25th May. 2021 @ Elab seminar

## Self-introduction, or a biased view of what theoretical cosmologists are recently interested in

### Yuichiro TADA

Mikura, YT, Yokoyama YT & Yokoyama Fujita, Kawasaki, YT, Takesako Clab, Nagoya U. EPL **132**, 3 (2020) **Highlights** PRD **100**, no. 2, 023537 (2019) JCAP **12**, 036 (2013)





## Yuichiro TADA (多田 祐一郎)

Ph.D. (Science) 2017 Mar. PD researcher 2017 Apr. JSPS fellow (PD) 2018 Apr. Designated Asst. Prof. (YLC) 2021 Apr.

#### **Approach to Cosmic Inflation in light of Stochastic Calc., Prim. Black Hole, and Grav. Wave Research Topic:**

**Stoc. Calc.** 

Our universe is thought to start with an accelerated expansion phase called Inflation. It can source various cosmic structures (e.g., galaxy) from quantum fluctuation, but its expansion mechanism has not been explained. As a possible sourced object, Primordial Black Hole (原始ブラックホール) has attracted attention as a candidate of Dark Matter (暗黒物質). Gravitational Wave (重力波) is also attractive because it can be directly detected now. I have proposed a powerful algorithm to evaluate their production by applying Stochastic Calculus (確率解析), and am approaching to the inflation mechanism from these viewpoint.





The University of Tokyo (IPMU & ICRR) Institut d'Astrophysique de Paris Nagoya University (C-lab.)

Nagoya University (IAR & C-lab.)







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## **Research Topic**

Our universe is though structures (e.g., galaxy sourced object, Primo Gravitational Wave (1) algorithm to evaluate mechanism from these







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> source various cosmic explained. As a possible Dark Matter (暗黒物質). oaching to the inflation





# Contents

#### Inflation **Primordial Black Hole** LIGO, DM, SMBH, OGLE, Planet 9, ... Planck (CMB obs.) $R^2$ -model or "Other Possibilities"? Starobinsky '80 Multistage Inflation Metric-Affine Gravity YT & Yokoyama '19, Inomata, Kawasaki, Mukaida, YT, Yanagida '17s Mikura, YT, Yokoyama '20 & '21 Stochastic Inflation

Fujita, Kawasaki, YT, Takesako '13, Pinol, Renaux-Petel, YT '19 & '21



**Current Status of Inflation** 





### **Thermal History** Why "Inflation"?



**Current Status of Inflation** 

# Dark Energy

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### Thermal History Why "Inflation"?



**Current Status of Inflation** 

# Dark Energy

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### Thermal History Why "Inflation"?



**Current Status of Inflation** 

# Dark Energy

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**Current Status of Inflation** 

$$\begin{array}{l} \overbrace{S_{z}[g] = \frac{1}{2} \int d^{4}x \sqrt{-g} f(R) \\ \Rightarrow S_{J}[g] = \frac{1}{2} \int d^{4}x \sqrt{-g} f(R) \\ \Rightarrow S_{J}[g, z] = \frac{1}{2} \int d^{4}x \sqrt{-g} \left[ f(z) + f'(z)(R - z) \right] \\ \Rightarrow S_{J}[g, z] = \frac{1}{2} \int d^{4}x \sqrt{-g} \left[ f(z) + f'(z)(R - z) \right] \\ \Rightarrow S_{J}[g, z] = \frac{1}{2} \int d^{4}x \sqrt{-g} \left[ f(z) + f'(z)(R - z) \right] \\ \Rightarrow S_{J}[g, z] = \int d^{4}x \sqrt{-g} \left[ f(z) + f'(z)(R - z) \right] \\ \Rightarrow S_{E}[\tilde{g}, \omega] = \int d^{4}x \sqrt{-\tilde{g}} \left[ \frac{1}{2} M_{\text{Pl}}^{2} R - \frac{3M_{\text{Pl}}^{2}}{4\omega^{2}} (\partial_{\mu}\omega)^{2} - \frac{M_{\text{Pl}}^{3}(\omega z - f(z)/M_{\text{Pl}})}{2\omega^{2}} \right] \\ \Rightarrow S_{E}[\tilde{g}, \omega] = \int d^{4}x \sqrt{-\tilde{g}} \left[ \frac{1}{2} M_{\text{Pl}}^{2} R - \frac{3M_{\text{Pl}}^{2}}{4\omega^{2}} (\partial_{\mu}\omega)^{2} - \frac{M_{\text{Pl}}^{3}(\omega z - f(z)/M_{\text{Pl}})}{2\omega^{2}} \right] \\ \end{cases}$$

 $R^2$  inflation  $f(R) = M_{Pl}^2 R + \beta R^2$ :  $S_E[g,\chi] = \int d^4 x \sqrt{-1}$ 

#### **Current Status of Inflation**

$$= \overline{g} \left[ \frac{1}{2} M_{\rm Pl}^2 R - \frac{1}{2} (\partial_\mu \chi)^2 - \frac{M_{\rm Pl}^2}{2\beta} \left( 1 - e^{-\sqrt{\frac{2}{3}} \frac{\chi}{M_{\rm Pl}}} \right)^2 \right]$$

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### Conformal inf. Kallosh & Linde '13

$$S = \int d^4 x \sqrt{-g} \left[ \frac{1}{12} (\chi^2 - \phi^2) R + \frac{1}{2} (\partial_\mu \chi)^2 - \frac{1}{2} (\partial_\mu \chi)^2 \right]$$

**local conformal** + **global (pseudo) SO(1,1)**   $\tilde{R} = e^{2\sigma}(R - e^{\sigma} \Box e^{-\sigma})$  $\partial_{\mu}\tilde{\phi} = e^{\sigma}\partial_{\mu}\phi + \phi\partial_{\mu}e^{\sigma}$ 

$$\begin{cases} g_{\mu\nu} \to \tilde{g}_{\mu\nu} = e^{-2\sigma(x)}g_{\mu\nu}, & \tilde{R} = e \\ \phi \to \tilde{\phi} = e^{\sigma(x)}\phi, & \partial_{\mu}\tilde{\phi} = e^{\sigma(x)}\phi \end{cases}$$

Gauge fixing: 
$$\chi^2 - \phi^2 = 6M_{\text{Pl}}^2$$
  
**pNG boson**  
 $\begin{cases} \sqrt{6}M_{\text{Pl}} \cosh \frac{\varphi}{\sqrt{6}M_{\text{Pl}}} \\ \sqrt{6}M_{\text{Pl}} \sinh \frac{\varphi}{\sqrt{6}M_{\text{Pl}}} \end{cases}$ 

**Current Status of Inflation** 

# ghost $p^{2} - \frac{1}{2}(\partial_{\mu}\phi)^{2} - \frac{1}{4}F(\frac{\phi}{\chi})(\chi^{2} - \phi^{2})^{2}$

$$V(\varphi) \propto \tanh^{2n} \frac{\varphi}{\sqrt{6}M_{\rm Pl}}$$

c.f.  $\alpha$  attractor:  $V(\varphi) \propto \tanh^{2n} \frac{\varphi}{\sqrt{6\alpha}M_{\rm Pl}}$ nontrivially embedded in superconformal

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# Metric-Affine Grav.

- Metric norm conserv. in parallel trs.  $\overset{\Gamma}{\nabla} g_{\mu\nu} = 0 \rightarrow \Gamma^{\mu}_{\nu\rho} = \frac{1}{2} g^{\mu\sigma} (g_{\nu\sigma,\rho} + g_{\rho\sigma,\nu} - g_{\nu\rho,\sigma})$ 

$$S = \int \mathrm{d}^4 x \sqrt{-g} \frac{1}{2} M_{\mathrm{Pl}}^2 g^{\mu\nu} R_{\mu\nu} \left( \Gamma(g) \right)$$

$$S = \int d^4x \sqrt{-g} f(R(g))$$

$$d^4x\sqrt{-g}\phi^2 R(g)$$

**Metric-Affine Gravity** 

Geometry =  $g_{\mu\nu}$  +  $R^{\mu}_{\nu\rho\sigma}(\Gamma)$  +  $T^{\mu}_{\nu\rho}$ 

## – Metric-Affine $S = \int d^4x \sqrt{-g} \frac{1}{2} M_{\rm Pl}^2 g^{\mu\nu} R_{\mu\nu}(\Gamma)$

EL constraint  $\frac{\delta S}{\delta \Gamma} = 0 \rightarrow \Gamma^{\mu}_{\nu\rho} = \frac{1}{2} g^{\mu\sigma} (g_{\nu\sigma,\rho} + g_{\rho\sigma,\nu} - g_{\nu\rho,\sigma})$ 

$$S = \int d^4x \sqrt{-g} f\left(g^{\mu\nu}R_{\mu\nu}(\Gamma)\right)$$
$$S = \int d^4x \sqrt{-g} \phi^2 g^{\mu\nu}R_{\mu\nu}(\Gamma)$$

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### Conf. inf. in MAG Mikura, YT, Yokoyama '20 & '21

$$S = \int d^4x \sqrt{-g} \left[ \frac{1}{12\alpha} (\chi^2 - \phi^2) R(g, \Gamma) + \frac{1}{2} (D_{\mu}) \right]$$

$$\begin{cases} g_{\mu\nu} \to \tilde{g}_{\mu\nu} = e^{-2\sigma(x)}g_{\mu\nu}, & \tilde{R} = \tilde{g}^{\mu\nu}R_{\mu\nu}(\tilde{\Gamma}) \\ \Gamma \to \tilde{\Gamma} = \Gamma, & \end{cases}$$

Gauge fixing:  $\chi^2 - \phi^2 = 6 \alpha M_{\rm Pl}^2$  $\begin{cases} \sqrt{6\alpha} M_{\rm Pl} \cosh \frac{\varphi}{\sqrt{6\alpha} M_{\rm Pl}} & \longrightarrow & V(\varphi) \propto \tanh^{2n} \frac{\varphi}{\sqrt{6\alpha} M_{\rm Pl}} \\ \sqrt{6\alpha} M_{\rm Pl} \sinh \frac{\varphi}{\sqrt{6\alpha} M_{\rm Pl}} & & & & & & & & \\ \end{cases}$ 

> c.f. MAG + conf. + O(2) $\rightarrow$  natural inf.  $V(\varphi) \propto 1 + \cos \frac{\varphi}{c}$

#### **Metric-Affine Gravity**

 $(\chi)^2 - \frac{1}{2} (D_\mu \phi)^2 - \frac{1}{4} F \left(\frac{\phi}{\chi}\right) (\chi^2 - \phi^2)^2$ 

 $= e^{2\sigma(x)}R$ 



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Primordial Black Hole



### **Primordial BH** Carr & Hawking '74

### Radiation D.



V. Martine



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### **Primordial BH** Carr & Hawking '74





# - almost arbitrary mass c.f. stellar BH $\gtrsim M_{\odot}$

almost zero spin







- Gravitational Lensing
- **Dynamical Friction**
- Accretion Y
- Prim. PTB
- Hawking Y



#### Massive than stellar BHs found

small spin



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**Editors' Suggestion** 

as primordial black holes (PBHs).

Featured in Physics

#### What If Planet 9 Is a Primordial Black Hole?

Jakub Scholtz<sup>1</sup> and James Unwin<sup>2</sup>

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Introduction.—As of this year, two gravitational anomalies of similar mass but very different origins remain to be explained. First, there is a growing body of observational anomalies connected to the orbits of trans-Neptunian objects (TNOs) [1–3]. These observations have been taken as evidence of a new ninth planet in our Solar System, called Planet 9 (P9), with mass  $M_9 \sim 5-15M_{\oplus}$  and orbiting around the Sun at a distance of 300–1000 AU [4]. Second, gravitational anomalies have also been recently observed by the Optical Gravitational Lensing Experiment (OGLE). OGLE reported an excess of six ultrashort microlensing events with crossing times of 0.1–0.3 days [5]. The lensing objects are located toward the galactic bulge, roughly 8 kpc away. These events correspond to lensing by objects of mass  $M \sim 0.5 M_{\oplus} - 20 M_{\oplus}$  [6] and could be interpreted as an unexpected population of free floating planets (FFPs) or

SUPPLEMENTARY MATERIAL

#### A. SIZE OF THE PBH The Schwarzschild radius of a black hole is given by $r_{\rm BH} = \frac{2GM_{\rm BH}}{c^2} \simeq 4.5 {\rm cm} \left(\frac{M_{\rm BH}}{5M_{\oplus}}\right)$ (15)

In Figure 1 we provide an exact scale image of a  $5M_{\oplus}$ PBH. The associated DM halo however extends to the stripping radius  $r_{t,\odot} \sim 8$ AU, this would imply a DM halo which extends roughly the distance from Earth to Saturn (both in real life and relative to the image).



FIG. 1. Exact scale (1:1) illustration of a  $5M_{\oplus}$  PBH. Note that a  $10M_{\oplus}$  PBH is roughly the size of a ten pin bowling ball

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#### **Multistage Inflation**

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**Multistage Inflation** 

 $\mathcal{P}_{\mathcal{R}} \sim 10^{-2}$ 

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## Double Inflation

Kumekawa, Moroi, Yanagida '94 Izawa, Kawasaki, Yanagida '97 Kawasaki, Sugiyama, Yanagida '98

 $V(\phi, \chi) = V_{\text{high}}(\phi) + V_{\text{hill}}(\chi) + \frac{c}{2}V_{\text{high}}(\phi)\frac{\chi^2}{M_{\text{Pl}}}$ Vc.f. SUGRA  $m_{\chi,\text{eff}}^2 = c V_{\text{high}} / M_{\text{Pl}}^2$ 



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 $\chi$ 

## Extreme case



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**Multistage Inflation** 

# Implication to String Theory?

dS swampland conjecture Ooguri & Vafa+ '18

"dS vacua will be unstable in UV-complete theories"

$$\epsilon_V = \frac{M_{\rm Pl}^2}{2} \left(\frac{V'}{V}\right)^2 \gtrsim \mathcal{O}(1) \quad \text{or} \quad \eta_V = M_{\rm Pl}^2 \frac{V''}{V} \lesssim -\mathcal{O}(1)$$

### \* CMB scale? Kogai & YT '20





**Multistage Inflation** 



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Stochastic Inflation



# Ptb. approach

coordinate so that ptb. only in Metric!

auxiliary  

$$d^{2}s = -\widehat{\mathcal{N}^{2}(t, \mathbf{x})}d^{2}t + a^{2}(t)e^{2\hat{\zeta}(t, \mathbf{x})}(\widehat{\beta^{i}(t, \mathbf{x})}dt + dx^{i})^{2}, \quad \phi(t, \mathbf{x}) = \phi_{0}(t)$$

$$S = \int d^{4}x \sqrt{-g} \left[\frac{1}{2}M_{\mathrm{Pl}}R - \frac{1}{2}(\partial_{\mu}\phi)^{2} - V(\phi)\right] = S^{(0)} + S^{(2)}[\zeta] + S^{(3)}[\zeta] + \cdots$$

$$\downarrow$$

$$\langle \zeta(t, \mathbf{x})\zeta(t, \mathbf{y}) \rangle = \langle 0_{\mathrm{in}} | \zeta(t, \mathbf{x})\zeta(t, \mathbf{y}) | 0_{\mathrm{in}} \rangle$$

$$= \left\langle \left[\overline{T}\exp\left(i\int^{t}H_{I}dt'\right)\right] \zeta^{I}(t, \mathbf{x})\zeta^{I}(t, \mathbf{y}) \left[T\exp\left(-i\int^{t}H_{I}dt'\right)\right] \right\rangle$$

Stochastic Inflation

### Stochastic Form. Starobinsky '86



![](_page_30_Picture_2.jpeg)

### (conserved) $\delta N$ Form. Starobinsky '85

![](_page_31_Figure_1.jpeg)

Stochastic Inflation

![](_page_31_Picture_4.jpeg)

# Stochastic $\delta N$

Fujita, Kawasaki, YT, Takesako '13 Vennin & Starobinsky '15

![](_page_32_Figure_2.jpeg)

![](_page_32_Picture_3.jpeg)

## c.f. stock price in finance When are you expected to achieve the goal?

## Finance in (Cosmic) Inflation!

![](_page_32_Picture_6.jpeg)

# Flat Inflection

![](_page_33_Figure_1.jpeg)

![](_page_33_Picture_2.jpeg)

![](_page_33_Figure_3.jpeg)

c.f. Ezquiaga, Garcia-Bellido, Vennin '20

![](_page_33_Picture_5.jpeg)

![](_page_33_Picture_6.jpeg)

# Conclusions

![](_page_34_Picture_1.jpeg)

![](_page_34_Picture_2.jpeg)

![](_page_34_Picture_3.jpeg)

![](_page_34_Picture_4.jpeg)

Stochastic approach to inflationary ptb.

![](_page_34_Picture_6.jpeg)

![](_page_34_Picture_10.jpeg)

![](_page_34_Picture_11.jpeg)

![](_page_35_Picture_0.jpeg)

Appendices

![](_page_35_Picture_2.jpeg)

# Higgs inf. $S = \int d^4x \sqrt{-g} \left[ \frac{1}{2} (M_{\rm Pl}^2 + M_{\rm Pl}^2) \right] d^4x \sqrt{-g} \left[ \frac{1}{2} (M_{\rm Pl}^2 + M_{\rm Pl}^2) \right] d^4x \sqrt{-g} \left[ \frac{1}{2} (M_{\rm Pl}^2 + M_{\rm Pl}^2) \right] d^4x \sqrt{-g} \left[ \frac{1}{2} (M_{\rm Pl}^2 + M_{\rm Pl}^2) \right] d^4x \sqrt{-g} \left[ \frac{1}{2} (M_{\rm Pl}^2 + M_{\rm Pl}^2) \right] d^4x \sqrt{-g} \left[ \frac{1}{2} (M_{\rm Pl}^2 + M_{\rm Pl}^2) \right] d^4x \sqrt{-g} \left[ \frac{1}{2} (M_{\rm Pl}^2 + M_{\rm Pl}^2) \right] d^4x \sqrt{-g} \left[ \frac{1}{2} (M_{\rm Pl}^2 + M_{\rm Pl}^2) \right] d^4x \sqrt{-g} \left[ \frac{1}{2} (M_{\rm Pl}^2 + M_{\rm Pl}^2) \right] d^4x \sqrt{-g} \left[ \frac{1}{2} (M_{\rm Pl}^2 + M_{\rm Pl}^2) \right] d^4x \sqrt{-g} \left[ \frac{1}{2} (M_{\rm Pl}^2 + M_{\rm Pl}^2) \right] d^4x \sqrt{-g} \left[ \frac{1}{2} (M_{\rm Pl}^2 + M_{\rm Pl}^2) \right] d^4x \sqrt{-g} \left[ \frac{1}{2} (M_{\rm Pl}^2 + M_{\rm Pl}^2) \right] d^4x \sqrt{-g} \left[ \frac{1}{2} (M_{\rm Pl}^2 + M_{\rm Pl}^2) \right] d^4x \sqrt{-g} \left[ \frac{1}{2} (M_{\rm Pl}^2 + M_{\rm Pl}^2) \right] d^4x \sqrt{-g} \left[ \frac{1}{2} (M_{\rm Pl}^2 + M_{\rm Pl}^2) \right] d^4x \sqrt{-g} \left[ \frac{1}{2} (M_{\rm Pl}^2 + M_{\rm Pl}^2) \right] d^4x \sqrt{-g} \left[ \frac{1}{2} (M_{\rm Pl}^2 + M_{\rm Pl}^2) \right] d^4x \sqrt{-g} \left[ \frac{1}{2} (M_{\rm Pl}^2 + M_{\rm Pl}^2) \right] d^4x \sqrt{-g} \left[ \frac{1}{2} (M_{\rm Pl}^2 + M_{\rm Pl}^2) \right] d^4x \sqrt{-g} \left[ \frac{1}{2} (M_{\rm Pl}^2 + M_{\rm Pl}^2) \right] d^4x \sqrt{-g} \left[ \frac{1}{2} (M_{\rm Pl}^2 + M_{\rm Pl}^2) \right] d^4x \sqrt{-g} \left[ \frac{1}{2} (M_{\rm Pl}^2 + M_{\rm Pl}^2) \right] d^4x \sqrt{-g} \left[ \frac{1}{2} (M_{\rm Pl}^2 + M_{\rm Pl}^2) \right] d^4x \sqrt{-g} \left[ \frac{1}{2} (M_{\rm Pl}^2 + M_{\rm Pl}^2) \right] d^4x \sqrt{-g} \left[ \frac{1}{2} (M_{\rm Pl}^2 + M_{\rm Pl}^2) \right] d^4x \sqrt{-g} \left[ \frac{1}{2} (M_{\rm Pl}^2 + M_{\rm Pl}^2) \right] d^4x \sqrt{-g} \left[ \frac{1}{2} (M_{\rm Pl}^2 + M_{\rm Pl}^2) \right] d^4x \sqrt{-g} \left[ \frac{1}{2} (M_{\rm Pl}^2 + M_{\rm Pl}^2) \right] d^4x \sqrt{-g} \left[ \frac{1}{2} (M_{\rm Pl}^2 + M_{\rm Pl}^2) \right] d^4x \sqrt{-g} \left[ \frac{1}{2} (M_{\rm Pl}^2 + M_{\rm Pl}^2) \right] d^4x \sqrt{-g} \left[ \frac{1}{2} (M_{\rm Pl}^2 + M_{\rm Pl}^2) \right] d^4x \sqrt{-g} \left[ \frac{1}{2} (M_{\rm Pl}^2 + M_{\rm Pl}^2) \right] d^4x \sqrt{-g} \left[ \frac{1}{2} (M_{\rm Pl}^2 + M_{\rm Pl}^2) \right] d^4x \sqrt{-g} \left[ \frac{1}{2} (M_{\rm Pl}^2 + M_{\rm Pl}^2) \right] d^4x \sqrt{-g} \left[ \frac{1}{2} (M_{\rm Pl}^2 + M_{\rm Pl}^2) \right] d^4x \sqrt{-g} \left[ \frac{1}{2} (M_{\rm Pl}^2 + M_{\rm Pl}^2) \right] d^4x \sqrt{-g} \left[ \frac{1}{2} (M_{\rm Pl}^2 + M_{\rm Pl}^2) \right] d^4x \sqrt{-g} \left[ \frac{1}{2} (M_{\rm Pl}^2 + M_{\rm Pl}^2) \right] d^4x \sqrt{-g} \left[ \frac{1}{2} (M_{\rm Pl}^2 + M$

- Metric  

$$V(\chi) \sim \frac{\lambda M_{\rm Pl}^4}{\xi^2} \left( 1 - \exp\left(-2\sqrt{\frac{\xi}{1+6\xi}}\frac{\chi}{M_{\rm Pl}}\right) \right)$$

$$\xi \to \infty : R^2 \text{ inflation}$$

![](_page_36_Figure_2.jpeg)

#### Metric-Affine Gravity

$$\begin{split} \xi \phi^2 R &= \frac{1}{2} (\partial_\mu \phi)^2 - \frac{\lambda}{4} \phi^4 \\ & - \text{ Metric-Affine} \\ & V(\chi) \sim \frac{\lambda M_{\text{Pl}}^4}{\xi^2} \left( 1 - 8 \exp\left(-2\sqrt{\xi} \frac{\chi}{M_{\text{Pl}}}\right) \right) \\ & \alpha \text{ attractor} \\ & \lambda_{\text{cut}} \sim \Lambda_{\text{inf}} \sim \frac{M_{\text{Pl}}}{\sqrt{\xi}} \\ & Y \text{ Ema+ 20} \end{split}$$

## Testability

![](_page_37_Figure_1.jpeg)

![](_page_37_Picture_2.jpeg)

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![](_page_37_Figure_4.jpeg)