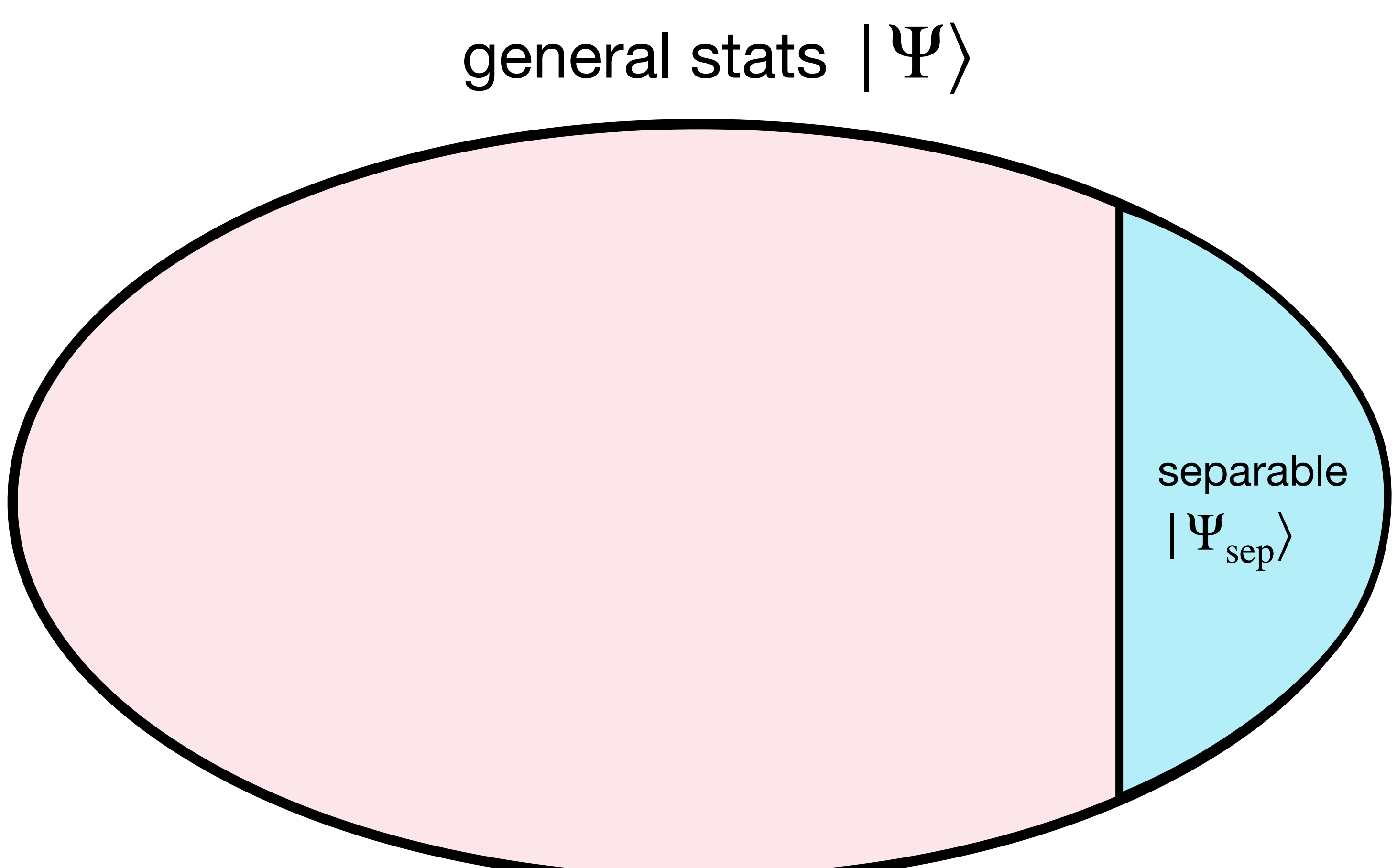
guarantees that
Bob measures -1,
100% anti-correlation

Not local

The origin of this bizarre feature is ***entanglement***.

general: $|\Psi\rangle \doteq c_{11}|+_z +_z\rangle + c_{12}|+_z -_z\rangle + c_{21}|-_z +_z\rangle + c_{22}|-_z -_z\rangle$

separable: $|\Psi_{\text{sep}}\rangle \doteq \left[c_1^\alpha|+_z\rangle + c_2^\alpha|-_z\rangle\right] \otimes \left[c_1^\beta|+_z\rangle + c_2^\beta|-_z\rangle\right]$



The origin of this bizarre feature is ***entanglement***.

general: $|\Psi\rangle \doteq c_{11}|+_z +_z\rangle + c_{12}|+_z -_z\rangle + c_{21}|-_z +_z\rangle + c_{22}|-_z -_z\rangle$

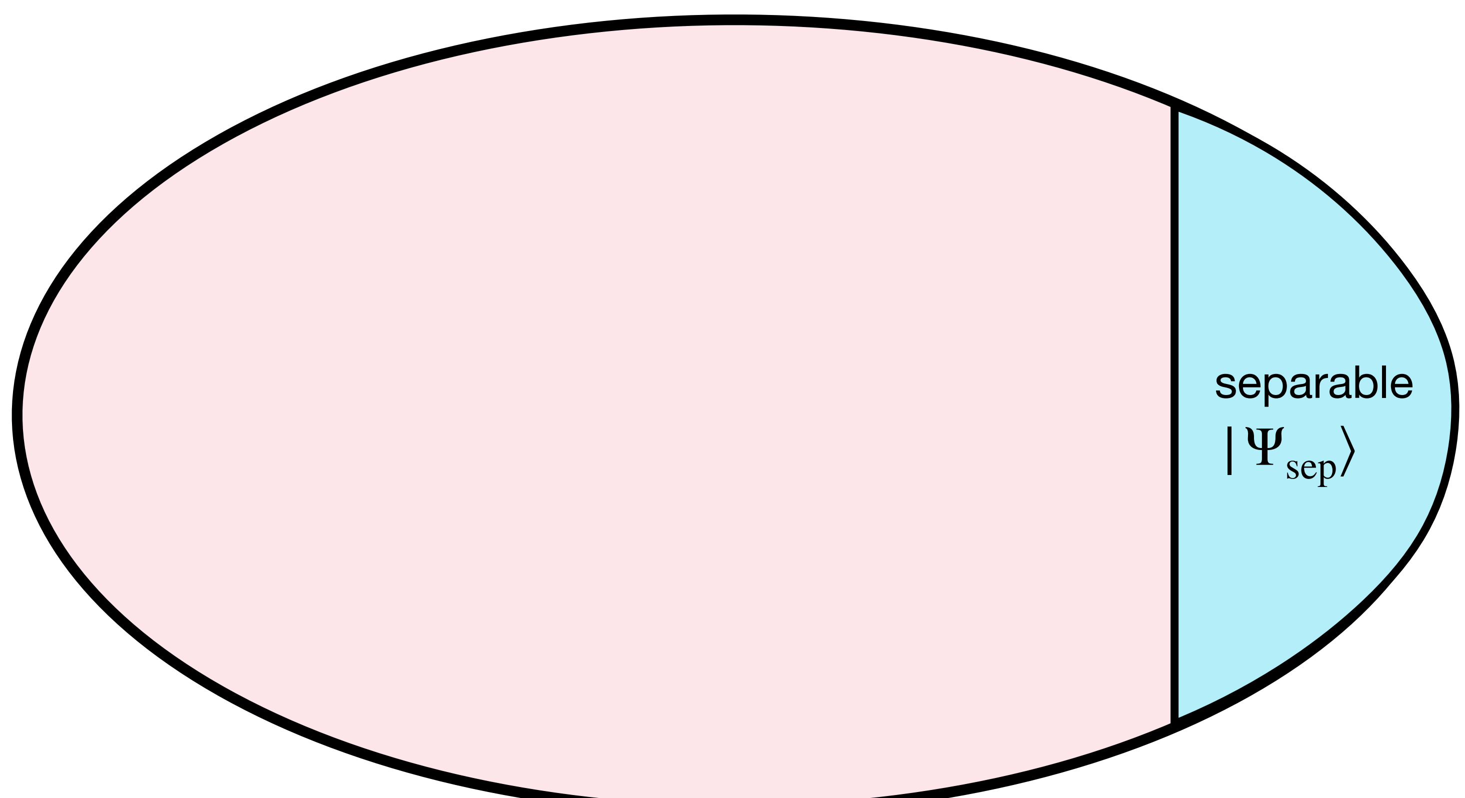
separable: $|\Psi_{\text{sep}}\rangle \doteq \boxed{\left[c_1^{\alpha}|+_z\rangle + c_2^{\alpha}|-_z\rangle\right]} \otimes \underline{\left[c_1^{\beta}|+_z\rangle + c_2^{\beta}|-_z\rangle\right]}$

Alice's measurement

Bob's local state is intact

$|+\rangle_z$

general stats $|\Psi\rangle$



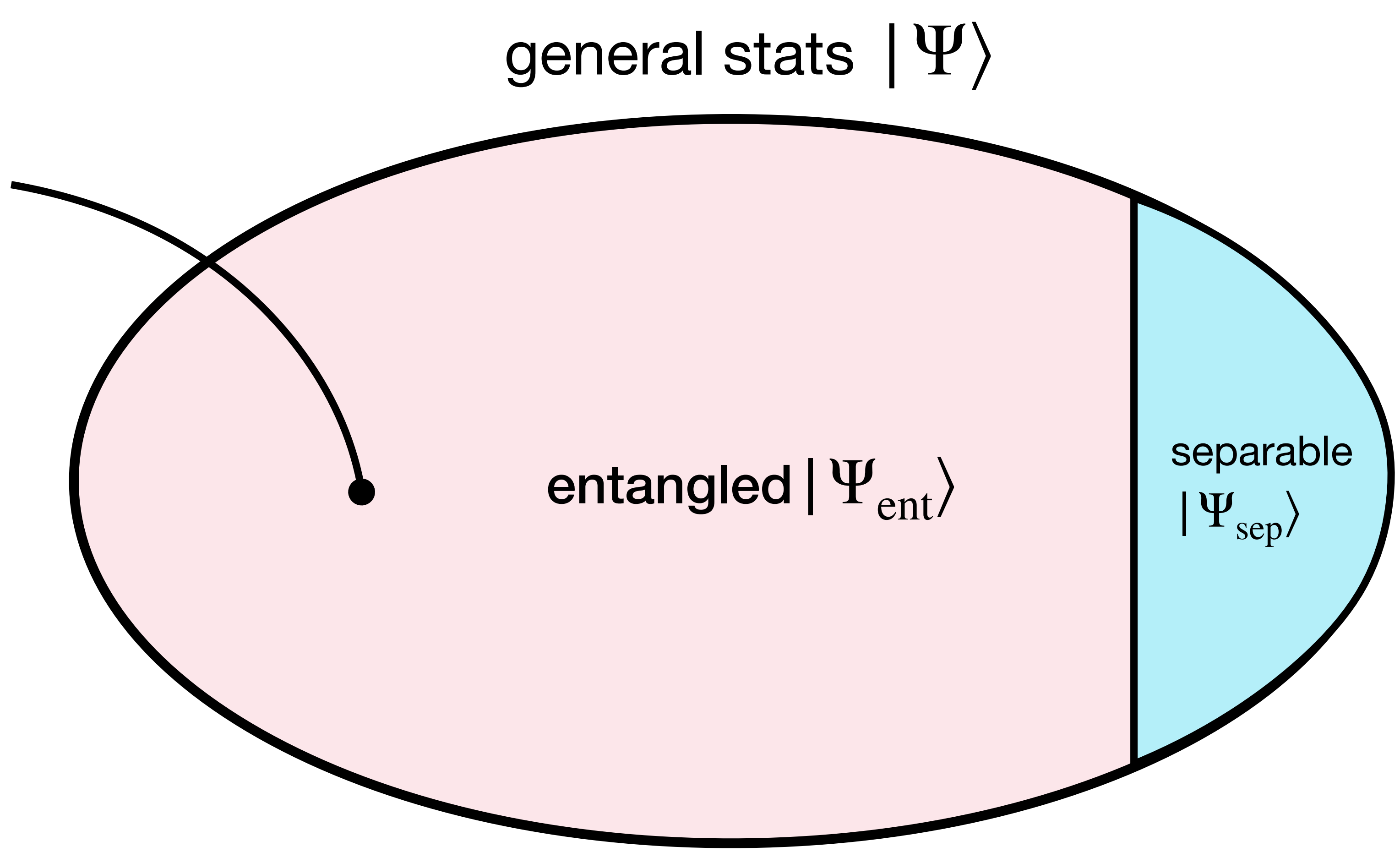
The origin of this bizarre feature is ***entanglement***.

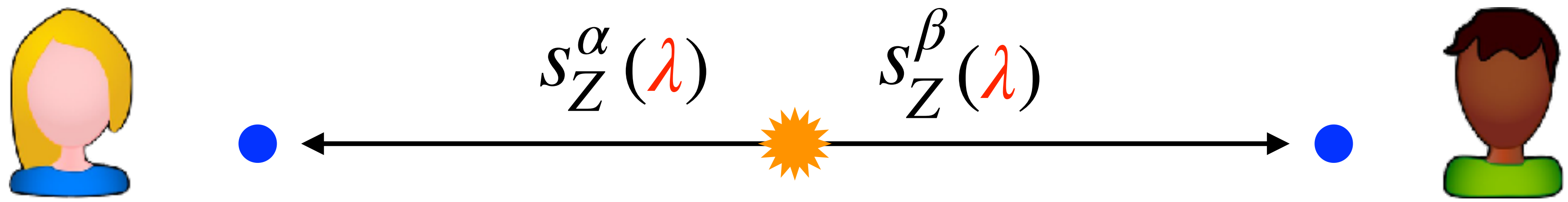
general: $|\Psi\rangle \doteq c_{11}|+_z+_z\rangle + c_{12}|+_z-_z\rangle + c_{21}|-_z+_z\rangle + c_{22}|-_z-_z\rangle$

separable: $|\Psi_{\text{sep}}\rangle \doteq \left[c_1^\alpha|+_z\rangle + c_2^\alpha|-_z\rangle\right] \otimes \left[c_1^\beta|+_z\rangle + c_2^\beta|-_z\rangle\right]$

entangled: $|\Psi_{\text{ent}}\rangle \cancel{\doteq} \left[c_1^\alpha|+\rangle_z + c_2^\alpha|-\rangle_z\right] \otimes \left[c_1^\beta|+\rangle_z + c_2^\beta|-\rangle_z\right]$

$$|\Psi^{(0,0)}\rangle \doteq \frac{|+-\rangle_z - |-+\rangle_z}{\sqrt{2}}$$



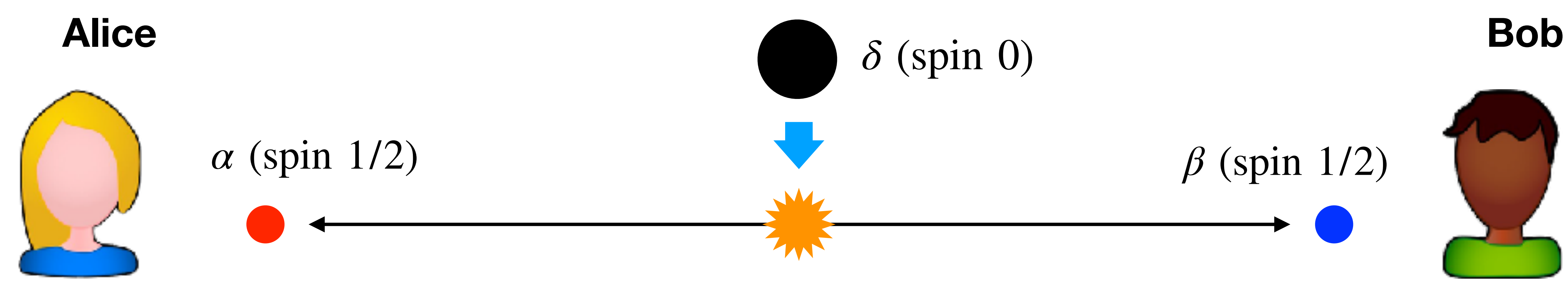


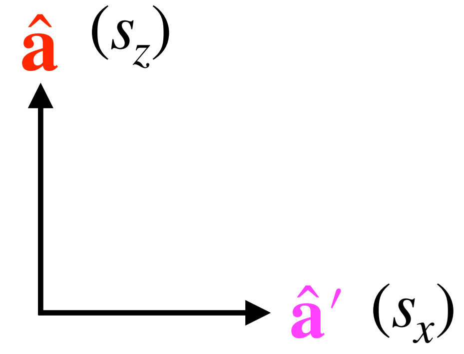
- Assuming the **reality**, Alice's result is predetermined before her measurement.
- The spin components of Bob's particle are also predetermined and not affected by Alice's measurement by the **locality** assumption.
- Without loss of generality, we can parametrise their spin components by a set of parameters λ, which appears with the probability $P(\lambda)$ in each decay.

$$P(\lambda) \geq 0, \qquad \sum_\lambda P(\lambda) = 1$$

- The spin correlation is given by

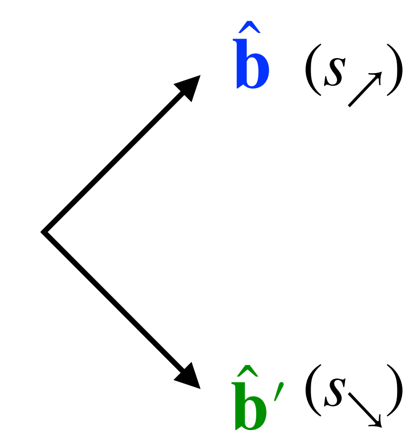
$$\langle s_Z^\alpha \cdot s_Z^\beta \rangle \;=\; \sum_\lambda P(\lambda)\, s_Z^\alpha(\lambda)\, s_Z^\beta(\lambda) = -1$$



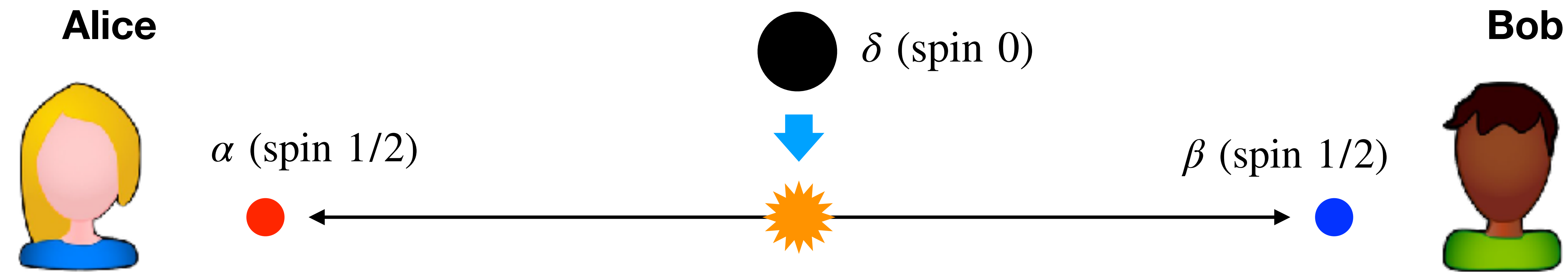


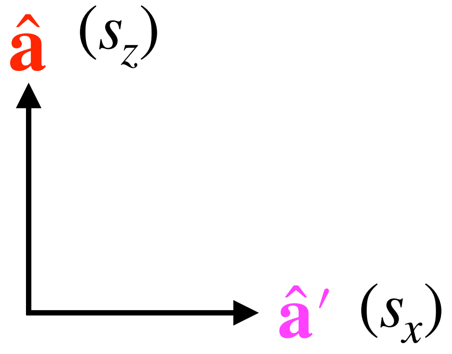
The experiment consists of 4 sessions:

1) Alice and Bob measure s_a^α and s_b^β, respectively. Repeat the measurement many times and calculate $\langle s_a \cdot s_b \rangle$.
2) Repeat (1) for a and b'.
3) Repeat (1) for a' and b.
4) Repeat (1) for a' and b'.



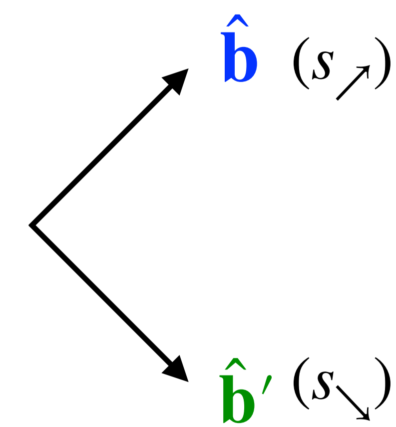
Finally, we calculate: $$R_{\text{CHSH}} \equiv \frac{1}{2} \left| \langle s_a s_b \rangle - \langle s_a s_{b'} \rangle + \langle s_{a'} s_b \rangle + \langle s_{a'} s_{b'} \rangle \right|$$





The experiment consists of 4 sessions:

1) Alice and Bob measure s_a^α and s_b^β, respectively. Repeat the measurement many times and calculate $\langle s_a \cdot s_b \rangle$.
2) Repeat (1) for a and b'.
3) Repeat (1) for a' and b.
4) Repeat (1) for a' and b'.



Finally, we calculate: $$R_{\mathrm{CHSH}} \equiv \frac{1}{2} \left| \langle s_a s_b \rangle - \langle s_a s_{b'} \rangle + \langle s_{a'} s_b \rangle + \langle s_{a'} s_{b'} \rangle \right|$$

Local and **Real** $\Rightarrow$ $\boxed{R_{\mathrm{CHSH}} \le 1}$

Bell (CHSH) inequality

[Clauser, Horne, Shimony, Holt, 1969]

Let's derive $\boxed{R_{\text{CHSH}} \equiv \frac{1}{2}\left|\langle ab\rangle - \langle ab'\rangle + \langle a'b\rangle + \langle a'b'\rangle\right| \leq 1}$

$$\left|\langle ab\rangle - \langle ab'\rangle\right| = \left|\sum_{\lambda} abP - \sum_{\lambda} ab'P\right|$$

$$\langle s_a^{\alpha} \cdot s_b^{\beta}\rangle = \langle ab\rangle = \sum_{\lambda} a(\lambda)\, b(\lambda)\, P(\lambda) = \sum_{\lambda} ab\, P$$

Let's derive $\boxed{R_{\mathrm{CHSH}} \equiv \frac{1}{2}\left|\langle ab\rangle - \langle ab'\rangle + \langle a'b\rangle + \langle a'b'\rangle\right| \leq 1}$

$$\left|\langle ab\rangle - \langle ab'\rangle\right| = \left|\sum_\lambda abP - \sum_\lambda ab'P\right|$$

$$\pm\, aba'b'P - (\pm\, aba'b'P) = 0$$

$$= \left|\sum_\lambda \left[ab(1 \pm a'b')P - ab'(1 \pm a'b)P\right]\right|$$

$$\langle s_a^\alpha \cdot s_b^\beta\rangle = \langle ab\rangle = \sum_\lambda a(\lambda)\, b(\lambda)\, P(\lambda) = \sum_\lambda ab\, P$$

Let's derive

$$\boxed{R_{\mathrm{CHSH}} \equiv \frac{1}{2}\left|\langle ab\rangle - \langle ab'\rangle + \langle a'b\rangle + \langle a'b'\rangle\right| \leq 1}$$

$$\left|\langle ab\rangle - \langle ab'\rangle\right| = \left|\sum_\lambda abP - \sum_\lambda ab'P\right|$$

$$\pm aba'b'P - (\pm aba'b'P) = 0$$

$$= \left|\sum_\lambda \left[ab(1 \pm a'b')P - ab'(1 \pm a'b)P\right]\right|$$

$$\leq \sum_\lambda \left[|ab|\,|1 \pm a'b'|\,P + |ab'|\,|1 \pm a'b|\,P\right]$$

$$\langle s_a^\alpha \cdot s_b^\beta\rangle = \langle ab\rangle = \sum_\lambda a(\lambda)\,b(\lambda)\,P(\lambda) = \sum_\lambda ab\,P$$

Let's derive $\boxed{R_{\text{CHSH}} \equiv \frac{1}{2}\left|\langle ab\rangle - \langle ab'\rangle + \langle a'b\rangle + \langle a'b'\rangle\right| \leq 1}$

$$\left|\langle ab\rangle - \langle ab'\rangle\right| = \left|\sum_\lambda abP - \sum_\lambda ab'P\right|$$

$$\pm\, aba'b'P - (\pm\, aba'b'P) = 0$$

$$= \left|\sum_\lambda \left[ab(1\pm a'b')P - ab'(1\pm a'b)P\right]\right|$$

$$\leq \sum_\lambda \left[\left|ab\right| \left|1\pm a'b'\right| P + \left|ab'\right| \left|1 \pm a'b\right| P\right]$$

$$|ab| = |ab'| = 1$$

$$|1\pm a'b'|, |1\pm a'b| \geq 0$$

$$= \sum_\lambda \left[\left(1\pm a'b'\right) P + \left(1\pm a'b\right) P\right]$$

$$= 2 \pm \left(\langle a'b'\rangle + \langle a'b\rangle\right)$$

$$\langle s_a^\alpha \cdot s_b^\beta\rangle = \langle ab\rangle = \sum_\lambda a(\lambda)\, b(\lambda)\, P(\lambda) = \sum_\lambda ab\, P$$

Let's derive

$$\boxed{R_{\text{CHSH}} \equiv \frac{1}{2}\left|\langle ab\rangle - \langle ab'\rangle + \langle a'b\rangle + \langle a'b'\rangle\right| \le 1}$$

$$\left|\langle ab\rangle - \langle ab'\rangle\right| = \left|\sum_\lambda abP - \sum_\lambda ab'P\right|$$

$$\pm aba'b'P - (\pm aba'b'P) = 0$$

$$= \left|\sum_\lambda \left[ab(1\pm a'b')P - ab'(1\pm a'b)P\right]\right|$$

$$\le \sum_\lambda \left[\left|ab\right|\,\left|1\pm a'b'\right|P + \left|ab'\right|\,\left|1\pm a'b\right|P\right]$$

$$|ab| = |ab'| = 1$$

$$|1\pm a'b'|, |1\pm a'b| \ge 0$$

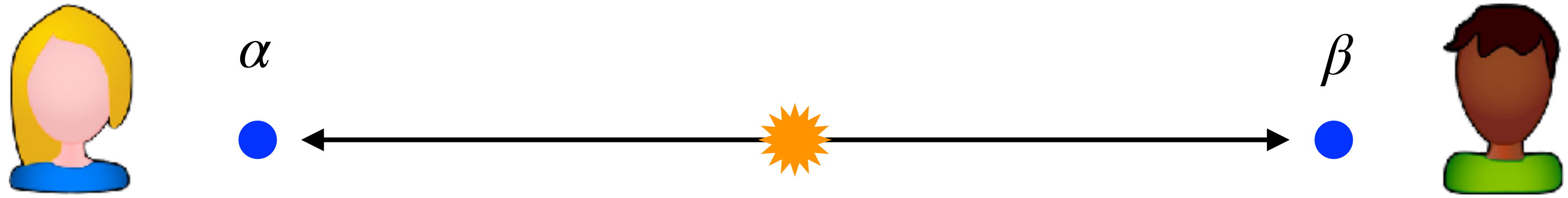
$$= \sum_\lambda \left[\left(1\pm a'b'\right)P + \left(1\pm a'b\right)P\right]$$

$$= 2 \pm \left(\langle a'b'\rangle + \langle a'b\rangle\right)$$

$$\tilde{R}_{\text{CHSH}} \equiv \frac{1}{2}\left(\left|\langle ab\rangle - \langle ab'\rangle\right| + \left|\langle a'b\rangle + \langle a'b'\rangle\right|\right) \le 1$$

$$\max_{(a,b,a',b')} R_{\text{CHSH}} = \max_{(a,b,a',b')} \tilde{R}_{\text{CHSH}}$$

$$\boxed{R_{\text{CHSH}} \le 1}$$

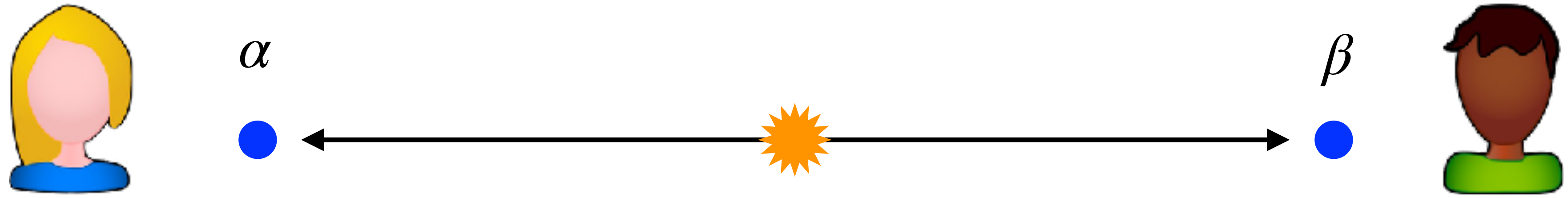


In QM, the state is: $|\Psi^{(0,0)}\rangle \doteq \dfrac{|+_z -_z\rangle - |-_z +_z\rangle}{\sqrt{2}}$

The spin correlation is: $\langle s_a s_b \rangle = \langle \Psi^{(0,0)} | (\mathbf{s}^\alpha \cdot \hat{\mathbf{a}})(\mathbf{s}^\beta \cdot \hat{\mathbf{b}}) | \Psi^{(0,0)} \rangle = (\hat{\mathbf{a}} \cdot \hat{\mathbf{b}})$

$$R_{\text{CHSH}} = \frac{1}{2} \left| \langle s_a s_b \rangle - \langle s_a s_{b'} \rangle + \langle s_{a'} s_b \rangle + \langle s_{a'} s_{b'} \rangle \right|$$

$$= \frac{1}{2} \left| (\hat{\mathbf{a}} \cdot \hat{\mathbf{b}}) - (\hat{\mathbf{a}} \cdot \hat{\mathbf{b}}') + (\hat{\mathbf{a}}' \cdot \hat{\mathbf{b}}) + (\hat{\mathbf{a}}' \cdot \hat{\mathbf{b}}') \right|$$

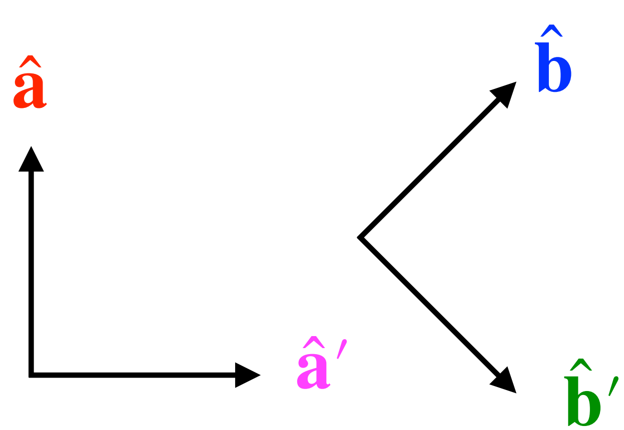


In QM, the state is: $\quad |\Psi^{(0,0)}\rangle \doteq \dfrac{|+_z -_z\rangle - |-_z +_z\rangle}{\sqrt{2}}$

The spin correlation is: $\quad \langle s_a s_b \rangle = \langle \Psi^{(0,0)} | (\mathbf{s}^\alpha \cdot \hat{\mathbf{a}})(\mathbf{s}^\beta \cdot \hat{\mathbf{b}}) | \Psi^{(0,0)} \rangle = (\hat{\mathbf{a}} \cdot \hat{\mathbf{b}})$

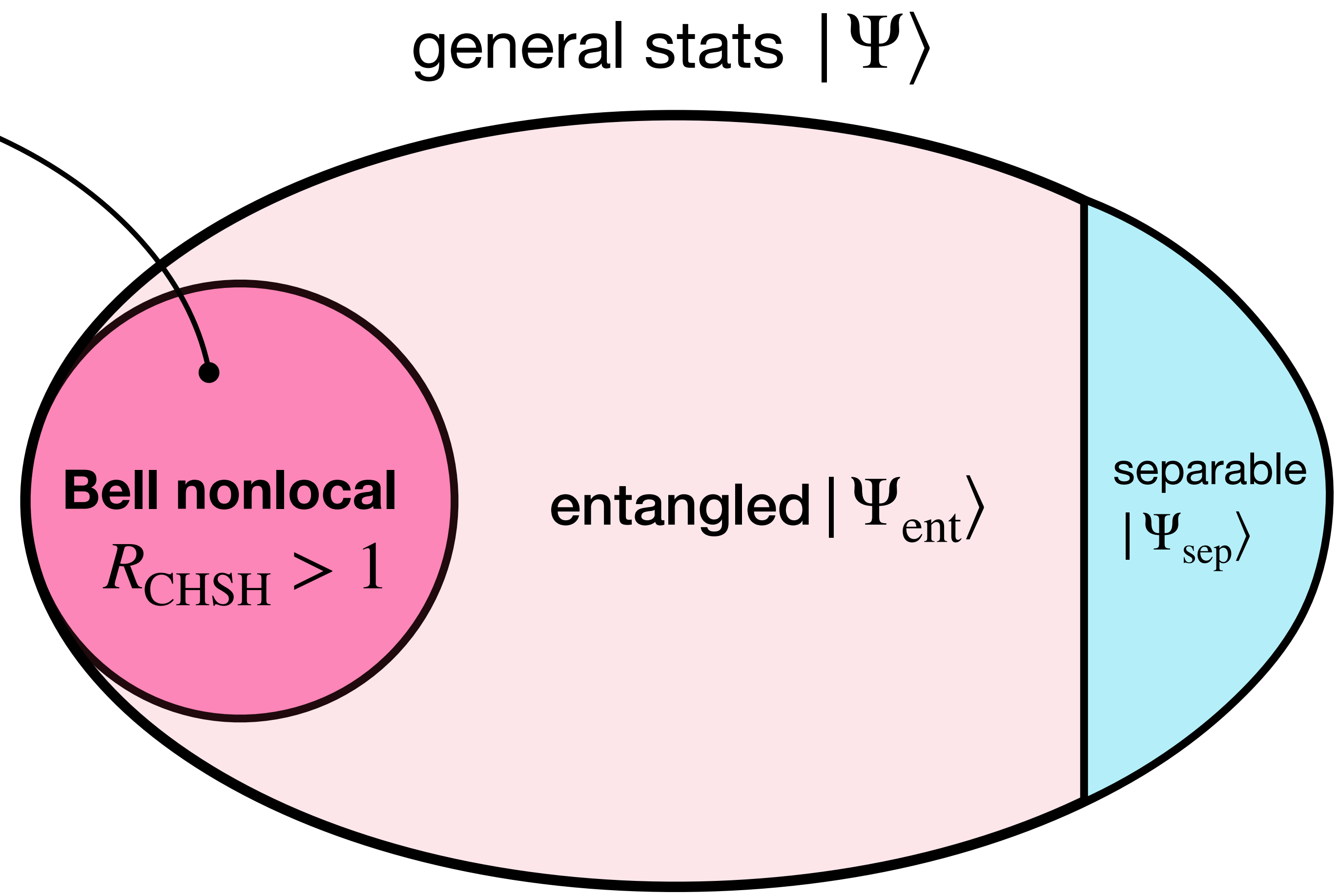
$$R_{\text{CHSH}} = \frac{1}{2} \left| \langle s_a s_b \rangle - \langle s_a s_{b'} \rangle + \langle s_{a'} s_b \rangle + \langle s_{a'} s_{b'} \rangle \right|$$

$$= \frac{1}{2} \left| \underbrace{(\hat{\mathbf{a}} \cdot \hat{\mathbf{b}})}_{\frac{1}{\sqrt{2}}} - \underbrace{(\hat{\mathbf{a}} \cdot \hat{\mathbf{b}}')}_{-\frac{1}{\sqrt{2}}} + \underbrace{(\hat{\mathbf{a}}' \cdot \hat{\mathbf{b}})}_{\frac{1}{\sqrt{2}}} + \underbrace{(\hat{\mathbf{a}}' \cdot \hat{\mathbf{b}}')}_{\frac{1}{\sqrt{2}}} \right| \quad = \sqrt{2}$$



Bell inequality violated !!

$$|\Psi^{(0,0)}\rangle \doteq \frac{|+-\rangle_z - |-+\rangle_z}{\sqrt{2}}$$



$$R_{\text{CHSH}} \leq \begin{cases} 1 & \text{(HV theories)} \\ \sqrt{2} & \text{(QM)} \end{cases}$$

✤ **Violation of Bell inequalities has been observed in low energy experiments:**

- **Entangled photon pairs** (from decays of Calcium atoms)

  Clauser, Horne, Shimony, Holt (1969), Freedman and Clauser (1972), A. Aspect et. al. (1981, 1982), Y. H. Shih, C. O. Alley (1988), L. K. Shalm et al. (2015) [5σ]

- **Entangled proton pairs** (from decays of ^{2}He)

  M. M. Lamehi-Rachti, W. Mitting (1972), H. Sakai (2006)

- $K^0\overline{K^0}, B^0\overline{B^0}$ **flavour oscillation** CPLEAR (1999), Belle (2004, 2007)

Bell inequality and entanglement have not been tested at high energy regime E ~ TeV

Can we test Bell inequality and entanglement at high energy colliders?

- Entanglement in $pp \to t\bar{t}$ @ LHC Y. Afik, J. R. M. de Nova (2020)

- Bell inequality test in $pp \to t\bar{t}$ @ LHC M. Fabbrichesi, R. Floreanini, G. Panizzo (2021); C. Severi, C. D. Boschi, F. Maltoni, M. Sioli (2021); J. A. Aguilar-Saavedra, J. A. Casas (2022)

- Bell inequality test in $H \to WW^*$ @ LHC A. J. Barr (2021)

- Quantum property test in $H \to \tau^+\tau^-$ @ high energy e^+e^- colliders ← **this talk**

Density operator

probability of having $|\Psi_1\rangle$

- For a statistical ensemble $\{\{p_1 : |\Psi_1\rangle\},\ \{p_2 : |\Psi_2\rangle\},\ \{p_3 : |\Psi_3\rangle\},\ \cdots\}$, we define the **density operator/matrix**

$$\hat{\rho} \equiv \sum_k p_k |\Psi_k\rangle\langle\Psi_k| \qquad \rho_{ab} \equiv \langle e_a|\hat{\rho}|e_b\rangle$$

$$0 \leq p_k \leq 1$$

$$\sum_k p_k = 1$$

$$\langle e_a|e_b\rangle = \delta_{ab}$$

- Probability and expectation values:

$$\hat{A}\,|a\rangle = a\,|a\rangle$$

$$P(a\,|\,\hat{A},\hat{\rho}) = \langle a\,|\,\rho\,|\,a\rangle$$ Probability for outcome a when $\hat{A}$ is measured on the state $\hat{\rho}$

$$\langle\hat{A}\rangle_\rho = \mathrm{Tr}\left[\hat{A}\hat{\rho}\right]$$ Expectation value for $\hat{A}$ on the state $\hat{\rho}$

Spin 1/2 biparticle system

- The spin system of α and β particles has 4 independent bases:

$$\left(\, |e_1\rangle, \, |e_2\rangle, \, |e_3\rangle, \, |e_4\rangle \, \right) \; = \; \left(\, |++\rangle, \, |+-\rangle, \, |-+\rangle, \, |--\rangle \, \right)$$

- ==> ρ_{ab} is a 4 x 4 matrix (hermitian, Tr=1, non-negative).

  It can be expanded as

$$\rho = \frac{1}{4}\left(\mathbf{1}_4 + B_i \cdot \sigma_i \otimes \mathbf{1} + \overline{B}_i \cdot \mathbf{1} \otimes \sigma_i + \underset{\uparrow \text{ 3 x 3 matrix}}{C_{ij}} \cdot \sigma_i \otimes \sigma_j\right) \qquad B_i, \overline{B}_i, C_{ij} \in \mathbb{R}$$

- For the spin operators $\hat{s}^\alpha$ and $\hat{s}^\beta$,

$$\langle \hat{s}_i^\alpha \rangle = \mathrm{Tr}\left[\hat{s}_i^\alpha \hat{\rho}\right] = B_i \qquad \langle \hat{s}_i^\beta \rangle = \mathrm{Tr}\left[\hat{s}_i^\beta \hat{\rho}\right] = \overline{B}_i$$

spin-spin correlation

$$\boxed{\langle \hat{s}_i^\alpha \hat{s}_j^\beta \rangle = \mathrm{Tr}\left[\hat{s}_i^\alpha \hat{s}_j^\beta \hat{\rho}\right] = C_{ij}}$$

$H \to \tau^+\tau^-$

$$\mathscr{L}_{\text{int}} = -\frac{m_\tau}{v_{\text{SM}}}\kappa H\,\bar{\psi}_\tau(\cos\delta + i\gamma_5\sin\delta)\,\psi_\tau$$

SM: $(\kappa, \delta) = (1,0)$

$H \to \tau^+\tau^-$

$$\mathscr{L}_{\rm int} = -\frac{m_\tau}{v_{\rm SM}}\kappa H\,\bar{\psi}_\tau(\cos\delta + i\gamma_5\sin\delta)\,\psi_\tau$$

SM: $(\kappa, \delta) = (1,0)$

$$\rho_{mn,\bar{m}\bar{n}} = \frac{\mathcal{M}^{*n\bar{n}}\mathcal{M}^{m\bar{m}}}{\sum_{m\bar{m}}|\mathcal{M}^{m\bar{m}}|^2}$$

$$\mathcal{M}^{m\bar{m}} = c\,\bar{u}^m(p)(\cos\delta + i\gamma_5\sin\delta)v^{\bar{m}}(\bar{p})$$

$$\rho_{mn,\bar{m}\bar{n}} = \frac{1}{2}\begin{pmatrix} 0 & 0 & 0 & 0 \\ 0 & 1 & e^{-i2\delta} & 0 \\ 0 & e^{i2\delta} & 1 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix}$$

$H \to \tau^+\tau^-$

$$\mathscr{L}_{\text{int}} = -\frac{m_\tau}{v_{\text{SM}}}\kappa H\,\bar{\psi}_\tau(\cos\delta + i\gamma_5 \sin\delta)\,\psi_\tau$$

SM: $(\kappa, \delta) = (1,0)$

$$\rho_{mn,\bar{m}\bar{n}} = \frac{\mathcal{M}^{*n\bar{n}}\mathcal{M}^{m\bar{m}}}{\sum_{m\bar{m}} |\mathcal{M}^{m\bar{m}}|^2}$$

$$\mathcal{M}^{m\bar{m}} = c\,\bar{u}^m(p)(\cos\delta + i\gamma_5 \sin\delta)v^{\bar{m}}(\bar{p})$$

$$\rho_{mn,\bar{m}\bar{n}} = \frac{1}{2}\begin{pmatrix} 0 & 0 & 0 & 0 \\ 0 & 1 & e^{-i2\delta} & 0 \\ 0 & e^{i2\delta} & 1 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix}$$

$$\rho = \frac{1}{4}\left(\mathbf{1}_4 + B_i \cdot \sigma_i \otimes \mathbf{1} \right.$$
$$\left. \overline{B}_i \cdot \mathbf{1} \otimes \sigma_i + C_{ij} \cdot \sigma_i \otimes \sigma_j\right)$$

$$B_i = \overline{B}_i = 0$$

$$C_{ij} = \begin{pmatrix} \cos 2\delta & \sin 2\delta & 0 \\ -\sin 2\delta & \cos 2\delta & 0 \\ 0 & 0 & -1 \end{pmatrix}$$

$H \to \tau^+\tau^-$

$$\mathscr{L}_{\text{int}} = -\frac{m_\tau}{v_{\text{SM}}}\kappa H \bar{\psi}_\tau(\cos\delta + i\gamma_5 \sin\delta)\psi_\tau$$

SM: $(\kappa, \delta) = (1,0)$

$$\rho_{mn,\bar{m}\bar{n}} = \frac{\mathcal{M}^{*n\bar{n}}\mathcal{M}^{m\bar{m}}}{\sum_{m\bar{m}} |\mathcal{M}^{m\bar{m}}|^2}$$

$$\mathcal{M}^{m\bar{m}} = c\,\bar{u}^m(p)(\cos\delta + i\gamma_5\sin\delta)v^{\bar{m}}(\bar{p})$$

$$\rho_{mn,\bar{m}\bar{n}} = \frac{1}{2}\begin{pmatrix} 0 & 0 & 0 & 0 \\ 0 & 1 & e^{-i2\delta} & 0 \\ 0 & e^{i2\delta} & 1 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix}$$

$$\rho = \frac{1}{4}\big(\mathbf{1}_4 + B_i \cdot \sigma_i \otimes \mathbf{1} + \overline{B}_i \cdot \mathbf{1} \otimes \sigma_i + C_{ij} \cdot \sigma_i \otimes \sigma_j\big)$$

$$B_i = \overline{B}_i = 0$$

$$C_{ij} = \begin{pmatrix} \cos 2\delta & \sin 2\delta & 0 \\ -\sin 2\delta & \cos 2\delta & 0 \\ 0 & 0 & -1 \end{pmatrix}$$

$$|\Psi_{H\to\tau\tau}(\delta)\rangle \propto |+-\rangle + e^{i2\delta}|-+\rangle$$

$\delta = 0$ (CP even)

$$|\Psi^{(1,m)}\rangle \propto \begin{pmatrix} |++\rangle \\ \boxed{|+-\rangle + |-+\rangle} \\ |--\rangle \end{pmatrix}$$

$\delta = \pi/2$ (CP odd)

$$|\Psi^{(0,0)}\rangle \propto \boxed{|+-\rangle - |-+\rangle}$$

Parity: $P = (\eta_f \eta_{\bar{f}}) \cdot (-1)^l$ with $\eta_f \eta_{\bar{f}} = -1$:

$$J^P = \begin{cases} 0^+ \implies & l = s = 1 \\ 0^- \implies & l = s = 0 \end{cases}$$

Entanglement

- If the state is separable (not entangled),

$$\rho = \sum_k p_k \rho_k^\alpha \otimes \rho_k^\beta \qquad\qquad 0 \le p_k \le 1$$

$$\sum_k p_k = 1$$

then, a modified matrix by the partial transpose

$$\rho^{T_\beta} \equiv \sum_k p_k \rho_k^\alpha \otimes [\rho_k^\beta]^T$$

is also a physical density matrix, i.e. Tr=1 and non-negative.

- For biparticle systems, entanglement $\Longleftrightarrow$ ρ^{T_β} to be non-positive.

Peres-Horodecki (1996, 1997)

- A simple sufficient condition for entanglement is:

$$\boxed{E \equiv C_{11} + C_{22} - C_{33} > 1}$$

$$(E = 2\cos 2\delta + 1 \quad \text{for} \quad H \to \tau^+\tau^-)$$

$$(E = 3 \quad \text{(maximally entangled)} \quad \text{for} \quad H \to \tau^+\tau^- \text{ in SM})$$

- In $\tau^{\pm} \to \pi^{\pm}\nu$, the direction of $\pi^{\pm}$, ($\vec{\pi}^{\pm}$), *measured at the rest frame of $\tau^{\pm}$* is

spin analyzing power [-1, 1]

$$\frac{d\Gamma}{d\Omega} \propto 1 + \alpha_{\tau\to\pi\nu} \cdot (\vec{\pi}^{\pm} \cdot \mathbf{s})$$

- $\vec{\pi}^{\pm}$ is a unit vector pointing to the direction of $\pi^{\pm}$ measured at the rest frame of $\tau^{\pm}$

- $\mathbf{s}$ is the spin of $\tau^{\pm}$ at its rest frame

- In $\tau^\pm \to \pi^\pm \nu$, the direction of $\pi^\pm$, $(\vec{\pi}^\pm)$, *measured at the rest frame of* $\tau^\pm$ is

spin analyzing power [-1, 1]

$$\frac{d\Gamma}{d\Omega} \propto 1 + \alpha_{\tau\to\pi\nu} \cdot (\vec{\pi}^\pm \cdot \mathbf{s})$$

- $\vec{\pi}^\pm$ is a unit vector pointing to the direction of $\pi^\pm$ measured at the rest frame of $\tau^\pm$

- $\mathbf{s}$ is the spin of $\tau^\pm$ at its rest frame

$$\boxed{\langle \hat{s}_i^{(\tau^-)} \hat{s}_j^{(\tau^+)} \rangle = C_{ij} = -\frac{9}{\alpha^2_{\tau\to\pi\nu}} \cdot \langle (\vec{\pi}^- \cdot \mathbf{e}_i)(\vec{\pi}^+ \cdot \mathbf{e}_j) \rangle}$$

measurable at colliders, but needs to reconstruct the $\tau^\pm$ rest frames

$$\boxed{\alpha_{\tau\to\pi\nu} = 1}$$

spin analyzing power for $\tau \to \pi\nu$ has the maximal value 1.

$$\boxed{\langle \hat{s}_i^{(\tau^-)} \hat{s}_j^{(\tau^+)} \rangle = -9 \cdot \langle (\vec{\pi}^- \cdot \mathbf{e}_i)(\vec{\pi}^+ \cdot \mathbf{e}_j) \rangle}$$

- For the unit vectors $(\hat{\mathbf{a}}, \hat{\mathbf{a}}', \hat{\mathbf{b}}, \hat{\mathbf{b}}')$, RHS of the Bell inequality can be measured as

$$R_{\mathrm{CHSH}} \equiv \frac{1}{2} \left| \langle s_a s_b \rangle - \langle s_a s_{b'} \rangle + \langle s_{a'} s_b \rangle + \langle s_{a'} s_{b'} \rangle \right|$$

$$= \frac{9}{2} \left| \left\langle (\vec{\pi}^- \cdot \hat{\mathbf{a}})(\vec{\pi}^+ \cdot \hat{\mathbf{b}}) \right\rangle - \left\langle (\vec{\pi}^- \cdot \hat{\mathbf{a}})(\vec{\pi}^+ \cdot \hat{\mathbf{b}}') \right\rangle + \left\langle (\vec{\pi}^- \cdot \hat{\mathbf{a}}')(\vec{\pi}^+ \cdot \hat{\mathbf{b}}) \right\rangle + \left\langle (\vec{\pi}^- \cdot \hat{\mathbf{a}}')(\vec{\pi}^+ \cdot \hat{\mathbf{b}}') \right\rangle \right|$$

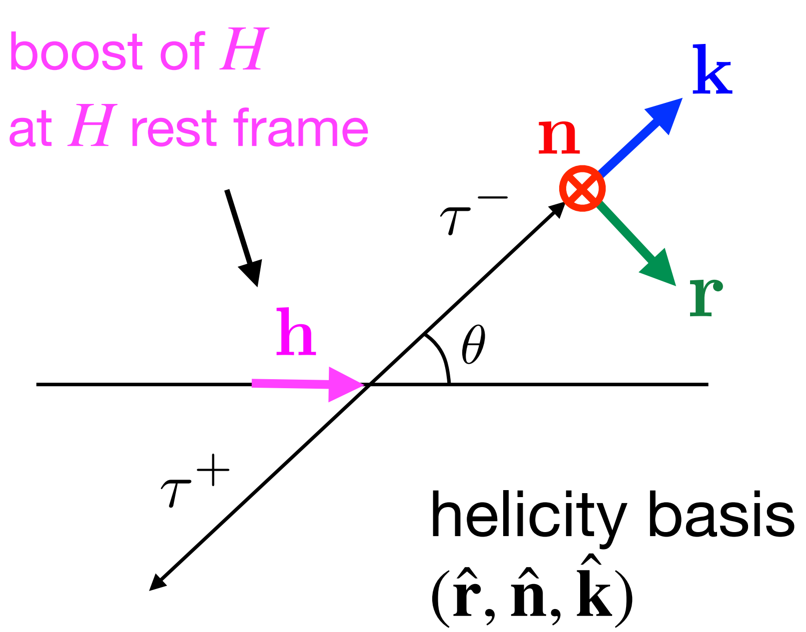
$$\boxed{\langle \hat{s}_i^{(\tau^-)} \hat{s}_j^{(\tau^+)} \rangle = -9 \cdot \langle (\vec{\pi}^- \cdot \mathbf{e}_i)(\vec{\pi}^+ \cdot \mathbf{e}_j) \rangle}$$

- For the unit vectors $(\hat{\mathbf{a}}, \hat{\mathbf{a}}', \hat{\mathbf{b}}, \hat{\mathbf{b}}')$, RHS of the Bell inequality can be measured as

$$R_{\mathrm{CHSH}} \equiv \frac{1}{2} \left| \langle s_a s_b \rangle - \langle s_a s_{b'} \rangle + \langle s_{a'} s_b \rangle + \langle s_{a'} s_{b'} \rangle \right|$$

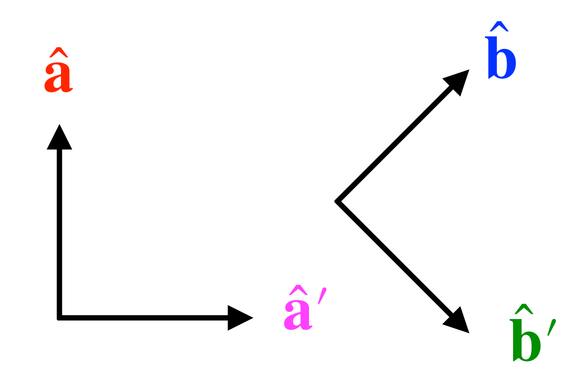
$$= \frac{9}{2} \left| \left\langle (\vec{\pi}^- \cdot \hat{\mathbf{a}})(\vec{\pi}^+ \cdot \hat{\mathbf{b}}) \right\rangle - \left\langle (\vec{\pi}^- \cdot \hat{\mathbf{a}})(\vec{\pi}^+ \cdot \hat{\mathbf{b}}') \right\rangle + \left\langle (\vec{\pi}^- \cdot \hat{\mathbf{a}}')(\vec{\pi}^+ \cdot \hat{\mathbf{b}}) \right\rangle + \left\langle (\vec{\pi}^- \cdot \hat{\mathbf{a}}')(\vec{\pi}^+ \cdot \hat{\mathbf{b}}') \right\rangle \right|$$

- We fix $(\hat{\mathbf{a}}, \hat{\mathbf{a}}', \hat{\mathbf{b}}, \hat{\mathbf{b}}')$ so that R_{CHSH} is maximised.



$$\hat{\mathbf{a}} = \mathbf{r} \qquad \hat{\mathbf{b}} = \frac{1}{\sqrt{2}}(\mathbf{n} + \mathbf{r})$$

$$\hat{\mathbf{a}}' = \mathbf{n} \qquad \hat{\mathbf{b}}' = \frac{1}{\sqrt{2}}(\mathbf{n} - \mathbf{r})$$



Separable state (compliment of entangled state):

$$P(a, b \,|\, A, B) = \sum_{\lambda} p_{\lambda} \langle a \,|\, \rho_{\lambda}^{\alpha} \,|\, a \rangle \cdot \langle b \,|\, \rho_{\lambda}^{\beta} \,|\, b \rangle \longleftarrow \rho = \sum_{\lambda} p_{\lambda}\, \rho_{k}^{\alpha} \otimes \rho_{\lambda}^{\beta}$$

$$\hat{A} \,|\, a \rangle = a \,|\, a \rangle$$

$$P(a \,|\, \hat{A}, \hat{\rho}) = \langle a \,|\, \rho \,|\, a \rangle$$

Probability for outcome a when $\hat{A}$ is measured on the state $\hat{\rho}$

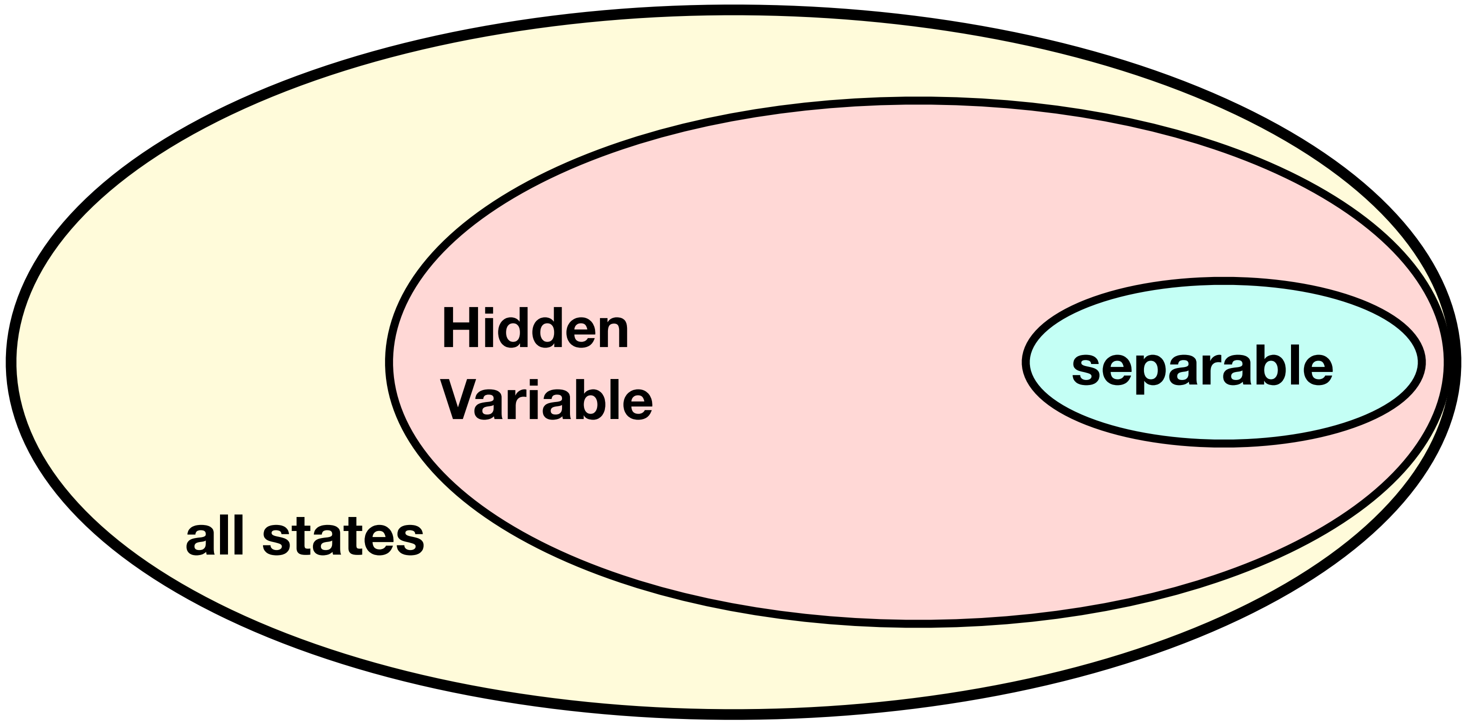
Separable state (compliment of entangled state):

$$P(a, b \,|\, A, B) = \sum_{\lambda} p_{\lambda} \langle a \,|\, \rho_{\lambda}^{\alpha} \,|\, a \rangle \cdot \langle b \,|\, \rho_{\lambda}^{\beta} \,|\, b \rangle \quad \longleftarrow \quad \rho = \sum_{\lambda} p_{\lambda}\, \rho_{k}^{\alpha} \otimes \rho_{\lambda}^{\beta}$$

Hidden Variable state (complement of Bell nonlocal state):

$$P(a, b \,|\, A, B) = \sum_{\lambda} p_{\lambda}\, P_{\alpha}(a \,|\, A, \lambda) \cdot P_{\beta}(b \,|\, B, \lambda)$$

arbitrary conditional probabilities



Separable state (compliment of entangled state):

$$P(a,b\,|\,A,B) = \sum_{\lambda} p_{\lambda} \langle a\,|\,\rho_{\lambda}^{\alpha}\,|\,a\rangle \cdot \langle b\,|\,\rho_{\lambda}^{\beta}\,|\,b\rangle \quad \longleftarrow \quad \rho = \sum_{\lambda} p_{\lambda}\, \rho_{k}^{\alpha} \otimes \rho_{\lambda}^{\beta}$$

Un-steerable state (not-steerable by Alice):

[Jones, Wiseman, Doherty 2007]

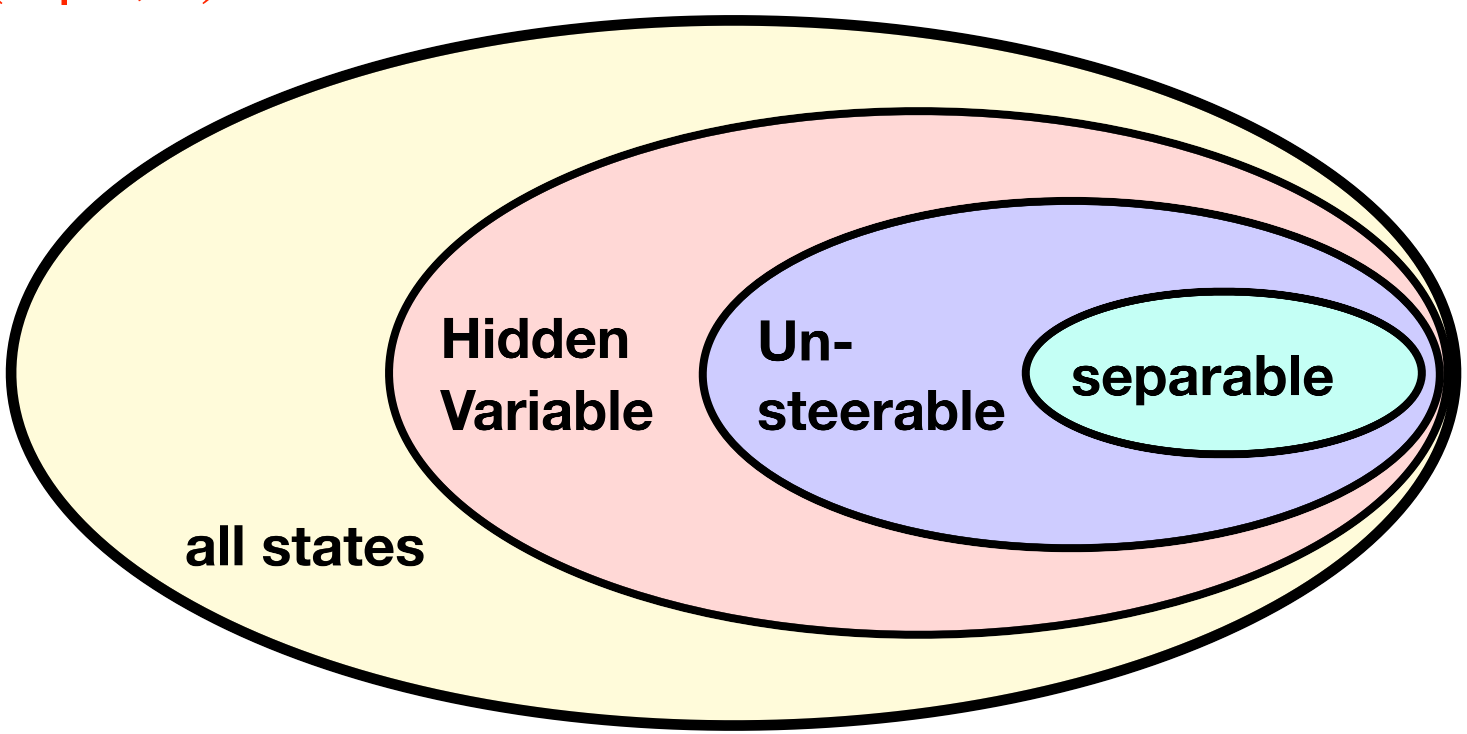
$$P(a,b\,|\,A,B) = \sum_{\lambda} p_{\lambda} P_{\alpha}(a\,|\,A,\lambda) \cdot \langle b\,|\,\rho_{\lambda}^{\beta}\,|\,b\rangle \quad \longleftarrow$$

If this description is possible, Alice cannot influence (`steer") Bob's local state

Hidden Variable state (complement of Bell nonlocal state):

$$P(a,b\,|\,A,B) = \sum_{\lambda} p_{\lambda} \underset{\uparrow}{P_{\alpha}}(a\,|\,A,\lambda) \cdot \underset{\uparrow}{P_{\beta}}(b\,|\,B,\lambda)$$

arbitrary conditional probabilities



Separable state (compliment of entangled state):

$$P(a,b\,|\,A,B) = \sum_\lambda p_\lambda \langle a\,|\,\rho^\alpha_\lambda\,|\,a\rangle \cdot \langle b\,|\,\rho^\beta_\lambda\,|\,b\rangle \longleftarrow \rho = \sum_\lambda p_\lambda\, \rho^\alpha_k \otimes \rho^\beta_\lambda$$

Un-steerable state (not-steerable by Alice): [Jones, Wiseman, Doherty 2007]

$$P(a,b\,|\,A,B) = \sum_\lambda p_\lambda P_\alpha(a\,|\,A,\lambda) \cdot \langle b\,|\,\rho^\beta_\lambda\,|\,b\rangle \longleftarrow$$

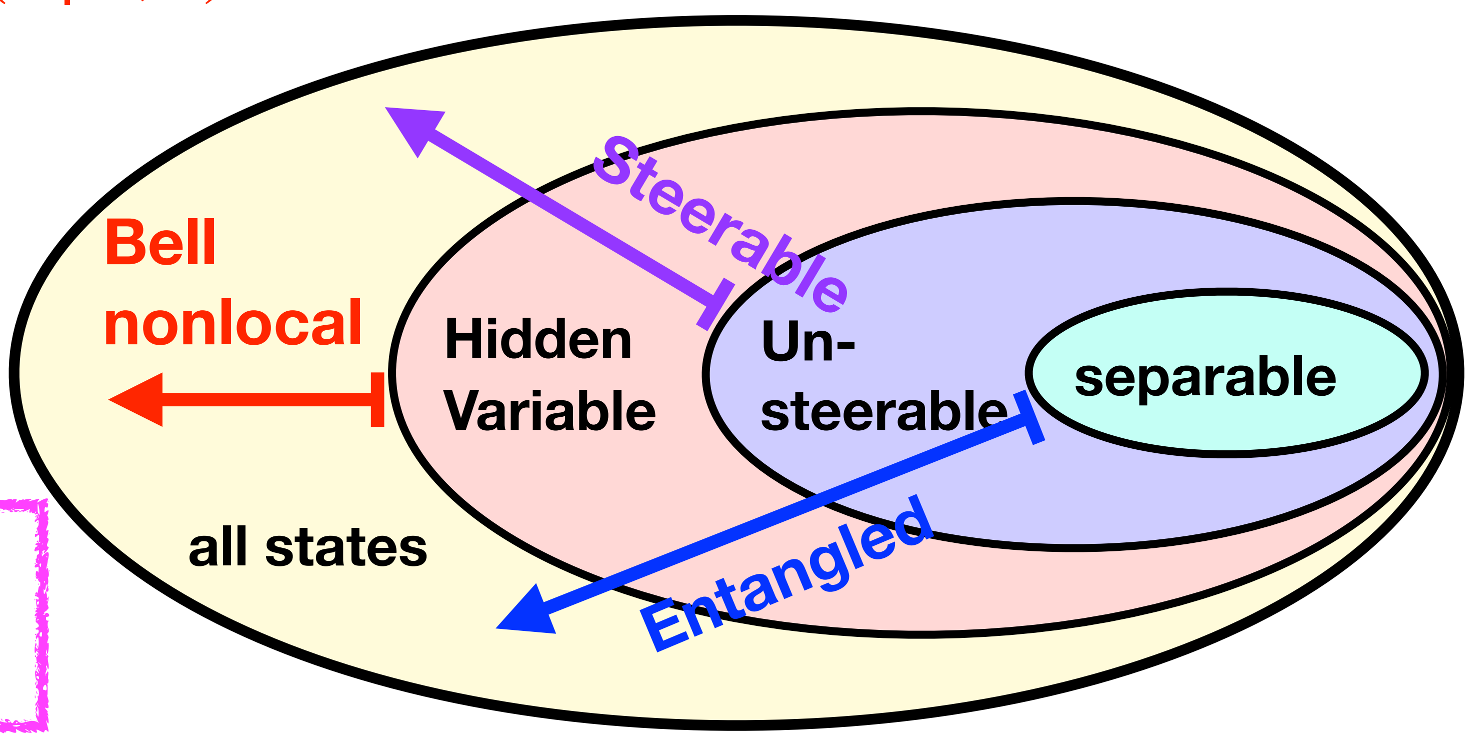
If this description is possible, Alice cannot influence (`steer") Bob's local state

Hidden Variable state (complement of Bell nonlocal state):

$$P(a,b\,|\,A,B) = \sum_\lambda p_\lambda P_\alpha(a\,|\,A,\lambda) \cdot P_\beta(b\,|\,B,\lambda)$$

$R_{\mathrm{CHSH}} > 1$

Bell nonlocal $\subset$ **Steerable** $\subset$ **Entangled**



Steerability

- For unpolarised cases, $\langle \hat{s}_i^A \rangle = \langle \hat{s}_i^B \rangle = 0$, a necessary and sufficient condition for steerability is given by: [Jevtic, Hall, Anderson, Zwierz, Wiseman 2015]

$$\mathcal{S}[\rho] \equiv \frac{1}{2\pi} \int d\Omega_{\mathbf{n}} \sqrt{\mathbf{n}^T C^T C \mathbf{n}} \qquad\qquad \mathcal{S}[\rho] > 1$$

- In $H \to \tau^+\tau^-$,

$$C_{ij} = \begin{pmatrix} \cos 2\delta & \sin 2\delta & 0 \\ -\sin 2\delta & \cos 2\delta & 0 \\ 0 & 0 & -1 \end{pmatrix} \Rightarrow \quad C^T C = \mathbf{1} \quad \Rightarrow \quad \mathcal{S}[\rho] = 2$$

(independent of δ)

Entanglement: [Peres-Horodecki 1996-7]

$$\boxed{E > 1} \qquad E \equiv C_{11} + C_{22} - C_{33}$$

$$\boxed{E(H \to \tau^+\tau^-) = 2\cos 2\delta + 1}$$

Steerability: [Jevtic, Hall, Anderson, Zwierz, Wiseman 2015]

$$\boxed{\mathcal{S}[\rho] > 1} \quad \text{(assuming } B_i = \overline{B}_i = 0\text{)} \qquad \mathcal{S}[\rho] \equiv \frac{1}{2\pi}\int d\Omega_{\mathbf{n}} \sqrt{\mathbf{n}^T C^T C\, \mathbf{n}}$$

$$\boxed{\mathcal{S}[\rho](H \to \tau^+\tau^-) = 2}$$

Bell-nonlocality: [Clauser, Horne, Shimony, Holt, 1969]

$$\boxed{R_{\rm CHSH} \equiv \frac{1}{2}\left|\langle s_a s_b\rangle - \langle s_a s_{b'}\rangle + \langle s_{a'} s_b\rangle + \langle s_{a'} s_{b'}\rangle\right| > 1}$$

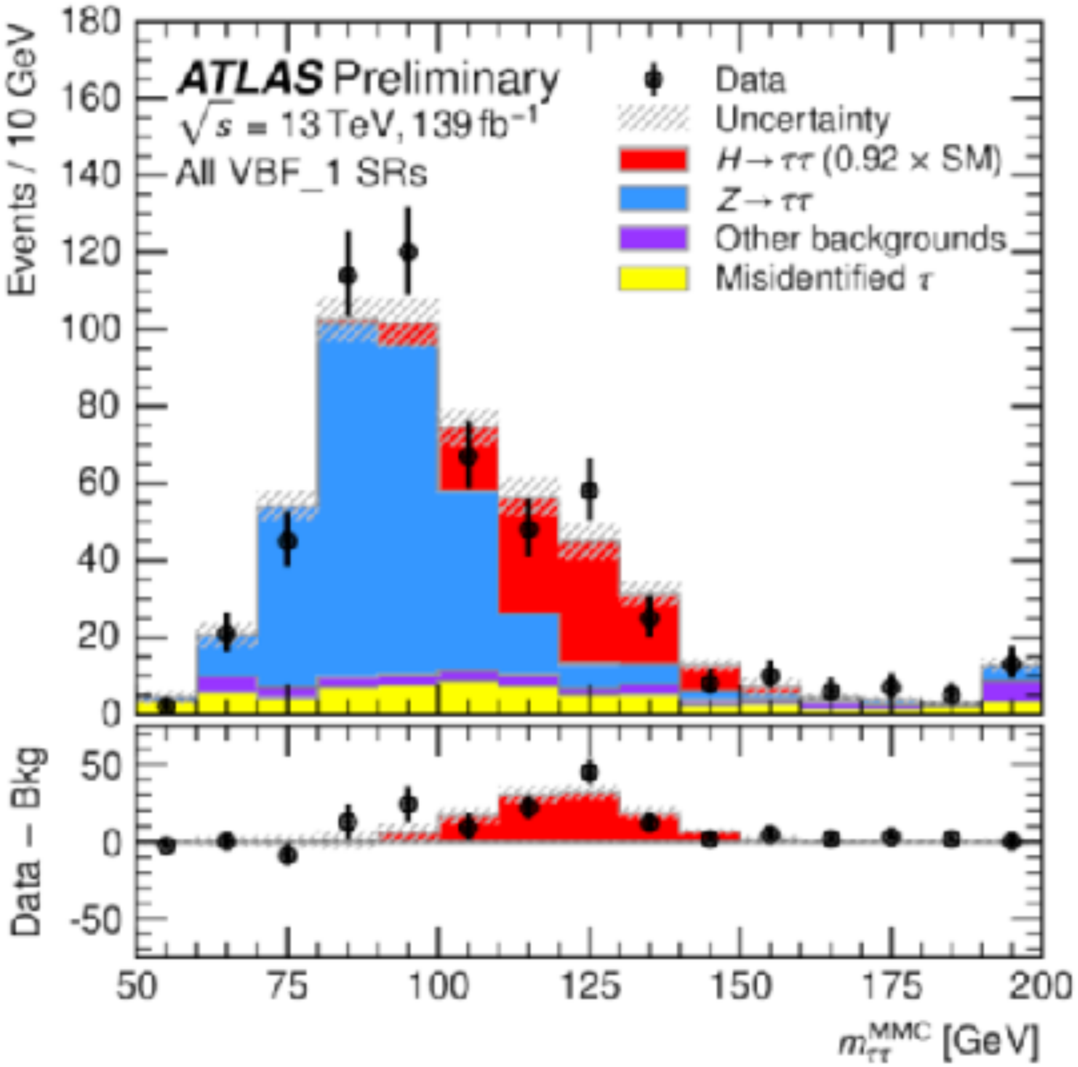
$$\boxed{R_{\rm CHSH}(H \to \tau^+\tau^-) = \sqrt{2}}$$

$$\boxed{\langle s_i s_j\rangle = C_{ij} = -9 \cdot \langle(\vec{\pi}^- \cdot \mathbf{e}_i)(\vec{\pi}^+ \cdot \mathbf{e}_j)\rangle}$$

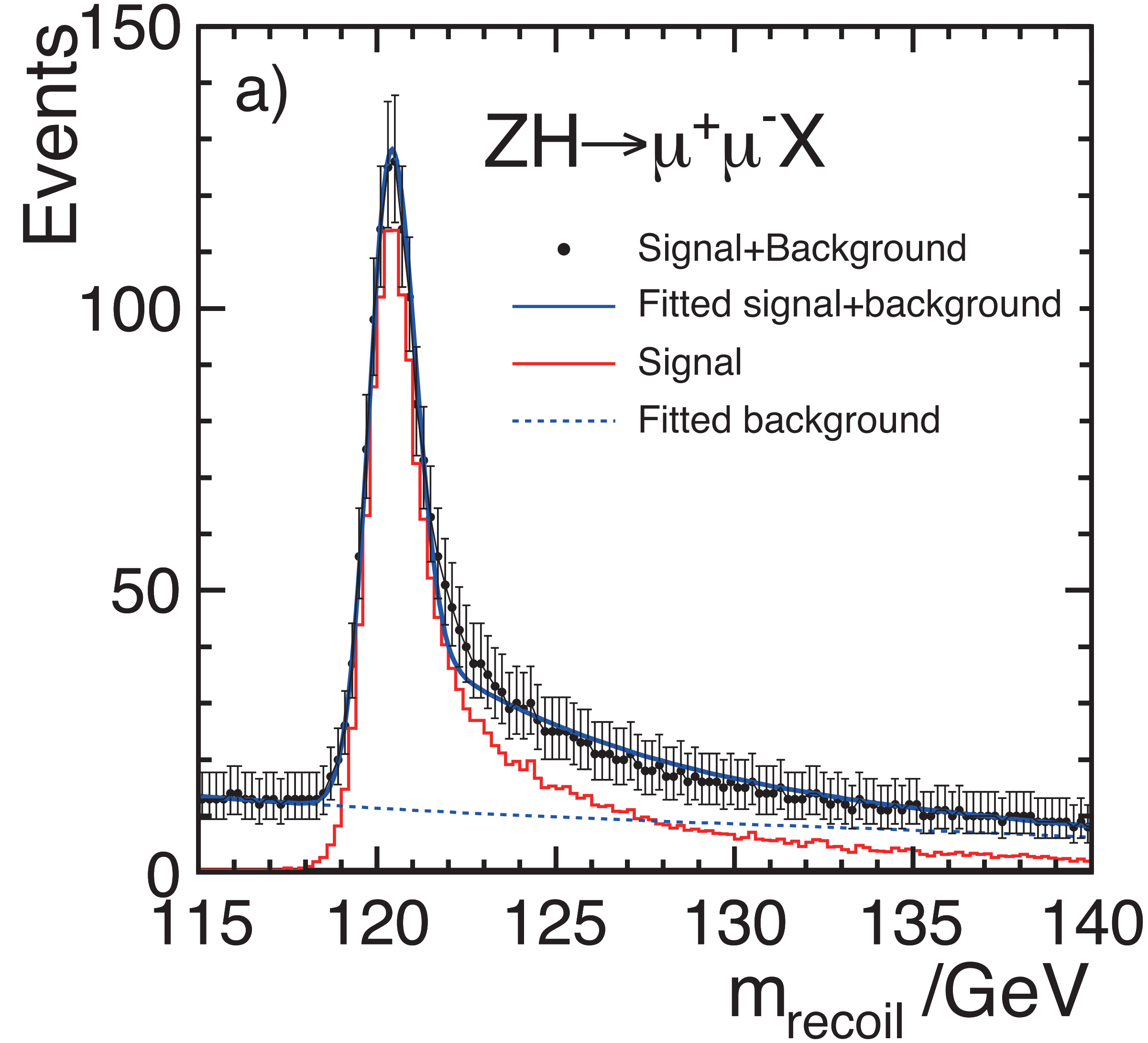
$H \to \tau^+\tau^-$ @ lepton colliders

- Background $Z/\gamma \to \tau^+\tau^-$ is much smaller for lepton colliders.
- We need to reconstruct each τ rest frame to measure $\vec{\pi}^{\pm}$. This is challenging at hadron colliders since partonic CoM energy is unknown for each event.

LHC



ILC



Simulation

$$e^+e^- \to Z + (Z^*/\gamma^*) \to f\bar{f} + \tau^+\tau^-$$

	ILC	FCC-ee
energy (GeV)	250	240
luminosity (ab^{-1})	3	5
beam resolution e^+ (%)	0.18	0.83×10^{-4}
beam resolution e^- (%)	0.27	0.83×10^{-4}
$\sigma(e^+e^- \to HZ)$ (fb)	240.1	240.3
# of signal ($\sigma \cdot \mathrm{BR} \cdot L \cdot \epsilon$)	385	663
# of background ($\sigma \cdot \mathrm{BR} \cdot L \cdot \epsilon$)	20	36

- Generate the SM events $(\kappa, \delta) = (1,0)$ with **MadGraph5**.

$$\boxed{e^+e^- \to HZ, \;\; Z \to f\bar{f}, \;\; H \to \tau^+\tau^-, \;\; \tau^\pm \to \nu\pi^\pm}$$

- **incorporate the detector effect** by smearing energies of visible particles with

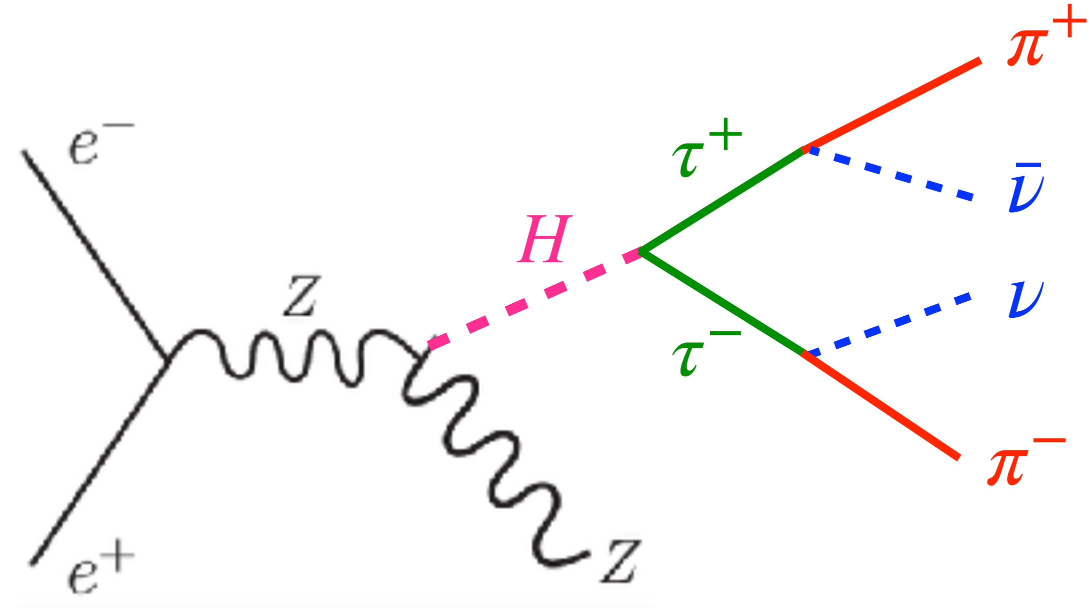
$$E^{\mathrm{true}} \to E^{\mathrm{obs}} = (1 + \sigma_E \cdot \omega) \cdot E^{\mathrm{true}} \qquad \sigma_E = 0.03$$

random number from the normal distribution

- **Event selection:** $|M_{\mathrm{recoil}} - 125\,\mathrm{GeV}| < 5\,\mathrm{GeV}$ $\qquad M_{\mathrm{recoil}} \equiv (P^\mu_{e^+e^-} - P^\mu_Z)^2$

- **100 pseudo-experiments** to estimate the statistical uncertainties

- To determine the tau momenta, we have to reconstruct the unobserved neutrino momenta $(p_x^{\nu}, p_y^{\nu}, p_z^{\nu})$, $(p_x^{\bar{\nu}}, p_y^{\bar{\nu}}, p_z^{\bar{\nu}})$.



- To determine the tau momenta, we have to reconstruct the unobserved neutrino momenta $(p_x^\nu, p_y^\nu, p_z^\nu)$, $(p_x^{\bar\nu}, p_y^{\bar\nu}, p_z^{\bar\nu})$.

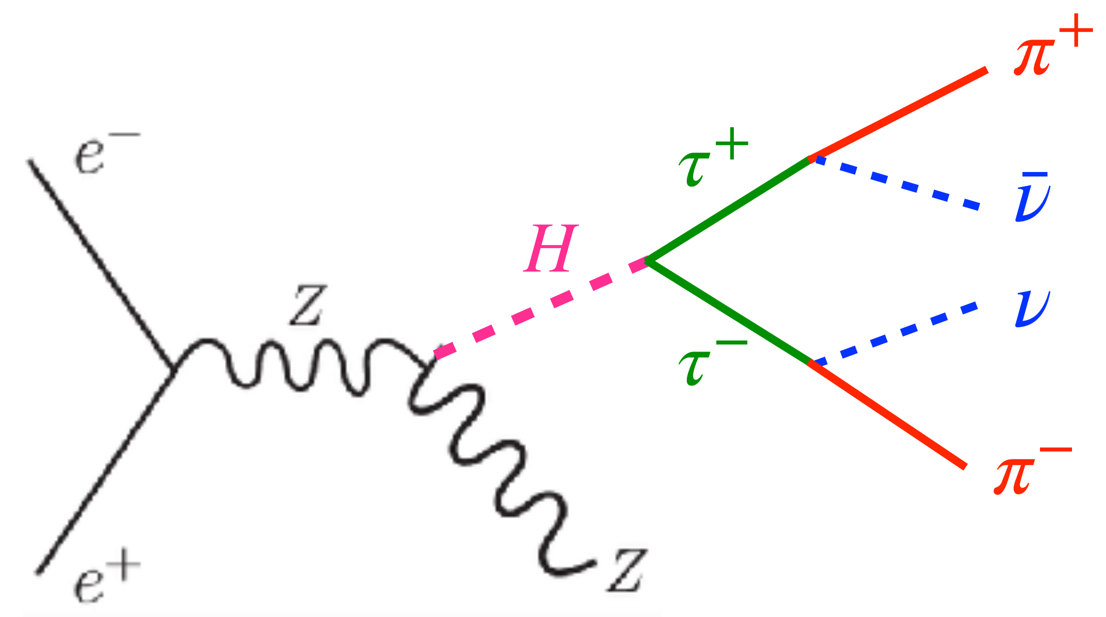
- **6** unknowns can be constrained by **2** mass-shell conditions and **4** energy-momentum conservation.

$$m_\tau^2 = (p_{\tau^+})^2 = (p_{\pi^+} + p_{\bar\nu})^2$$

$$m_\tau^2 = (p_{\tau^-})^2 = (p_{\pi^-} + p_{\nu})^2$$

$$(p_{ee} - p_Z)^\mu \;=\; p_H^\mu \;=\; \left[(p_{\pi^-} + p_\nu) + (p_{\pi^+} + p_{\bar\nu})\right]^\mu$$

We have 2-fold solutions.



Result 1

2211.10513

	ILC	FCC-ee
C_{ij}	$\begin{pmatrix} -0.600 \pm 0.210 & 0.003 \pm 0.125 & 0.020 \pm 0.149 \\ 0.003 \pm 0.125 & -0.494 \pm 0.190 & 0.007 \pm 0.128 \\ 0.048 \pm 0.174 & 0.0007 \pm 0.156 & 0.487 \pm 0.193 \end{pmatrix}$	$\begin{pmatrix} -0.559 \pm 0.143 & -0.010 \pm 0.095 & -0.014 \pm 0.122 \\ -0.010 \pm 0.095 & -0.494 \pm 0.139 & -0.002 \pm 0.111 \\ 0.012 \pm 0.124 & 0.020 \pm 0.105 & 0.434 \pm 0.134 \end{pmatrix}$
E_k	-1.057 ± 0.385	-0.977 ± 0.264
$\mathcal{C}[\rho]$	0.030 ± 0.071	0.005 ± 0.023
$\mathcal{S}[\rho]$	1.148 ± 0.210	1.046 ± 0.163
R^*_{CHSH}	0.769 ± 0.189	0.703 ± 0.134

SM values: $C_{ij}^{\text{SM}} = \begin{pmatrix} 1 & & \\ & 1 & \\ & & -1 \end{pmatrix}$

$$E_{\text{SM}}[\rho] = 3$$

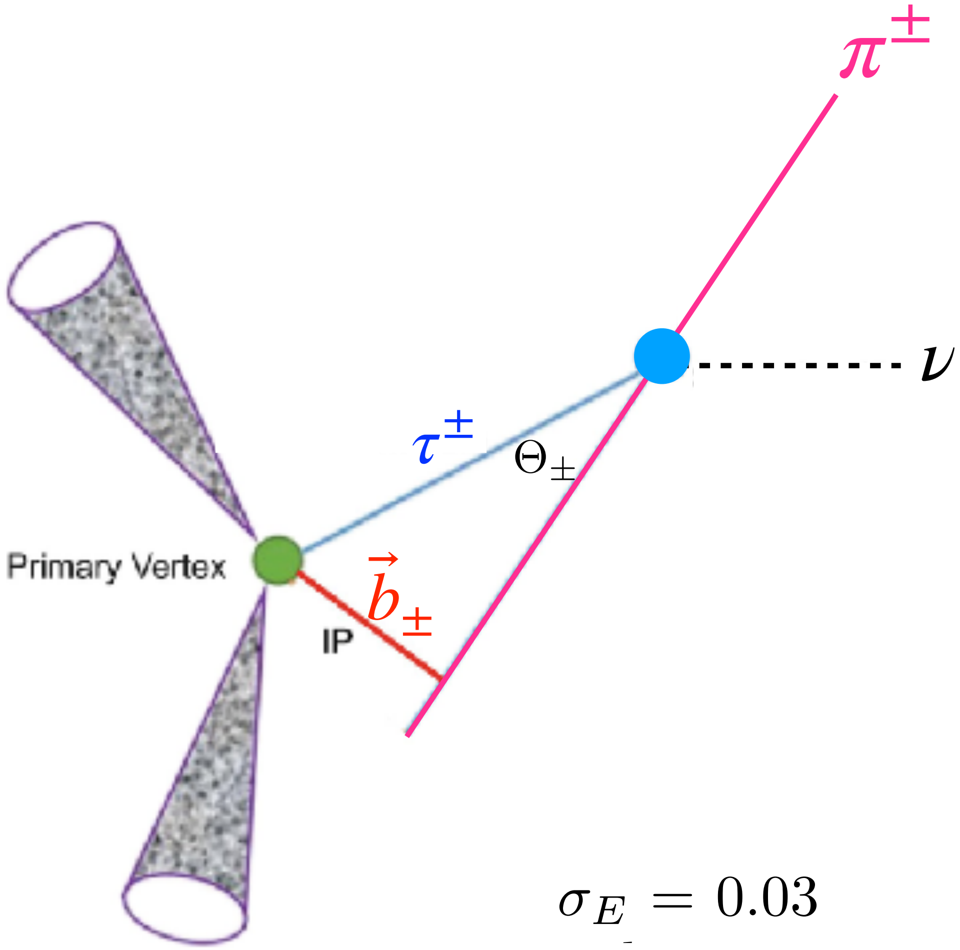
$$\mathcal{S}_{\text{SM}}[\rho] = 2$$

$$R_{\text{CHSH}}^{\text{SM}} = \sqrt{2} \simeq 1.414$$

Entanglement $\Longrightarrow E > 1$

Steerablity $\Longrightarrow \mathcal{S}[\rho] > 1$

Bell-nonlocal $\Longrightarrow R_{\text{CHSH}} > 1$



$\sigma_E = 0.03$

Use impact parameter information

- We use the information of impact parameter $\vec{b}_{\pm}$ measurement of $\pi^{\pm}$ to "correct" the observed energies of $\tau^{\pm}$ and Z decay products
- We check whether the reconstructed τ momenta are consistent with the measured impact parameters.
- We construct the likelihood function and search for the most likely τ momenta.

$$E^{\rm true} \to E^{\rm obs} = (1 + \sigma_E \cdot \omega) \cdot E^{\rm true}$$

$$\vec{b}^{\rm reco}_{\pm}(\mathbf{e}_{\tau^{\pm}}) = |\vec{b}_{\pm}| \cdot \left[\mathbf{e}_{\tau^{\pm}} \cdot \sin^{-1}\Theta_{\pm} - \mathbf{e}_{\pi^{\pm}} \cdot \tan^{-1}\Theta_{\pm}\right]$$

$$\boxed{E_\alpha(\delta_\alpha) = (1 + \sigma^E_\alpha \cdot \delta_\alpha) \cdot E^{\rm obs}_\alpha} \qquad (\alpha = \pi^+, \pi^-, x, \bar{x})$$

$$\vec{\Delta}^{i_s}_{b_\pm}(\boldsymbol{\delta}) \equiv \vec{b}_{\pm} - \vec{b}^{\rm reco}_{\pm}\left(\mathbf{e}^{i_s}_{\tau^+}(\boldsymbol{\delta})\right)$$

(2-fold solutions: $i_s = 1, 2$)

$$L^{i_s}_{\pm}(\boldsymbol{\delta}) = \frac{[\Delta^{i_s}_{b_\pm}(\boldsymbol{\delta})]^2_x + [\Delta^{i_s}_{b_\pm}(\boldsymbol{\delta})]^2_y}{\sigma^2_{b_T}} + \frac{[\Delta^{i_s}_{b_\pm}(\boldsymbol{\delta})]^2_z}{\sigma^2_{b_z}} \qquad (\alpha = \pi^+, \pi^-, x, \bar{x}) \qquad (\sigma_{b_T} = 2\mu m, \ \ \sigma_{b_z} = 5\mu m)$$

$$L^{i_s}(\boldsymbol{\delta}) = L^{i_s}_{+}(\boldsymbol{\delta}) + L^{i_s}_{-}(\boldsymbol{\delta}) + \delta^2_{\pi^+} + \delta^2_{\pi^-} + \delta^2_{x} + \delta^2_{\bar{x}} \, .$$

← **We choose δ and i_s to minimises this.**

Result 2

2211.10513

	ILC	FCC-ee
C_{ij}	$\begin{pmatrix} 0.830 \pm 0.176 & 0.020 \pm 0.146 & -0.019 \pm 0.159 \\ -0.034 \pm 0.160 & 0.981 \pm 0.1527 & -0.029 \pm 0.156 \\ -0.001 \pm 0.158 & -0.021 \pm 0.155 & -0.729 \pm 0.140 \end{pmatrix}$	$\begin{pmatrix} 0.925 \pm 0.109 & -0.011 \pm 0.110 & 0.038 \pm 0.095 \\ -0.009 \pm 0.110 & 0.929 \pm 0.113 & 0.001 \pm 0.115 \\ -0.026 \pm 0.122 & -0.019 \pm 0.110 & -0.879 \pm 0.098 \end{pmatrix}$
E_k	2.567 ± 0.279	2.696 ± 0.215
$\mathcal{C}[\rho]$	0.778 ± 0.126	0.871 ± 0.084
$\mathcal{S}[\rho]$	1.760 ± 0.161	1.851 ± 0.111
R^*_{CHSH}	1.103 ± 0.163	1.276 ± 0.094

SM values: $C_{ij}^{\mathrm{SM}} = \begin{pmatrix} 1 & & \\ & 1 & \\ & & -1 \end{pmatrix}$

$$E_{\mathrm{SM}}[\rho] = 3$$

$$\mathcal{S}_{\mathrm{SM}}[\rho] = 2$$

$$R_{\mathrm{CHSH}}^{\mathrm{SM}} = \sqrt{2} \simeq 1.414$$

Entanglement $\Longrightarrow E > 1$

Steerablity $\Longrightarrow \mathcal{S}[\rho] > 1$

Bell-nonlocal $\Longrightarrow R_{\mathrm{CHSH}} > 1$

Result 2

2211.10513

	ILC		FCC-ee	
C_{ij}	$\begin{pmatrix} 0.830 \pm 0.176 & 0.020 \pm 0.146 & -0.019 \pm 0.159 \\ -0.034 \pm 0.160 & 0.981 \pm 0.1527 & -0.029 \pm 0.156 \\ -0.001 \pm 0.158 & -0.021 \pm 0.155 & -0.729 \pm 0.140 \end{pmatrix}$		$\begin{pmatrix} 0.925 \pm 0.109 & -0.011 \pm 0.110 & 0.038 \pm 0.095 \\ -0.009 \pm 0.110 & 0.929 \pm 0.113 & 0.001 \pm 0.115 \\ -0.026 \pm 0.122 & -0.019 \pm 0.110 & -0.879 \pm 0.098 \end{pmatrix}$	
E_k	2.567 ± 0.279	$\sim 5\sigma$	2.696 ± 0.215	$\gg 5\sigma$
$\mathcal{C}[\rho]$	0.778 ± 0.126	$\sim 5\sigma$	0.871 ± 0.084	$\gg 5\sigma$
$\mathcal{S}[\rho]$	1.760 ± 0.161	$\sim 3\sigma$	1.851 ± 0.111	$\sim 5\sigma$
R^*_{CHSH}	1.103 ± 0.163		1.276 ± 0.094	$\sim 3\sigma$

SM values: $C_{ij}^{\mathrm{SM}} = \begin{pmatrix} 1 & & \\ & 1 & \\ & & -1 \end{pmatrix}$

$$E_{\mathrm{SM}}[\rho] = 3$$

$$\mathcal{S}_{\mathrm{SM}}[\rho] = 2$$

$$R^{\mathrm{SM}}_{\mathrm{CHSH}} = \sqrt{2} \simeq 1.414$$

Entanglement $\implies E > 1$

Steerablity $\implies \mathcal{S}[\rho] > 1$

Bell-nonlocal $\implies R_{\mathrm{CHSH}} > 1$

Result 2

2211.10513

	ILC		FCC-ee	
C_{ij}	$\begin{pmatrix} 0.830 \pm 0.176 & 0.020 \pm 0.146 & -0.019 \pm 0.159 \\ -0.034 \pm 0.160 & 0.981 \pm 0.1527 & -0.029 \pm 0.156 \\ -0.001 \pm 0.158 & -0.021 \pm 0.155 & -0.729 \pm 0.140 \end{pmatrix}$		$\begin{pmatrix} 0.925 \pm 0.109 & -0.011 \pm 0.110 & 0.038 \pm 0.095 \\ -0.009 \pm 0.110 & 0.929 \pm 0.113 & 0.001 \pm 0.115 \\ -0.026 \pm 0.122 & -0.019 \pm 0.110 & -0.879 \pm 0.098 \end{pmatrix}$	
E_k	2.567 ± 0.279	$\sim 5\sigma$	2.696 ± 0.215	$\gg 5\sigma$
$\mathcal{C}[\rho]$	0.778 ± 0.126	$\sim 5\sigma$	0.871 ± 0.084	$\gg 5\sigma$
$\mathcal{S}[\rho]$	1.760 ± 0.161	$\sim 3\sigma$	1.851 ± 0.111	$\sim 5\sigma$
R^*_{CHSH}	1.103 ± 0.163		1.276 ± 0.094	$\sim 3\sigma$

SM values: $C_{ij}^{\text{SM}} = \begin{pmatrix} 1 & & \\ & 1 & \\ & & -1 \end{pmatrix}$

$$E_{\text{SM}}[\rho] = 3$$

$$\mathcal{S}_{\text{SM}}[\rho] = 2$$

$$R_{\text{CHSH}}^{\text{SM}} = \sqrt{2} \simeq 1.414$$

Entanglement $\Longrightarrow E > 1$

Steerablity $\Longrightarrow \mathcal{S}[\rho] > 1$

Bell-nonlocal $\Longrightarrow R_{\text{CHSH}} > 1$

Superiority of FCC-ee over ILC is due to a better beam resolution

	ILC	FCC-ee
energy (GeV)	250	240
luminosity (ab^{-1})	3	5
beam resolution e^+ (%)	0.18	$0.83 \cdot 10^{-4}$
beam resolution e^- (%)	0.27	$0.83 \cdot 10^{-4}$

CP measurement

- Under CP, the spin correlation matrix transforms: $C \xrightarrow{CP} C^T$
- This can be used for a *model-independent* test of CP violation. We define:

$$A \equiv (C_{rn} - C_{nr})^2 + (C_{nk} - C_{kn})^2 + (C_{kr} - C_{rk})^2 \geq 0$$

- Observation of $A \neq 0$ immediately confirms CP violation.
- From our simulation, we observe

$$A = \begin{cases} 0.204 \pm 0.173 & \text{(ILC)} \\ 0.112 \pm 0.085 & \text{(FCC-ee)} \end{cases} \longleftarrow$$

consistent with absence of CPV

- This model independent bounds can be translated to the constraint on the CP-phase δ

$$\mathscr{L}_{\rm int} \propto H \bar{\psi}_\tau (\cos\delta + i\gamma_5 \sin\delta)\, \psi_\tau \;\Rightarrow\; C_{ij} = \begin{pmatrix} \cos 2\delta & \sin 2\delta & 0 \\ -\sin 2\delta & \cos 2\delta & 0 \\ 0 & 0 & -1 \end{pmatrix} \;\Rightarrow\; A(\delta) = 4\sin^2 2\delta$$

CP measurement

- Focusing on the region near $|\delta| = 0$, we find the 1-σ bounds:

$$|\delta| < \begin{cases} 8.9^o & \text{(ILC)} \\ 6.4^o & \text{(FCC-ee)} \end{cases}$$

- Other studies:

$$\Delta\delta \sim 11.5^o \quad \text{(HL-LHC)}$$ [Hagiwara, Ma, Mori 2016]

$$\Delta\delta \sim 4.3^o \quad \text{(ILC)}$$ [Jeans and G. W. Wilson 2018]

Summary

- High energy tests of entanglement and Bell inequality has recently attracted an attention.
- $\tau^+\tau^-$ pairs from $H \to \tau^+\tau^-$ form the EPR triplet state $|\Psi^{(1,0)}\rangle = \dfrac{|+,-\rangle + |-,+\rangle}{\sqrt{2}}$, and maximally entangled.
- We investigated feasibility of quantum property tests @ ILC and FCC-ee.
- Quantum test requires to a precise reconstruction of the tau rest frames and IP information is crucial to achieve this.
- Spin correlation is sensitive to CP-phase and we can measure the CP-phase as a byproduct of the quantum property measurement.

	Entanglement	Steering	Bell-inquality	CP-phase
ILC	$\sim 5\sigma$	$\sim 3\sigma$		8.9^o
FCC-ee	$\gg 5\sigma$	$\sim 5\sigma$	$\sim 3\sigma$	6.4^o

Norway grants

NATIONAL SCIENCE CENTRE POLAND

The research leading to the results presented in this talk has received funding from the Norwegian Financial Mechanism for years 2014-2021, grant nr 2019/34/H/ST2/00707

Understanding the Early Universe:
interplay of theory and collider experiments

Joint research project between the University of Warsaw & University of Bergen

$$
\begin{aligned}
\sigma(e^+e^- \to HZ)\big|_{\sqrt{s}=240\mathrm{GeV}} &= 240.3\,\mathrm{fb} \\
BR(H \to \tau^+\tau^-) &= 0.0632 \\
BR(\tau^- \to \pi^-\nu_\tau) &= 0.109 \\
BR(Z \to jj, \mu\mu, ee) &= 0.766 \\
\sigma(e^+e^- \to HZ)^{\mathrm{unpol}}_{240} \cdot BR_{H\to\tau\tau} \cdot [BR_{\tau\to\pi\nu}]^2 \cdot BR_{Z\to jj,\mu\mu,ee} &= 0.1382\,\mathrm{fb}
\end{aligned}
$$