Flow equation, black hole, and singularity

27 July 2020 @ Nagoya U

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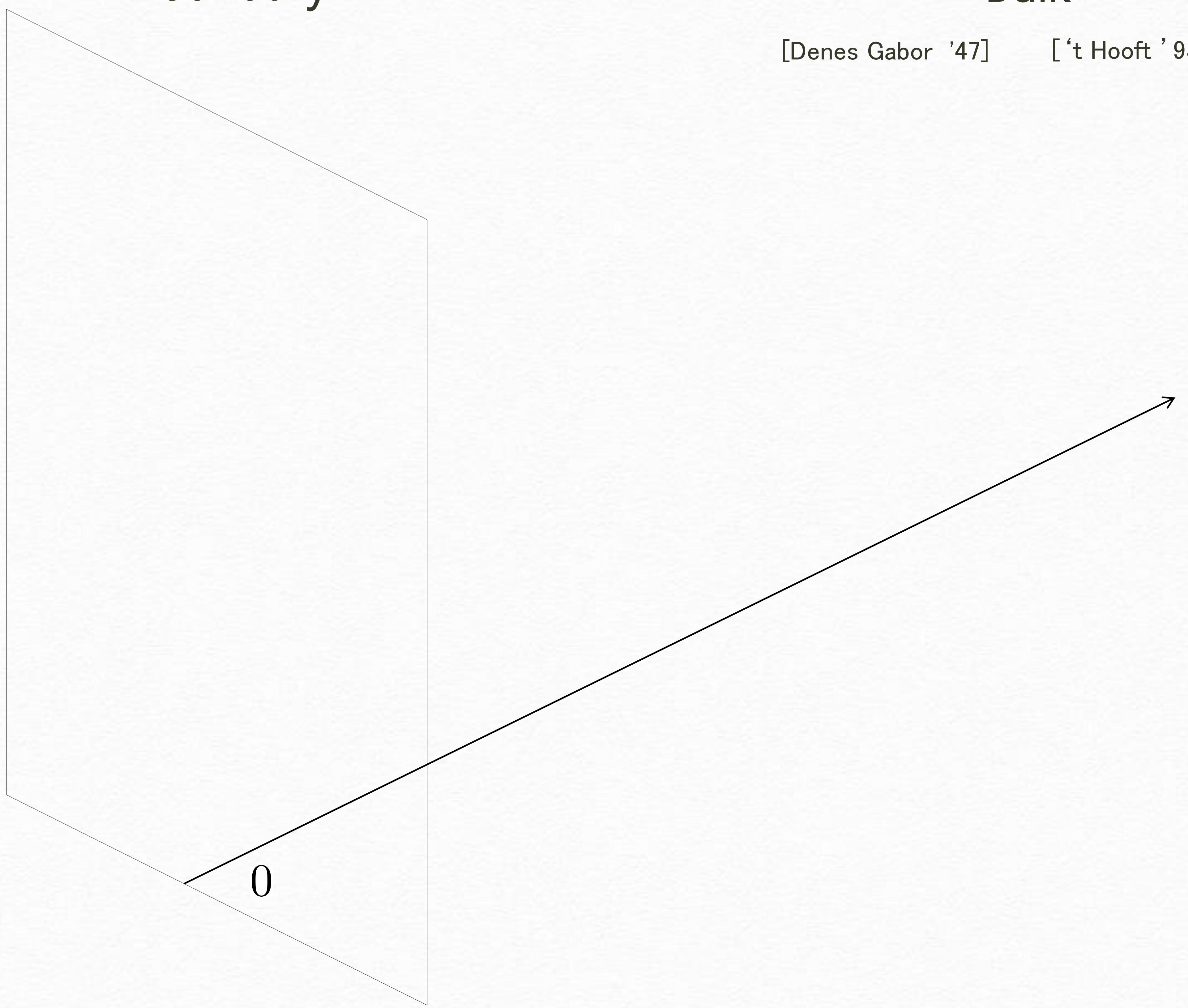
Refs. S.Aoki-T.Onogi-SY arXiv:2004.03779 [hep-th]

S.Aoki-T.Onogi-SY arXiv:2005.13233 [gr-qc]

Holography

Boundary ⟷ Bulk

[Denes Gabor '47] ['t Hooft '93, Susskind '94]



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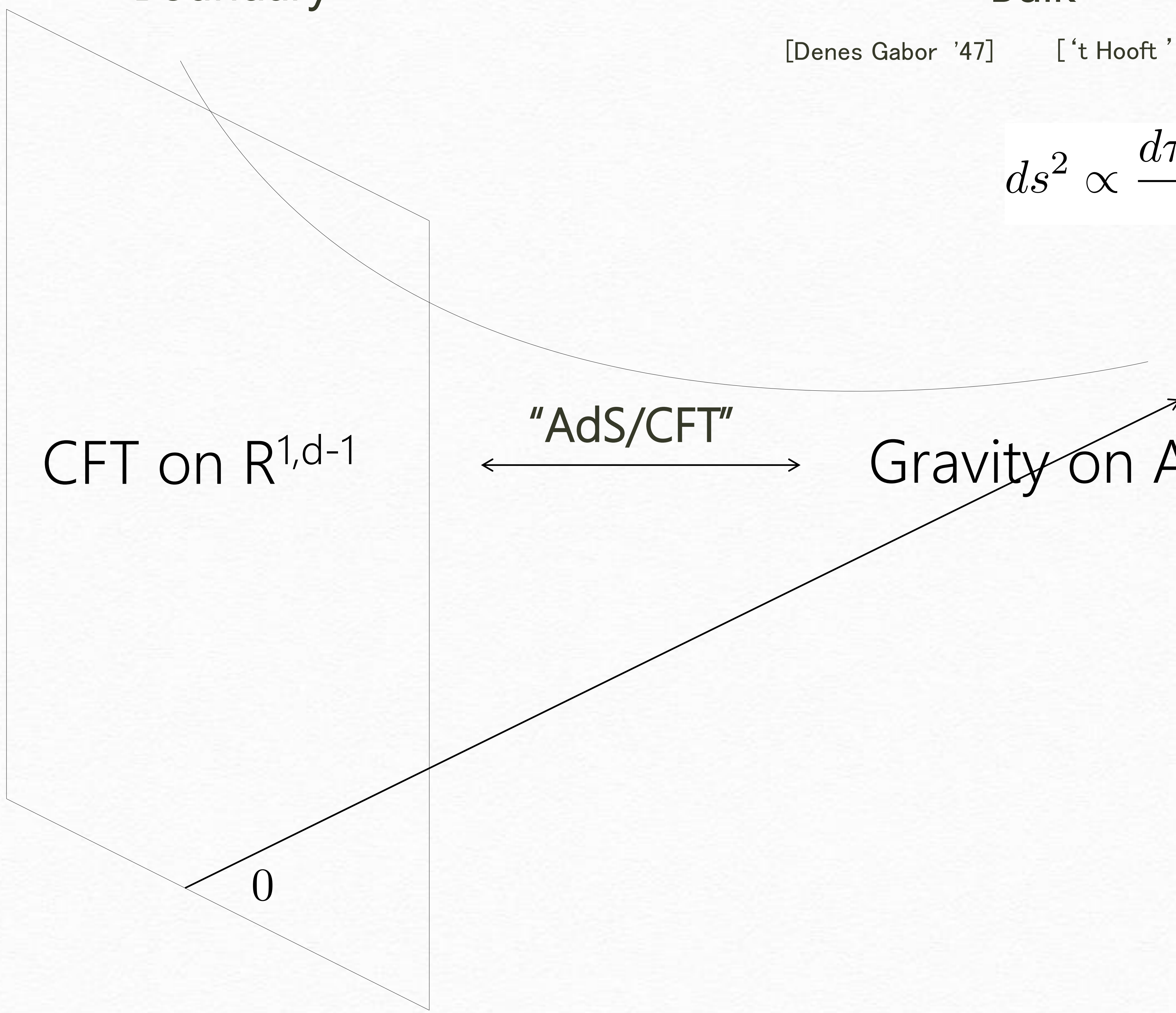
$$ds^2 \propto \frac{d\tau^2 + (dx^\mu)^2}{\tau^2}$$



CFT on $R^{1,d-1}$ ⟷ "AdS/CFT" ⟷ Gravity on AdS_{d+1}

[Maldacena '97]

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How?

τ

CFT on $R^{1,d-1}$ "AdS/CFT" Gravity on AdS_{d+1}

[Banks–Douglas–Horowitz–Martinec '98]...

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[Banks-Douglas-Horowitz-Martinec '98]...

[Maldacena '97]

- Relevant RG defromation

[Girardello-Petrini-Porrati-Zaffaroni '98] [Distler-Zamora '98]

- Wilsonian RG scale

[Heemskerk-Polchinski '10]

- Relative coordinate in bilocal field

[Das-Jevicki '03]

- Entanglement entropy, cMERA

[Ryu-Takayanagi '06], [Swingle '09]

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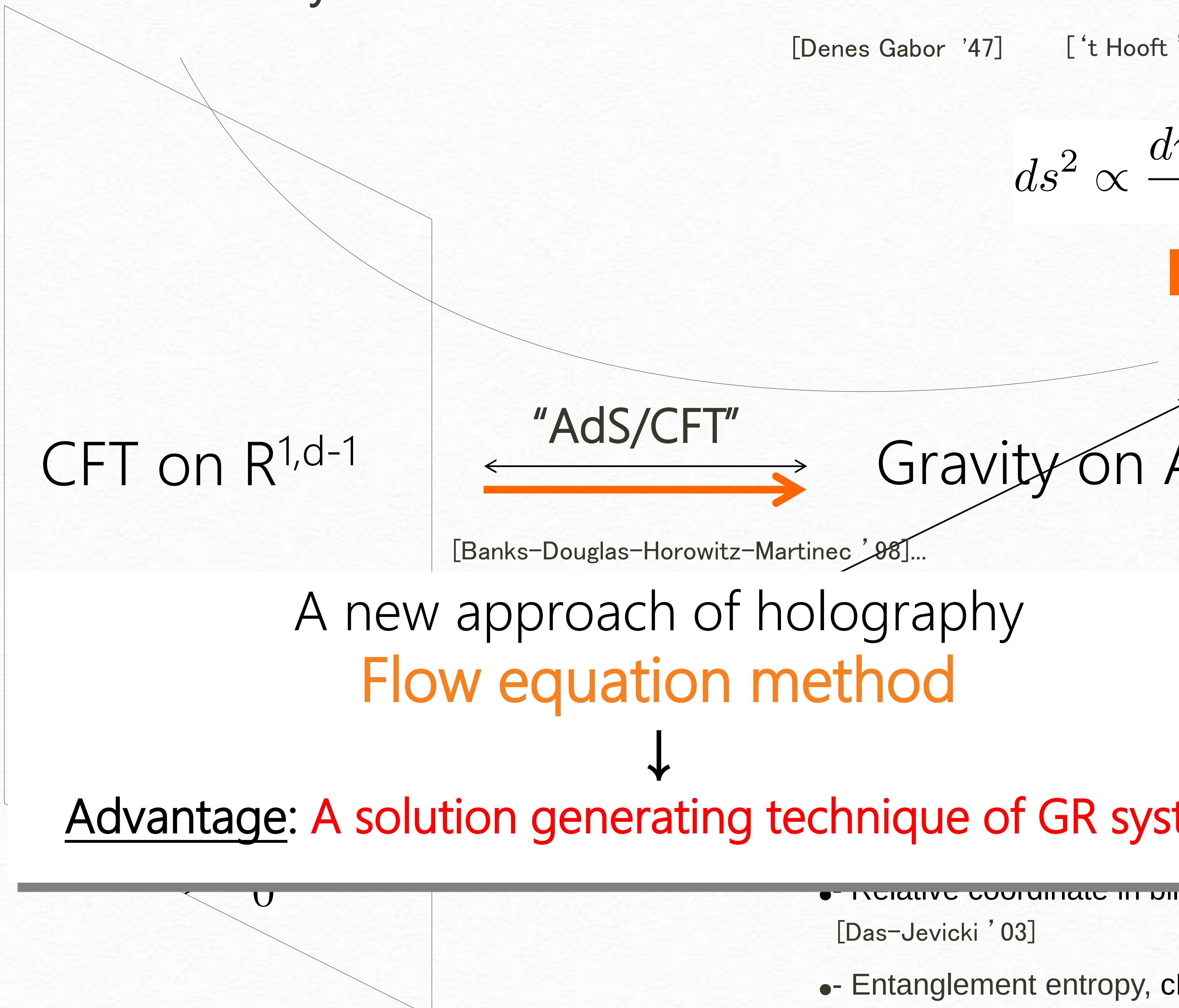

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[Maldacena '97]

A new approach of holography

Flow equation method

↓

<u>Advantage</u>: A solution generating technique of GR system

ra '98]

- Relative coordinate in bilocal field

[Das–Jevicki '03]

- Entanglement entropy, cMERA

[Ryu–Takayanagi '06], [Swingle '09]

Plan

✓ 1. Introduction

2. Flow equation and bulk construction

3. Holographic geometry at finite T

4. Conserved charge in GR

5. Summary

(Gradient) flow equation

(Gradient) flow equation

1. was introduced to help numerics of lattice QCD.

cf. def of stress energy tensor

[Albanese et al. (APE) '87] [Narayanan-Neuberger '06] [Luscher '10,'13]

2. describes a non-local course-graining of an operator.

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2. describes a non-local course-graining of an operator.

Consider a CFT_d which contains a primary scalar φ

$$\langle \phi(x_1)\phi(x_2)\rangle = \frac{1}{x_{12}^{2\Delta}}$$

$$x_{12} := x_1 - x_2$$

Free flow equation

$$\boxed{\frac{\partial \phi(x;\eta)}{\partial \eta} = \partial^2 \phi(x;\eta). \quad \phi(x;0) = \phi(x)}$$

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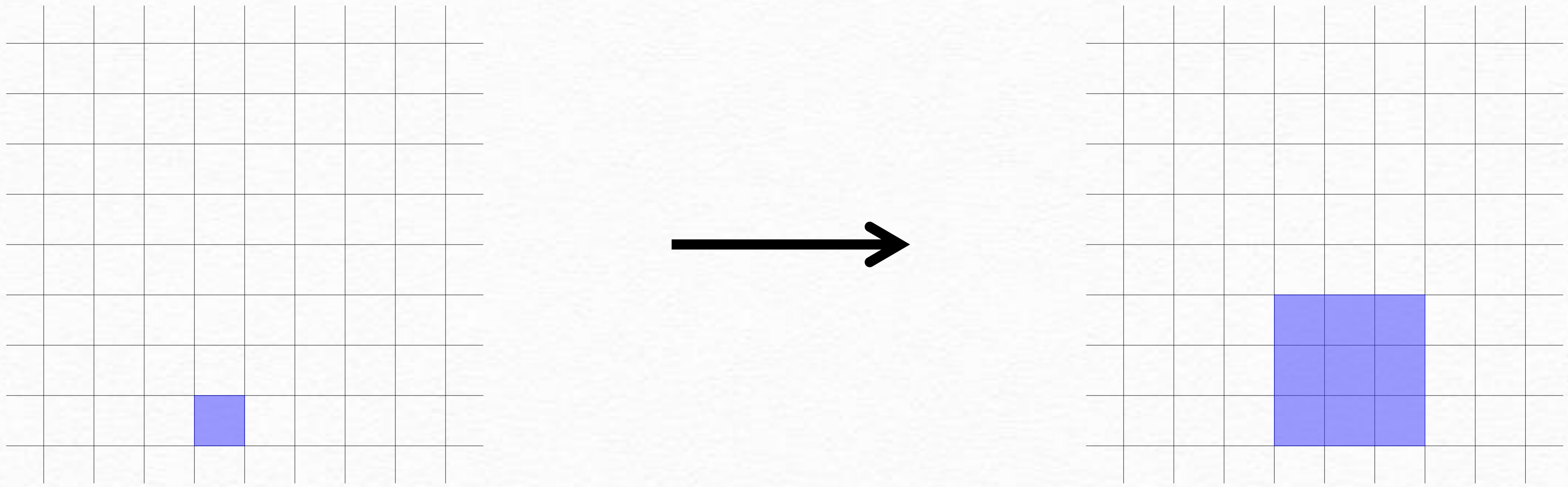
The solution: $\phi(x;\eta) = \int d^d y\, K(x-y;\eta)\phi(y).$ $\quad K(x-y;\eta) = \frac{e^{-(x-y)^2/4\eta}}{(4\pi\eta)^{d/2}}$

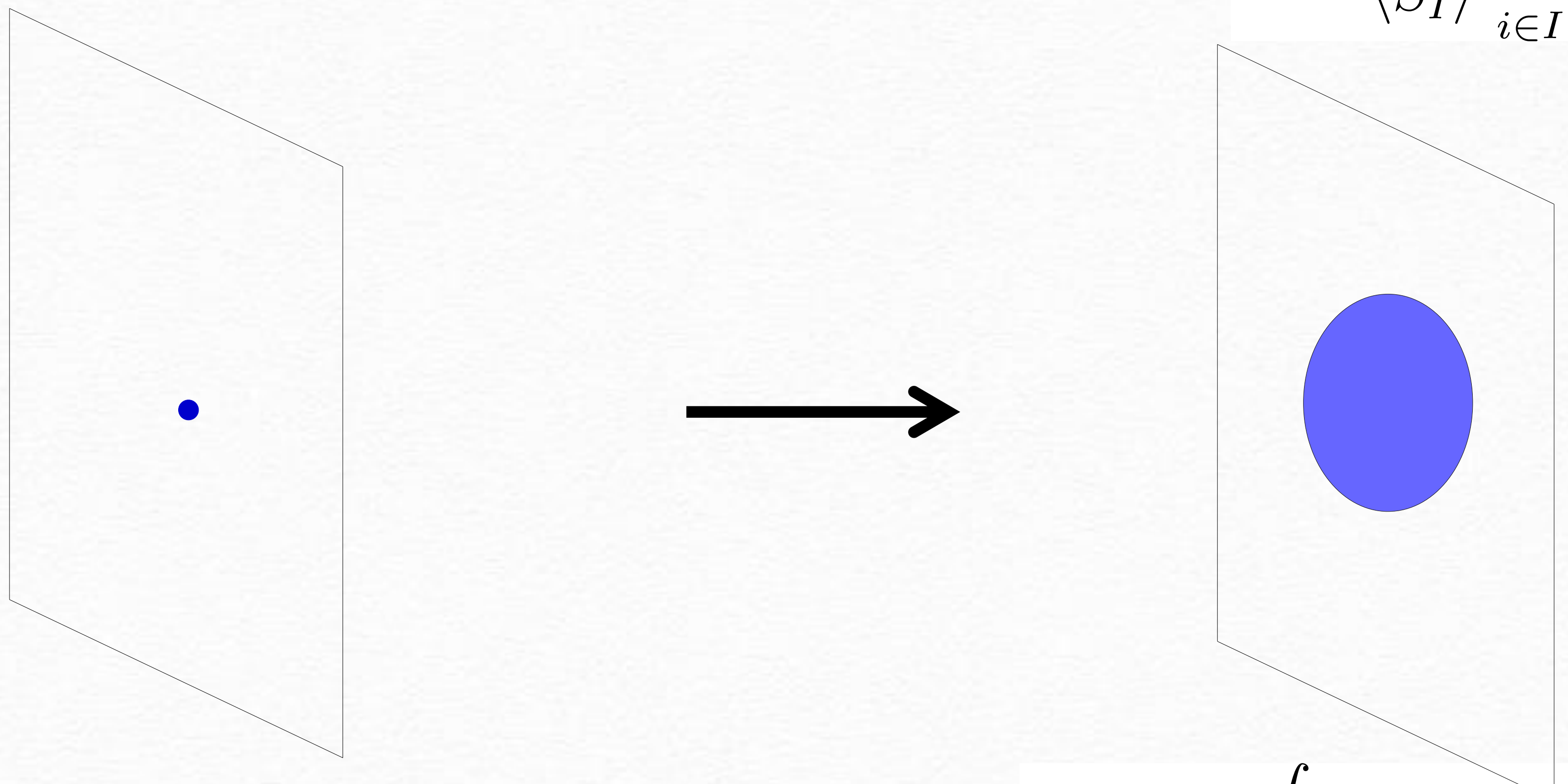
➡ Reminiscent of the block spin transformation!?

Block spin v.s. Flowed operator



s_i

$$S_I = \frac{1}{\langle S_I \rangle} \sum_{i \in I} s_i$$



$\phi(x)$

"Point particle"

$$\phi(x;\eta) = \int d^d y \, K(x-y;\eta)\phi(y).$$

"Gaussian wave packet"

(Gradient) flow equation

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➡ Reminiscent of the block spin transformation!?

<u>Claim</u>: Contact singularity in 2pt function is resolved.

$$\langle\phi(x_1;\eta_1)\phi(x_2;\eta_2)\rangle = \frac{1}{\eta_+^{\Delta}}F(\frac{x_{12}^2}{\eta_+};1) \qquad \eta_+ := \eta_1 + \eta_2$$

$$F(v;1) = \frac{1}{(4\pi)^{\frac{d}{2}}}\int_0^1 du(1-u)^{d/2-\Delta-1}e^{-vu/4}u^{\Delta-1} \qquad \frac{d-2}{2} \le \Delta < \frac{d-1}{2}$$

Construction of holographic space

[Aoki-Kikuchi-Onogi ’15]

[Aoki-Balog-Onogi-Weisz ’16,’17]

[Aoki-SY ’17]

Metric operator and holographic metric

[Aoki–Kikuchi–Onogi '15] [Aoki–Balog–Onogi–Weisz '16,'17]

Def. (Normalized operator)

$$\sigma^a(x;\eta) := \frac{\phi^a(x;t)}{\sqrt{\langle \phi^b(x;\eta)^2 \rangle}}$$

$$\langle \sigma^a(x;\eta)\sigma^b(x;\eta) \rangle = \frac{1}{n}\delta^{ab}$$

NOTE: This is well-defined due to the absence of the contact singularity.

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Def. (Metric operator)

$$\hat{g}_{AB}(x;\eta) := \lim_{(x',\eta')\to(x,\eta)} \frac{\partial}{\partial X^A}\sigma^a(x;\eta)\frac{\partial}{\partial X'^B}\sigma^a(x';\eta'),$$

$$(X^A) = (x^\mu, \tau) \text{ with } \tau := \sqrt{\eta/\alpha}$$

NOTE:

1. An induced metric can be interpreted as the information metric. [Aoki–SY '17]
2. The metric operator is singlet under the transformation related to the index.

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Def. (Metric of the holographic space)

$$g_{AB}(X) := \langle \hat{g}_{AB}(x;\eta)\rangle \,,$$

Explicit computation of holographic metric

Let us compute the induced metric in this free O(n) vector model.

$$\langle \sigma(x_1;\eta_1)\sigma(x_2;\eta_2)\rangle = \left(\frac{2\sqrt{\eta_1\eta_2}}{\eta_+}\right)^{\Delta} F\left(\frac{x_{12}^2}{\eta_+}\right) \qquad F(u) := F_0(u;1)/F_0(0;1)$$

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$$\rightarrow \quad g_{\eta\eta}(X) = \lim \partial_\eta \partial_{\eta'} \left(\frac{2\sqrt{\eta_1\eta_2}}{\eta_+}\right)^{\Delta} F(\frac{x_{12}^2}{\eta_+}) = 2^{\Delta}\frac{\Delta}{\eta^2 2^{\Delta+2}} = \frac{\Delta}{4\eta^2}$$

$$g_{ij}(X) = \lim \partial_i \partial_{j'} \left(\frac{2\sqrt{\eta_1\eta_2}}{\eta_+}\right)^{\Delta} F(\frac{x_{12}^2}{\eta_+}) = \lim \partial_{j'} \left(\frac{2\sqrt{\eta_1\eta_2}}{\eta_+}\right)^{\Delta} \frac{2x_{12}^i}{\eta_+} F'(\frac{x_{12}^2}{\eta_+})$$

$$= \lim \left(\frac{2\sqrt{\eta_1\eta_2}}{\eta_+}\right)^{\Delta} \frac{-2\delta_{ij}}{\eta_+} F'(\frac{x_{12}^2}{\eta_+}) = \frac{-\delta_{ij}}{\eta} F'(0)$$

$$\because \quad \partial_\eta \frac{\sqrt{4\eta\eta'}^{\Delta}}{\eta_+^{\Delta}} = \frac{\sqrt{4\eta\eta'}^{\Delta}}{\eta_+^{\Delta+1}} \frac{\Delta}{2}(-1+\frac{\eta'}{\eta}) \to 0,$$

$$\partial_{\eta'}\partial_\eta \frac{\sqrt{4\eta\eta'}^{\Delta}}{\eta_+^{\Delta}} \to \frac{\sqrt{4\eta\eta'}^{\Delta}}{\eta_+^{\Delta+1}} \frac{\Delta}{2}\frac{1}{\eta} \to \frac{\Delta}{2}\frac{1}{2\eta^2}$$

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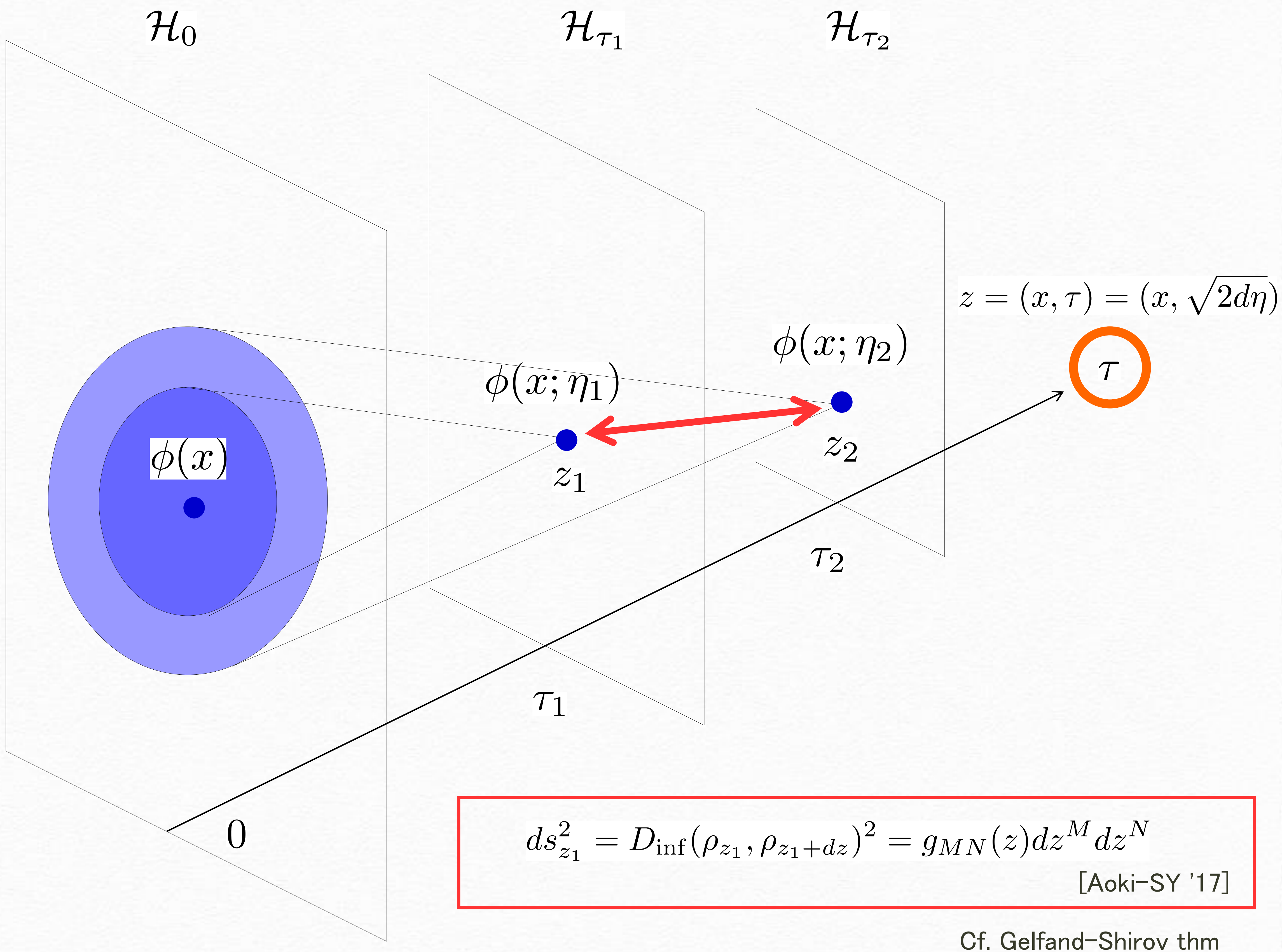
$$\rightarrow \quad ds^2 = \frac{\Delta}{4\eta^2}d\eta^2 + F'(0)\frac{-1}{\eta}(dx^i)^2 = \frac{\Delta}{z^2}dz^2 + F'(0)\frac{-1}{\alpha z^2}(dx^i)^2 = \Delta(\frac{dz^2+(dx^i)^2}{z^2})$$

$\eta = \alpha z^2$ with $\alpha = -\frac{F'(0)}{\Delta} = 1/2d$ **AdS metric!!** $L_{AdS}^2 = \Delta = \frac{d-2}{2}$

Smearing → Extra direction



$$ds^2_{z_1} = D_{\inf}(\rho_{z_1}, \rho_{z_1+dz})^2 = g_{MN}(z)dz^M dz^N$$

[Aoki–SY '17]

Cf. Gelfand–Shirov thm

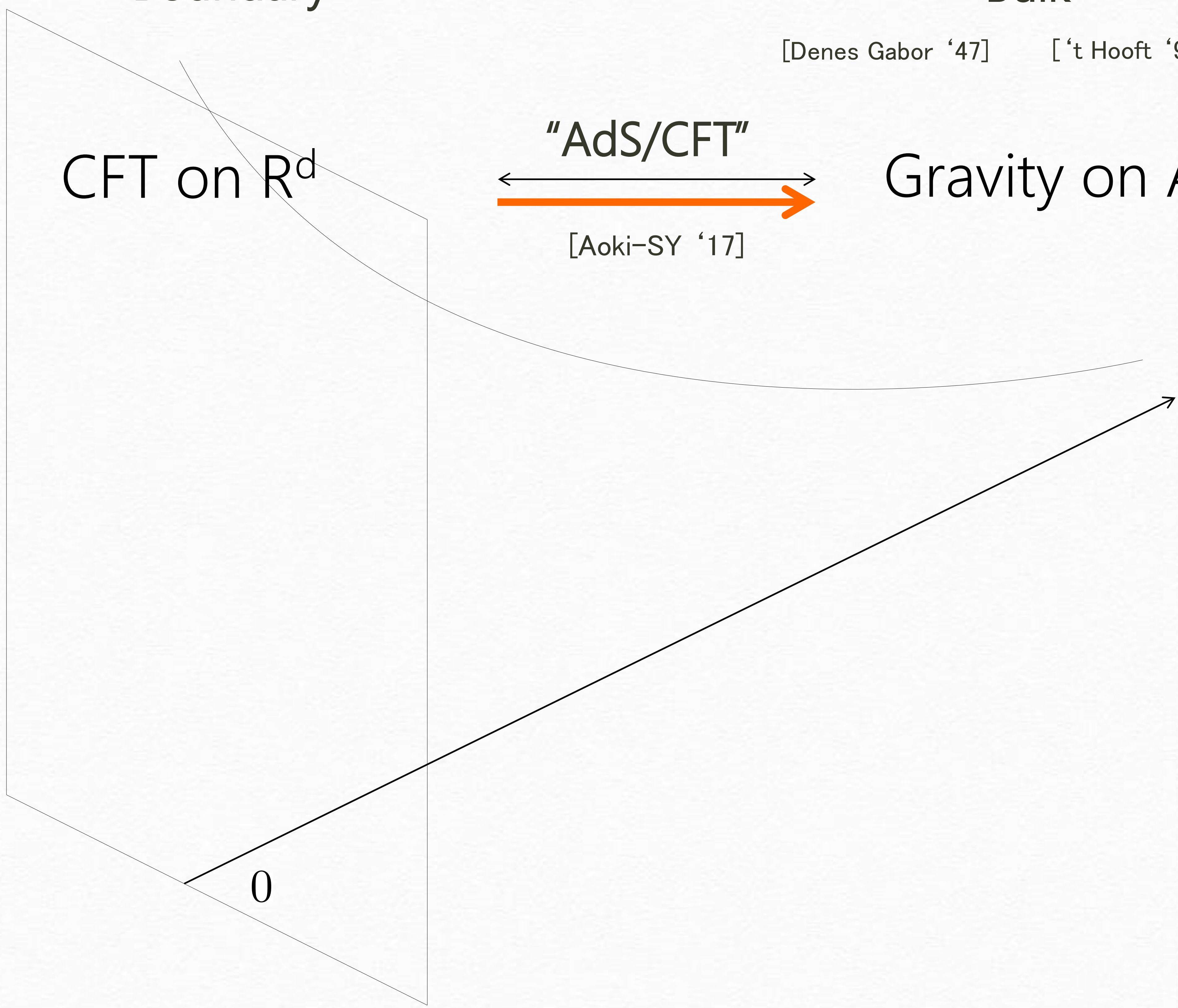
Holography

Boundary ⟷ Bulk

[Denes Gabor '47] ['t Hooft '93, Susskind '94]

CFT on R^d $\xleftrightarrow{\text{"AdS/CFT"}}$ Gravity on AdS_{d+1}

[Aoki-SY '17] [Maldacena ' 97]



Holography

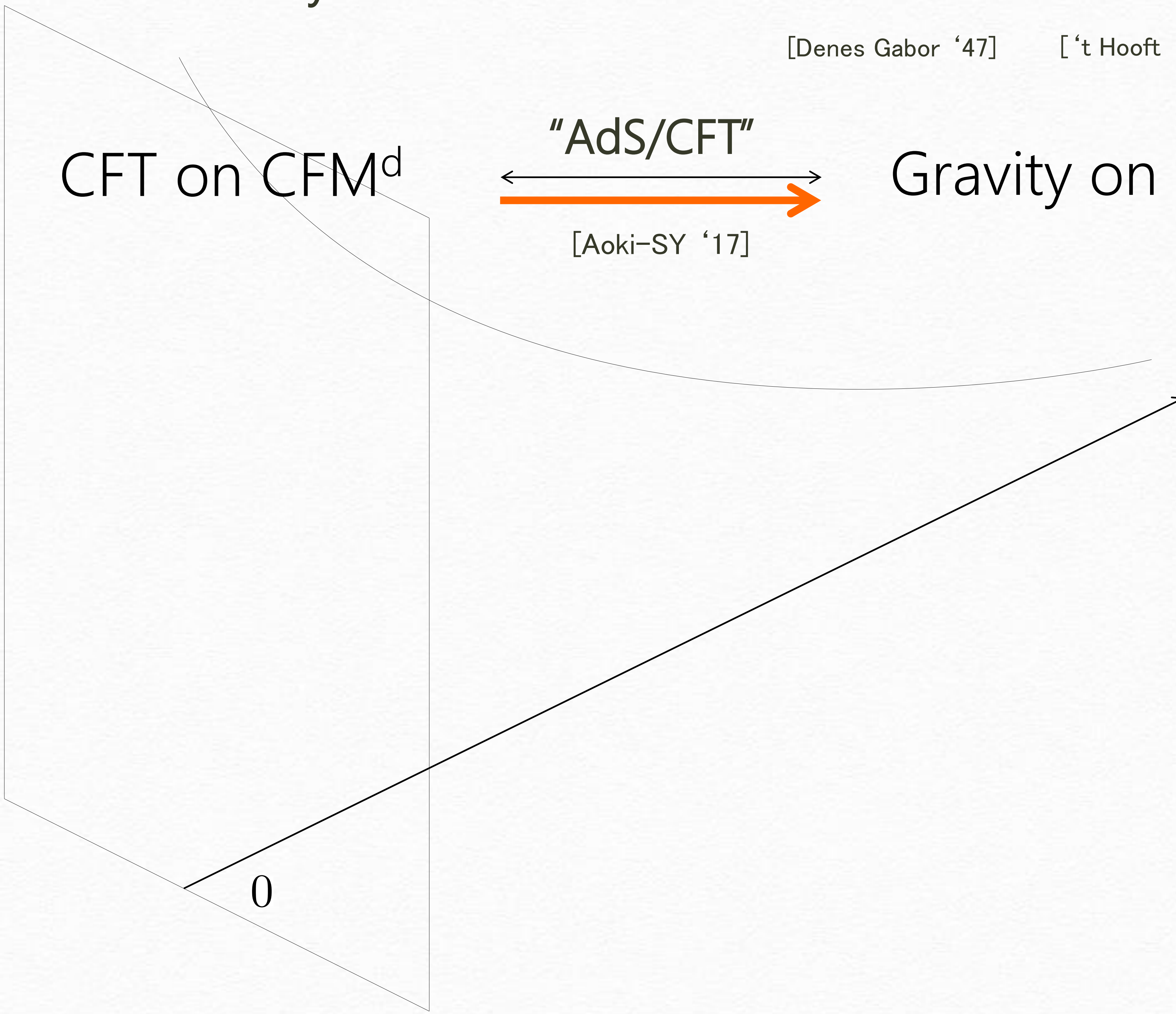
Boundary $\longleftrightarrow$ Bulk

[Denes Gabor '47] ['t Hooft '93, Susskind '94]

"AdS/CFT"

CFT on CFM^d $\longleftrightarrow$ Gravity on AdS$_{d+1}$

[Aoki-SY '17] [Maldacena '97]



Holography

Boundary ⟷ Bulk

Flow equation

[Denes Gabor ‘47] [‘t Hooft ‘93, Susskind ‘94]

CFT on CFM^{d} ⟷ “AdS/CFT” ⟷ Gravity on AdS$_{d+1}$

[Aoki–SY ‘17]

[Maldacena ’97]

NR CFT on R$^{1,d-1}$ ⟷ “AdS/CMP” ⟷ Gravity on Sch$^{\ \ \ d+2}_{\tau}$

[Aoki–SY–Yoshida ‘19]

[Son ’08, Balasubramanian–McGreevy ‘08]

Lifshitz FT on R$^{1,d-1}$ ⟷ Gravity on Lifs$_{d+1}$

[Aoki–SY–Yoshida ‘19]

[Kachru–Liu–Mulligan ‘08]

NR CFT on R$^{1,d-1}$ ⟷ Gravity on NRH$_{d+2}$

[Aoki–SY–Yoshida ‘19]

[Aoki–Balog–SY–Yoshida ‘19]

Holography

Boundary	Flow equation	Bulk
		[Denes Gabor '47] ['t Hooft '93, Susskind '94]
CFT on CFM^d	"AdS/CFT" [Aoki–SY '17]	Gravity on AdS_{d+1} [Maldacena '97]
NR CFT on $R^{1,d-1}$	"AdS/CMP" [Aoki–SY–Yoshida '19]	Gravity on Sch^{τ}_{d+2} [Son '08, Balasubramanian–McGreevy '08]
Lifshitz FT on $R^{1,d-1}$	[Aoki–SY–Yoshida '19]	Gravity on $Lifs_{d+1}$ [Kachru–Liu–Mulligan '08]
NR CFT on $R^{1,d-1}$	[Aoki–SY–Yoshida '19]	Gravity on NRH_{d+2} [Aoki–Balog–SY–Yoshida '19]
Sp(N) VM on $R^{1,d-1}$	"dS/CFT" [SY '20]	Gravity on dS_{d+1} [Strominger '01]

Holography

Boundary	$\longleftrightarrow$	Bulk
	Flow equation	[Denes Gabor '47] ['t Hooft '93, Susskind '94]
CFT on CFM^{d}	"AdS/CFT" [Aoki–SY '17]	Gravity on AdS$_{d+1}$ [Maldacena '97]
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CFT on S^{1} x R^{d-1}	[Aoki–Onogi–SY '20]	Gravity on AdSBH$_{d+1}$ **Today!**

Plan

✓ 1. Introduction

✓ 2. Flow equation and bulk construction

3. Holographic geometry at finite T

4. Conserved charge in GR

5. Summary

Expectation

Confine/Deconfine v.s. Hawking-Page

Boundary $\longleftrightarrow$ Bulk

N=4 SYM on $R^1 \times S^d$ $\xleftrightarrow{\text{“AdS/CFT”}}$ Type IIB on $gAdS_{d+1}$

[Maldacena '97]

Confine/Deconfine v.s. Hawking-Page

Boundary $\longleftrightarrow$ Bulk

N=4 SYM on $S^1 \times S^d$ $\xleftrightarrow[\text{[Maldacena '97]}]{\text{"AdS/CFT"}}$ Type IIB on $gAdS_{d+1}$

T<Tc: Confinment phase $\longleftrightarrow$ T<Tc: Thermal gAdS

[Witten '98]

T>Tc: Deconfinment phase $\longleftrightarrow$ T>Tc: gAdSBH

$$T_c = \frac{c}{R}$$

Confine/Deconfine v.s. Hawking-Page

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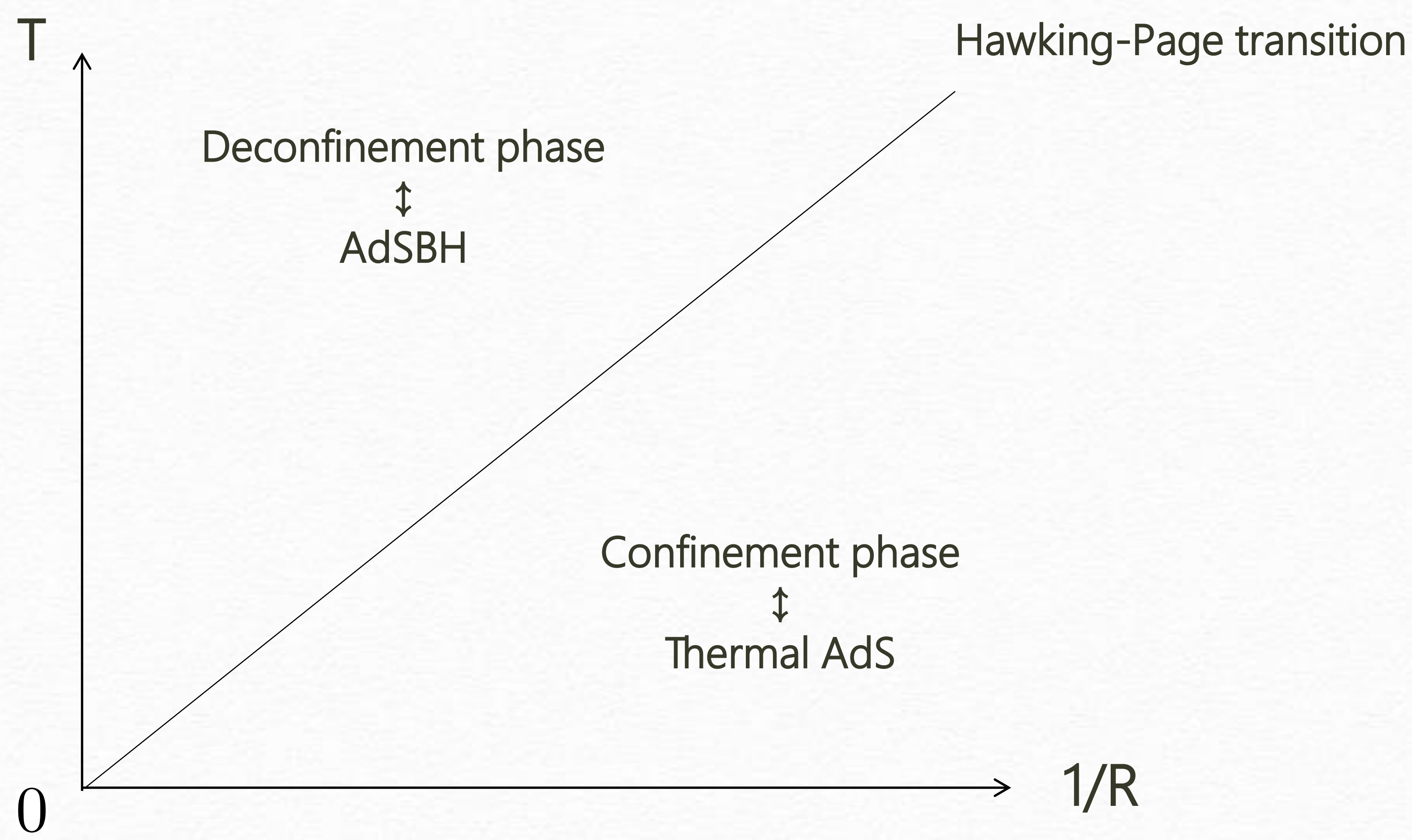

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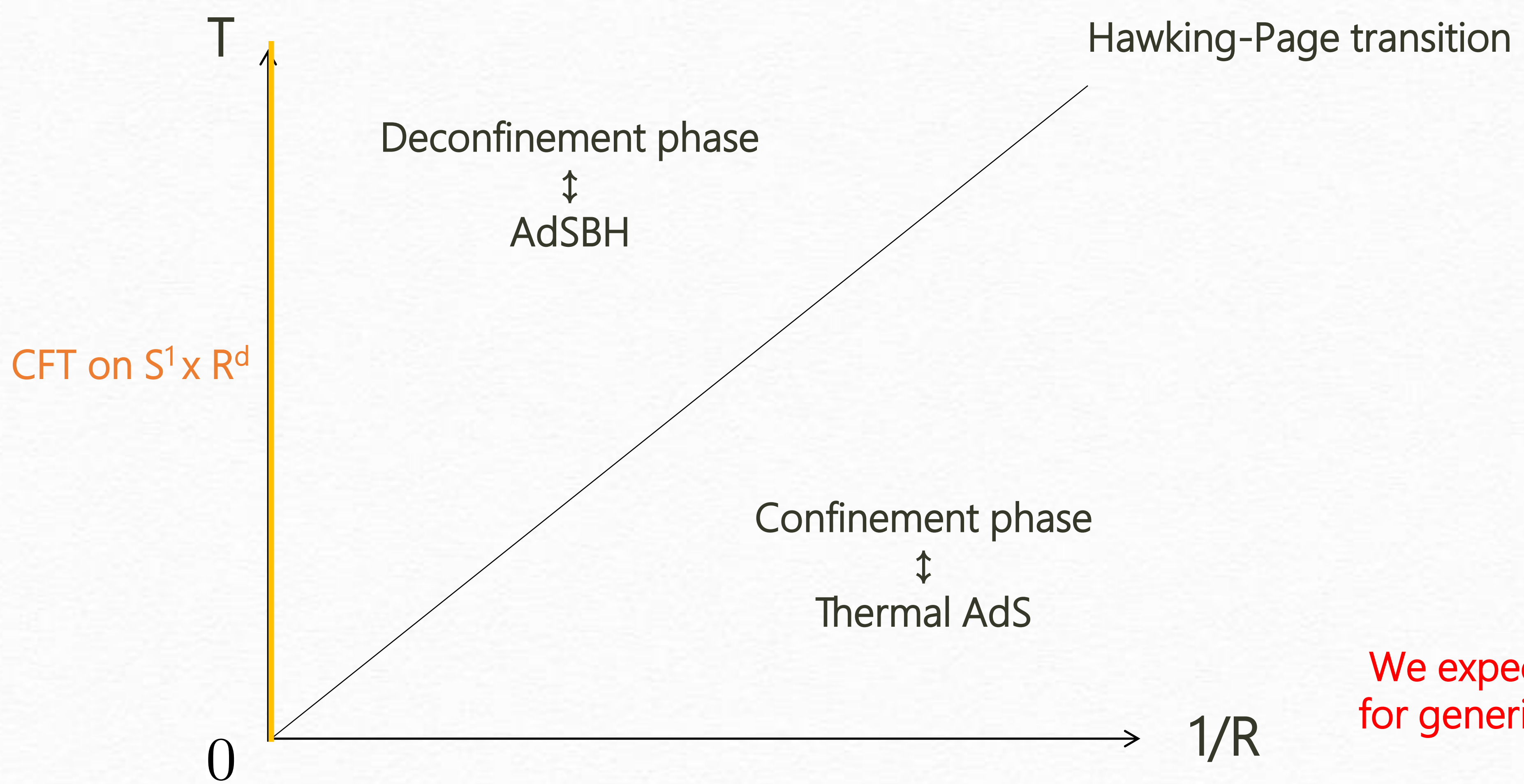

We expect this behavior for generic CFT on $S^1 \times S^d$

Vector-Model/Higher-Spin duality

Boundary $\longleftrightarrow$ Bulk

VM on S^1 x R^d $\xleftrightarrow{\text{"VM/HS"}}$ HS on AdSBH_{d+1}

[Klebanov–Polyakov '02]

Euclidean AdSBH_{d+1} metric

$$ds^2 = \frac{\mathrm{L}_{\mathrm{AdS}}^2}{\tau^2}\left(f^{-1}(\mathrm{z})\mathrm{d}\tau^2 + f(\mathrm{z})(\mathrm{d}x^0)^2 + \sum_{i=1}^{d-1}(\mathrm{d}x^i)^2\right)$$

where $$f(z) = 1 - \frac{z_H^d}{z^d}$$

Vector-Model/Higher-Spin duality

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VM on S^1 x R^d $\xleftrightarrow{\text{"VM/HS"}}$ HS on $AdSBH_{d+1}$

[Klebanov–Polyakov '02]

Euclidean $AdSBH_{d+1}$ metric

$$ds^2 = \frac{L_{AdS}^2}{\tau^2}\left(f^{-1}(z)d\tau^2 + f(z)(dx^0)^2 + \sum_{i=1}^{d-1}(dx^i)^2\right)$$

where $$f(z) = 1 - \frac{z_H^d}{z^d}$$

Question

Can we reproduce this BH geometry from a free VM on S^1 x R^d ?

Free scalars at finite T and holographic space

[Aoki-Onogi-SY ’20]

Free real scalars on S^1 x R^{d-1}

Prop (2pt function at finite temperature)

$$F_0(u) = \int_0^1 dv\, v^{d/2-2} e^{-\frac{u}{4}v}$$

Free flow equation

$$\langle \phi^a(x_1)\phi^b(x_2)\rangle_0 \sim \frac{\delta^{ab}}{[x_{12}^2]^{\frac{d-2}{2}}} \longrightarrow \langle \phi^a(x_1;t_1)\phi^b(x_2;t_2)\rangle_0 = \frac{\delta^{ab}}{[4t_+]^{\frac{d-2}{2}}\Gamma(\frac{d-2}{2})} F_0\left(\frac{x_{12}^2}{t_+}\right),$$

$$\frac{\partial}{\partial t}\phi^a(x;t) = \partial^2\phi^a(x;t), \quad \phi^a(x;0) = \phi^a(x),$$

Free real scalars on $S^1 \times R^{d-1}$

Prop (2pt function at finite temperature)

$$F_0(u) = \int_0^1 dv\, v^{d/2-2} e^{-\frac{u}{4}v}$$

$$\langle \phi^a(x_1)\phi^b(x_2)\rangle_0 \sim \frac{\delta^{ab}}{[x_{12}^2]^{\frac{d-2}{2}}} \xrightarrow{\text{Free flow equation}} \langle \phi^a(x_1;t_1)\phi^b(x_2;t_2)\rangle_0 = \frac{\delta^{ab}}{[4t_+]^{\frac{d-2}{2}}\Gamma(\frac{d-2}{2})} F_0\left(\frac{x_{12}^2}{t_+}\right),$$

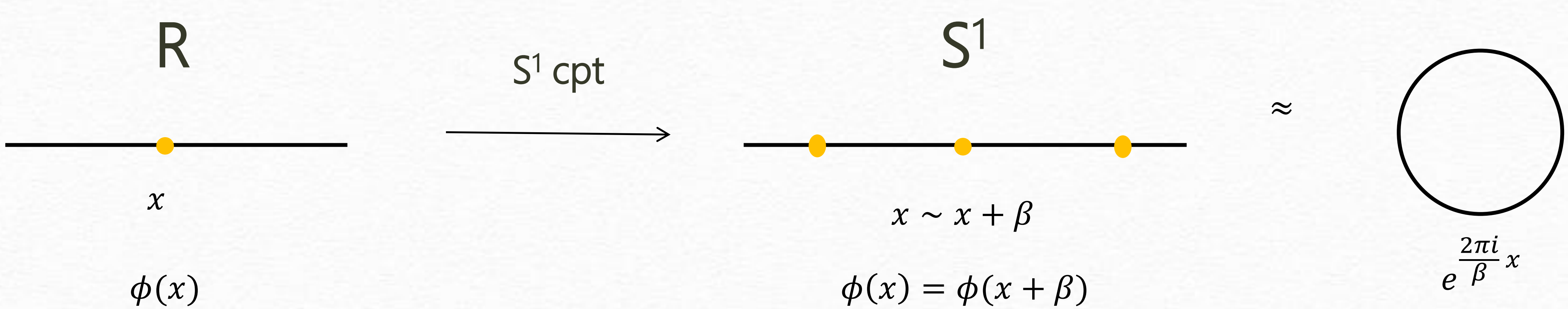
S^1 cpt $\downarrow$ $\qquad$ $\downarrow$ S^1 cpt

$$\frac{\partial}{\partial t}\phi^a(x;t) = \partial^2\phi^a(x;t), \quad \phi^a(x;0) = \phi^a(x),$$

$$\langle \phi^a(x^0,\vec{x})\phi^b(y^0,\vec{y})\rangle_T = \sum_{n=-\infty}^{\infty} \langle \phi^a(x^0,\vec{x})\phi^b(y^0+\frac{n}{T},\vec{y})\rangle_0.$$

$\xrightarrow{\text{Free flow equation}}$

$$\langle \phi^a(x^0,\vec{x};t)\phi^b(y^0,\vec{y};s)\rangle_T = \sum_{n=-\infty}^{\infty} \langle \phi^a(x^0,\vec{x};t)\phi^b(y^0+\frac{n}{T},\vec{y};s)\rangle_0.$$



Free real scalars on S^1 x R^{d-1}

Def. (Normalized operator)

$$\sigma^a(x^0,\vec{x};t) = \frac{\phi^a(x^0,\vec{x};t)}{\sqrt{\langle\phi^2(x^0,\vec{x};t)\rangle_0}},$$

Def. (Metric of the holographic space)

$$g_{MN}(X) = \sum_{a=1}^{N}\langle\partial_M\sigma^a(x^0,\vec{x};t)\partial_N\sigma^a(x^0,\vec{x};t)\rangle_T,$$

Result

$$\begin{aligned}
g_{00}(X) =&\frac{L_{\rm AdS}^2}{\tau^2}\frac{d}{2}\left[F(d,z) - z\frac{d}{dz}F(d,z)\right],\\
g_{\tau\tau}(X) =&\frac{L_{\rm AdS}^2}{\tau^2}\left[\frac{d-2}{2}F(d-2,z) - \frac{1}{2}z\frac{d}{dz}F(d-2,z) + \frac{1}{4}\left(z\frac{d}{dz}\right)^2F(d-2,z)\right],\\
g_{ij}(X) =&\delta_{ij}\frac{L_{\rm AdS}^2}{\tau^2}\frac{d}{2}F(d,z),
\end{aligned}$$

where $F(s,w) = \int_0^1 dv\, v^{s/2-1}\,\theta_3\left(e^{-\frac{dv}{4z^2}}\right),\quad \theta_3(q) := 1 + 2\sum_{n=1}^{\infty}q^{\frac{1}{2}n^2}$ $\quad z = T\tau = T\sqrt{2dt}$

Free real scalars on $S^1 \times R^{d-1}$

(Asymptotic behavior)

$$g_{00}(X) = \frac{L_{\mathrm{AdS}}^2}{\tau^2} f_0(z), \quad g_{\tau\tau}(X) = \frac{L_{\mathrm{AdS}}^2}{\tau^2} f_\tau(z), \quad g_{ij}(X) = \delta_{ij} \frac{L_{\mathrm{AdS}}^2}{\tau^2} f_i(z).$$

In the small z region, we have

$$\begin{aligned} f_0(z) &= 1 - z^d (d-1) A_d - \frac{d}{2}\left(1 - z\frac{\partial}{\partial z}\right) \delta F_{\mathrm{UV}}(d, z), \\ f_\tau(z) &= 1 + z^{d-2}\frac{(d-2)}{2} A_{d-2} - \left(\frac{(d-2)}{2} - \frac{1}{2} z\frac{\partial}{\partial z} + \frac{1}{4}\left(z\frac{\partial}{\partial z}\right)^2\right) \delta F_{\mathrm{UV}}(d-2, z), \\ f_i(z) &= 1 + z^d A_d - \frac{d}{2} \delta F_{\mathrm{UV}}(d, z), \end{aligned}$$

$$A_s := (4/d)^{\frac{s}{2}} s\, \Gamma(s/2) \zeta(s).$$

In the large z region, we obtain

$$\begin{aligned} f_0(z) &= \frac{d}{2}\left(1 - z\frac{\partial}{\partial z}\right) \delta F_{\mathrm{IR}}(d, z) \\ f_\tau(z) &= z\sqrt{\frac{4\pi}{d}}\left(\frac{2d-5}{2d-6}\right) + \frac{1}{2}\left((d-2) - z\frac{\partial}{\partial z} + \frac{1}{2}\left(z\frac{\partial}{\partial z}\right)^2\right) \delta F_{\mathrm{IR}}(d-2, z). \\ f_i(z) &= z\sqrt{\frac{4\pi}{d}}\left(\frac{d}{d-1}\right) + \frac{d}{2} \delta F_{\mathrm{IR}}(d, z). \end{aligned}$$

Free real scalars on $S^1 \times R^{d-1}$

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➡ Asymptotic AdSBH !! ➡ higher spin matter? ➡ Quantum effect?

In the large z region, we obtain

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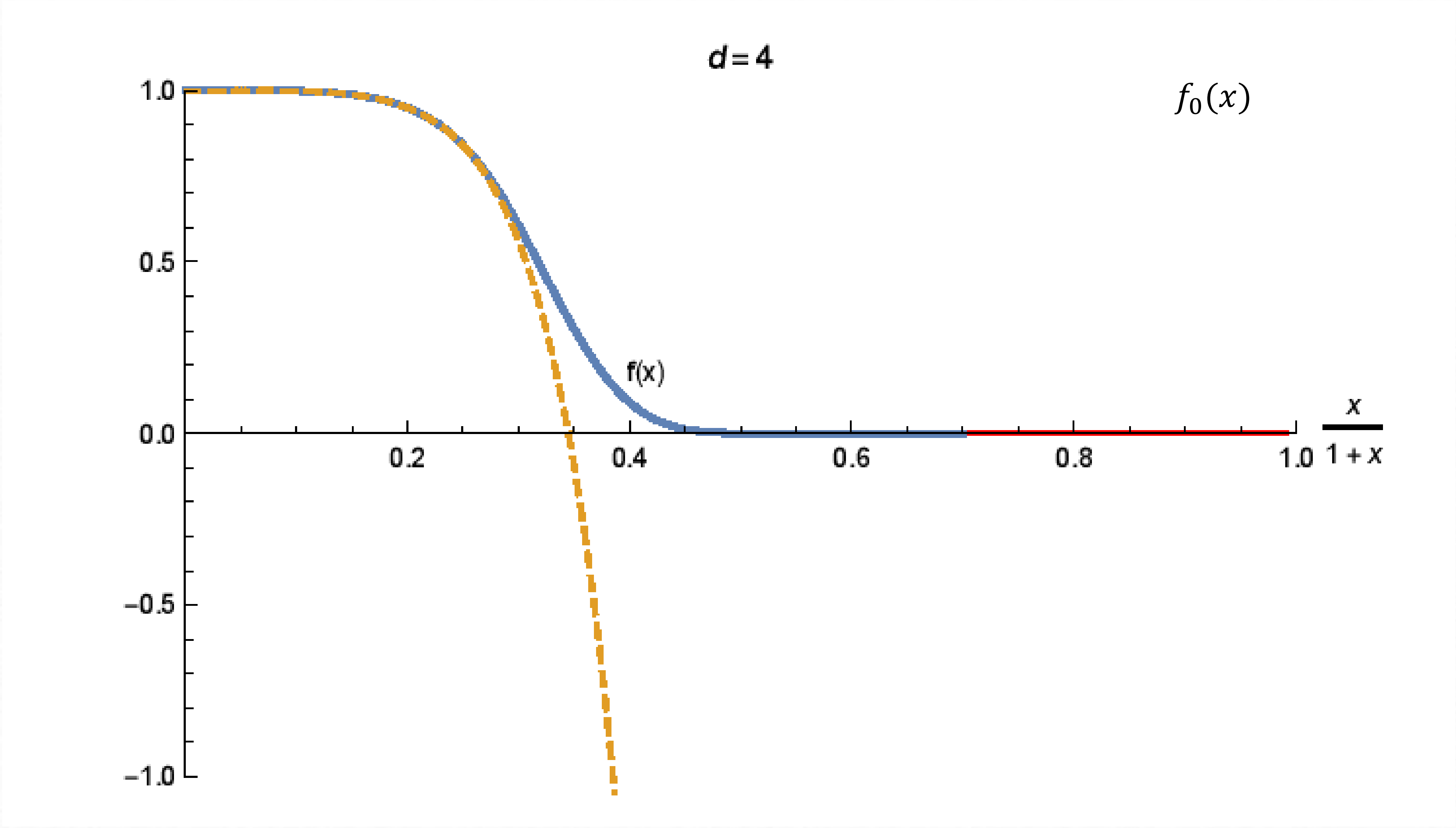
Free real scalars on $S^1 \times R^{d-1}$



1. Asymptotic AdSBH!

2. No horizon!! (Un-smeared information is less and less.)

Free real scalars on S^1 x R^{d-1}

(Asymptotic behavior of matter energy momentum tensor)

$$T_{AB} = \frac{1}{8\pi G_N}\left(G_{AB} + \Lambda\, g_{AB}\right)$$

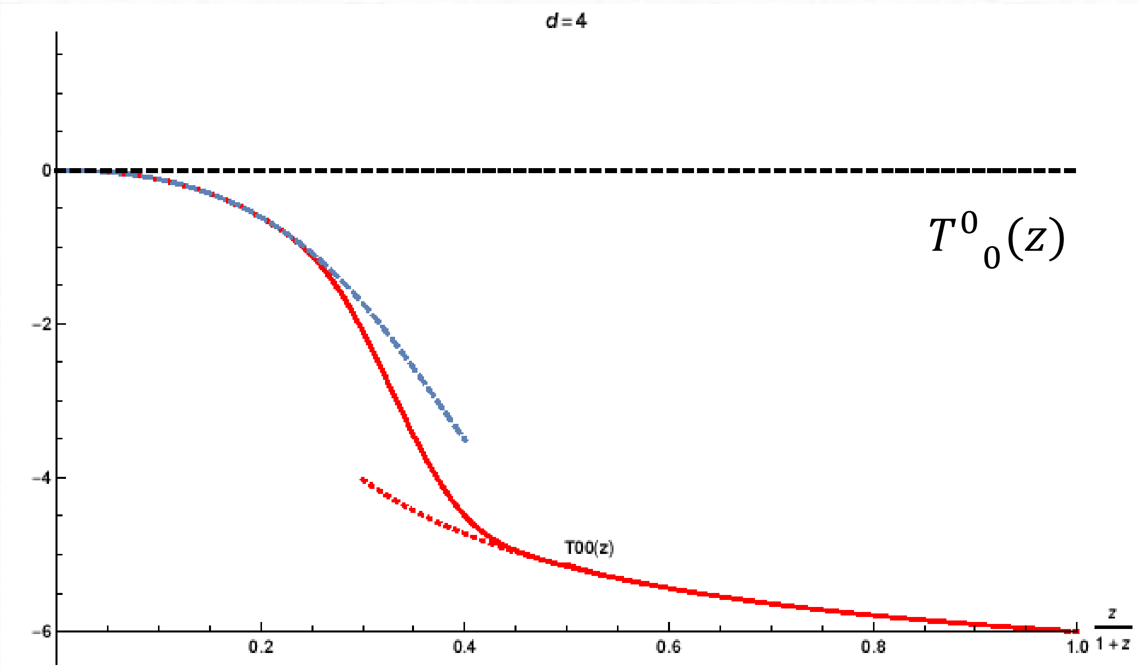
At small z, we have

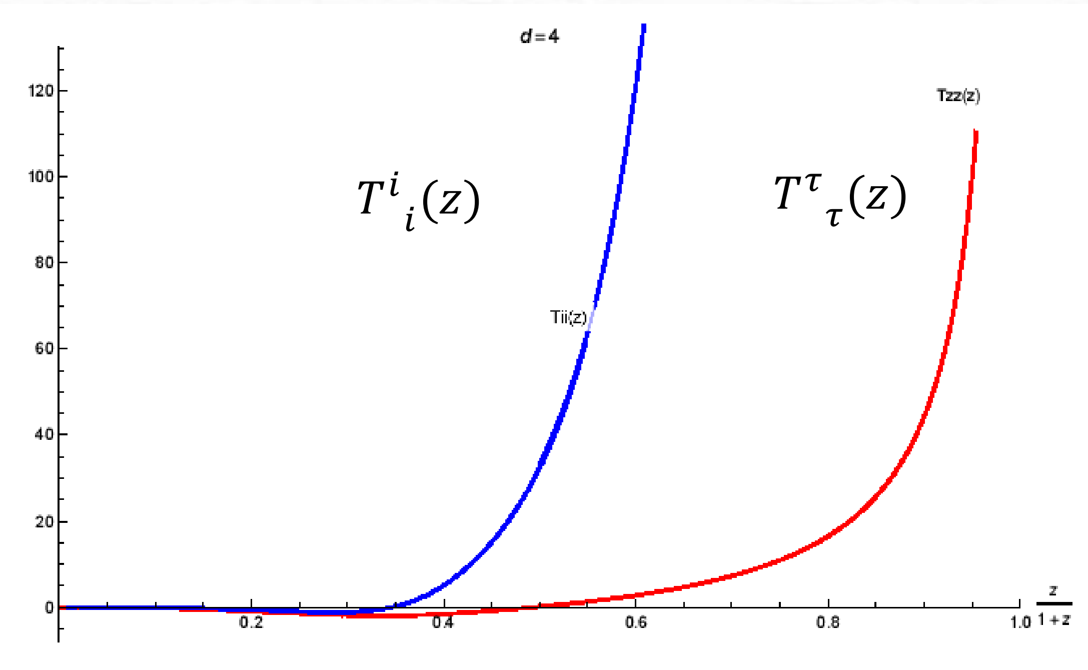
$$\begin{aligned}
\kappa^2 T^0{}_0(z) &\simeq -(d-1)Bz^{d-2}, \qquad B := \frac{d-2}{2}\frac{A_{d-2}}{L_{\mathrm{AdS}}^2}, \\
\kappa^2 T^\tau{}_\tau(z) &\simeq -\frac{d(d-1)}{2}Bz^{d-2}, \quad \kappa^2 T^i{}_j(z) \simeq -\delta^i_j(d-1)Bz^{d-2},
\end{aligned}$$

In the large z region, we obtain

$$\begin{aligned}
\kappa^2 T^0{}_0(z) &\simeq -\frac{1}{L_{\mathrm{AdS}}^2}\frac{d(d-1)}{2}\left(1 - \frac{(d-3)(d+2)}{4(2d-5)\sqrt{\pi d}}\frac{1}{z}\right), \\
\kappa^2 T^\tau{}_\tau(z) &\simeq \frac{1}{L_{\mathrm{AdS}}^2}\frac{2(d-1)(d-3)\pi^2}{(2d-5)\sqrt{\pi d}}z, \quad \kappa^2 T^i{}_j(z) \simeq \delta^i_j \frac{1}{L_{\mathrm{AdS}}^2}\frac{16(d-3)\pi^4}{d(2d-5)\sqrt{\pi d}}z^3.
\end{aligned}$$

Free real scalars on S^1 x R^{d-1}

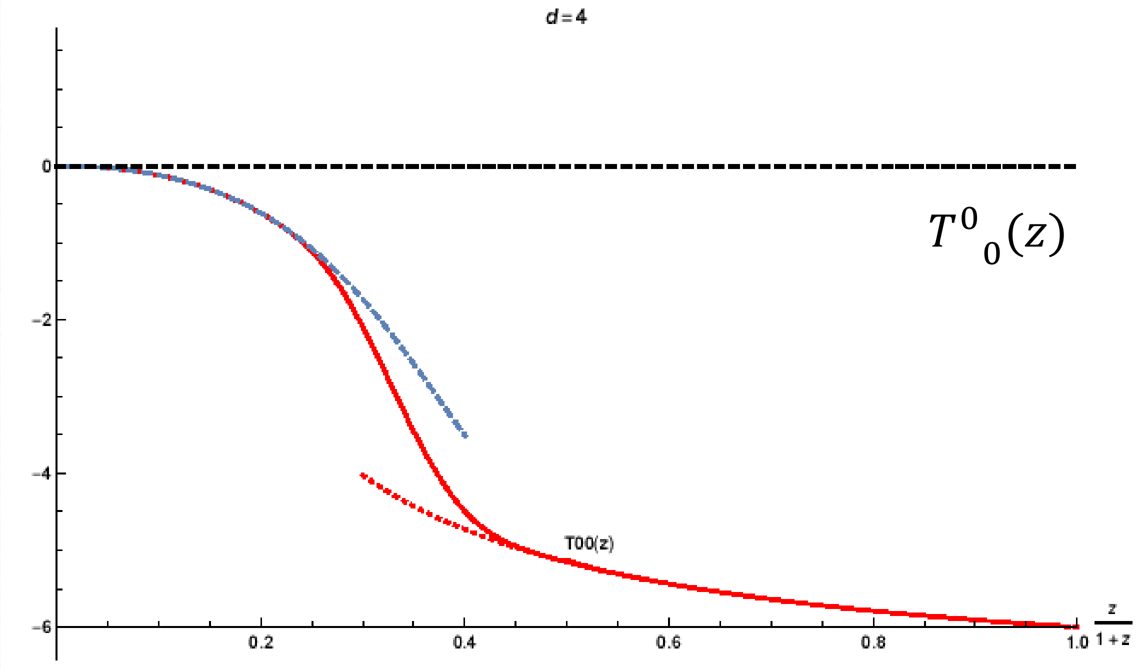


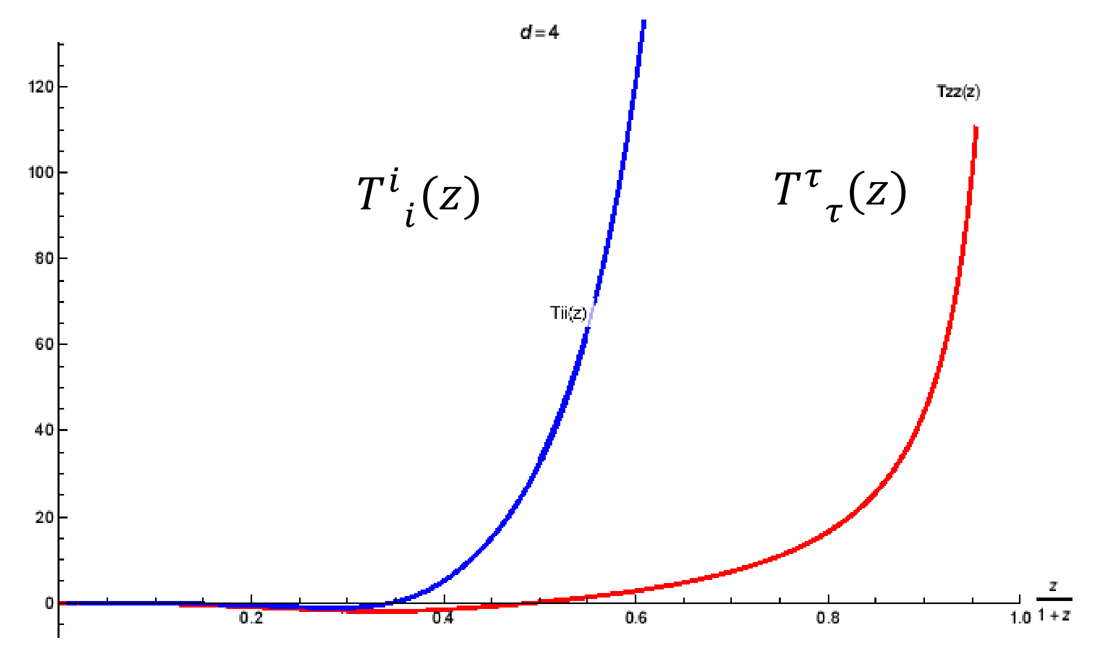


3. No curvature singularity!!

3′. Milder curvature singularity!!

Free real scalars on $S^1 \times R^{d-1}$





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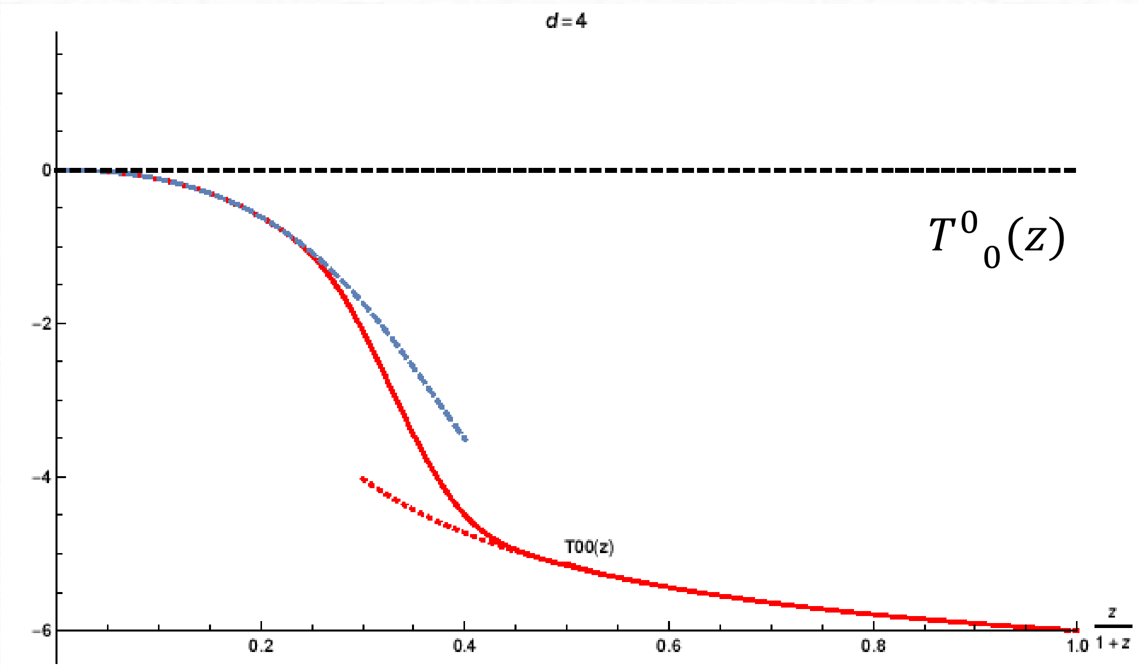
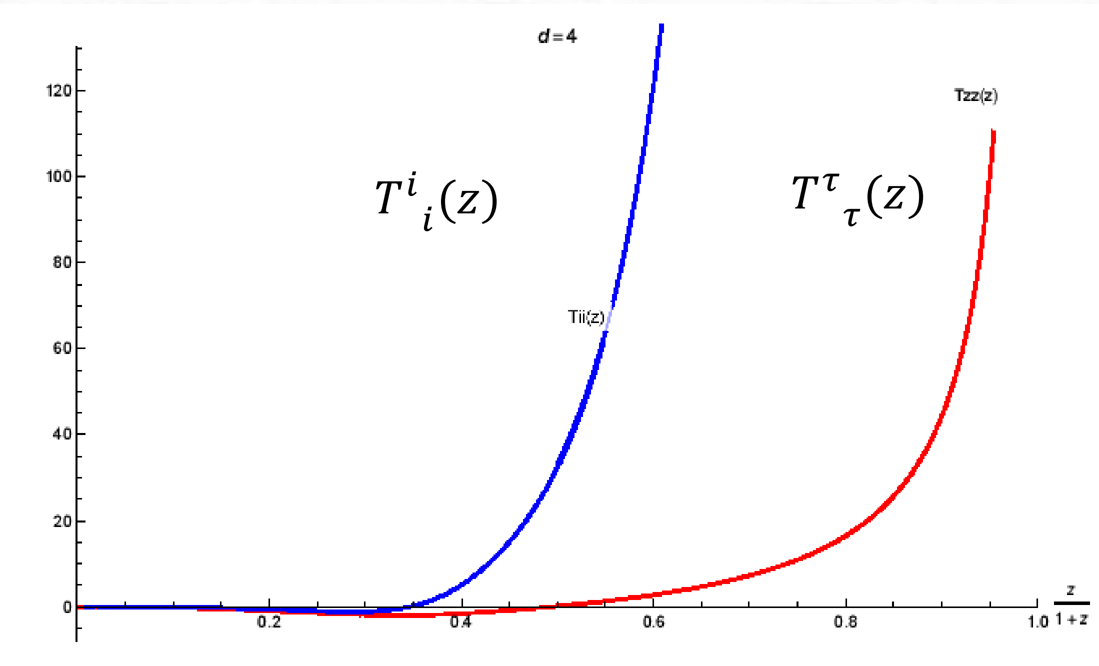
Question

How to evaluate the energy of this geometry?

Free real scalars on $S^1 \times R^{d-1}$





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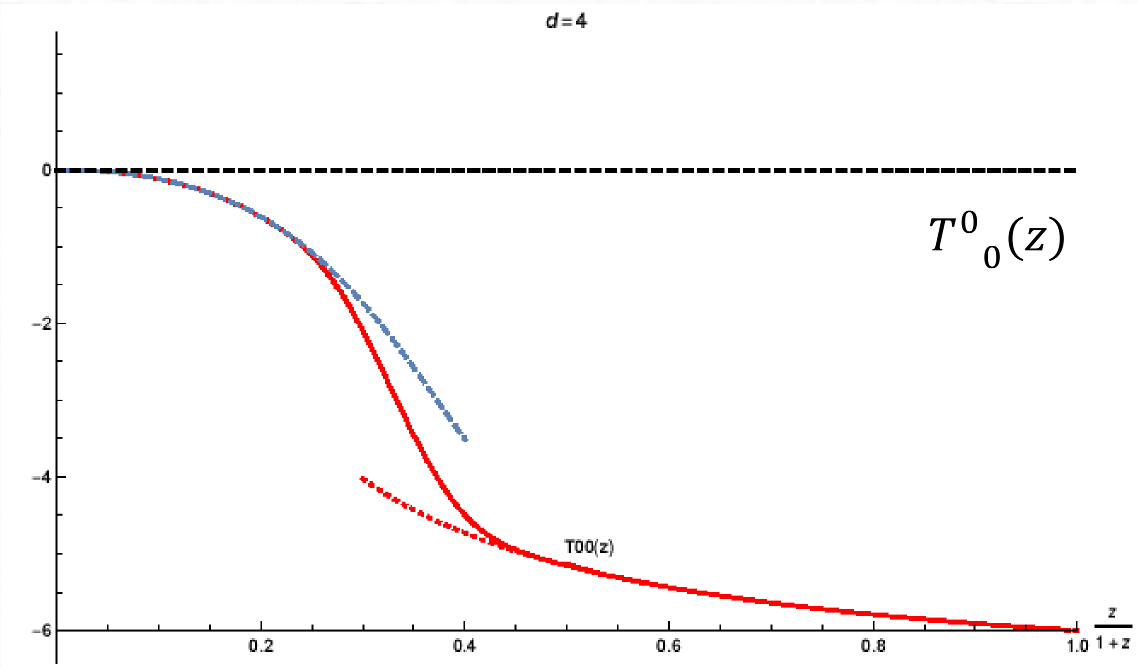
How to evaluate the energy of this geometry?

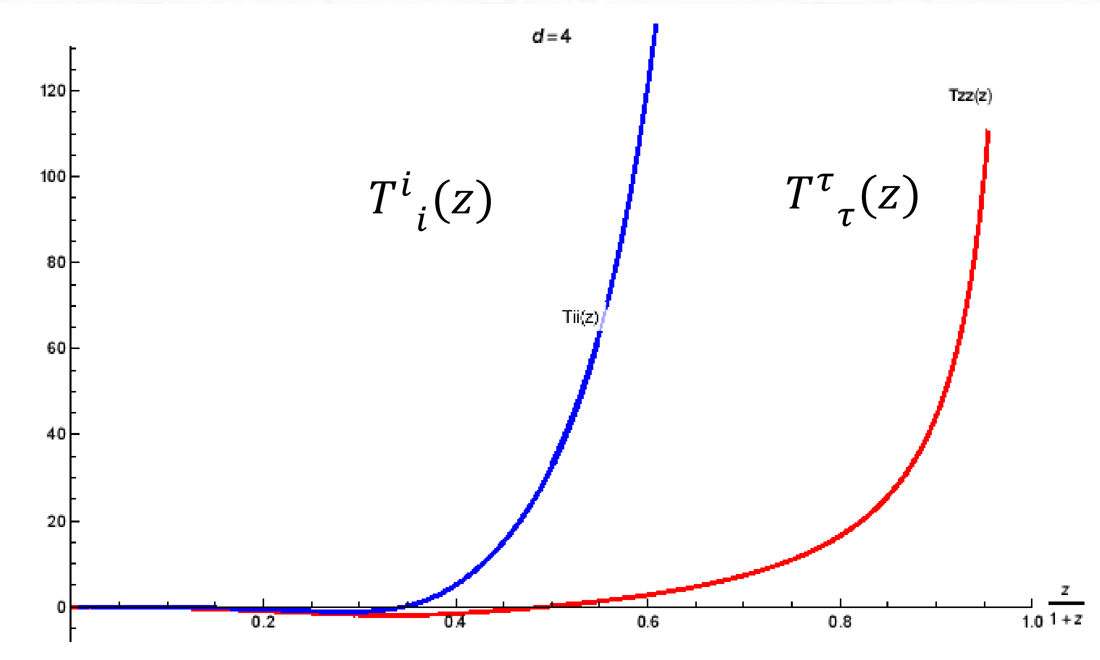
Standard: Quasi-local energy. (Energy defined locally on asymptotic region of spacetime)

$\int d^{d-1}x\, H\,?$ + GH term + conter term? Surface integral ➡ cannot be precise…

[ADM '62] [Brown-York '92] [Hawking-Horowitz '95] [Horowitz-Mayers '98] [Balasubramanian-Kraus '98]…

Free real scalars on $S^1 \times R^{d-1}$





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[ADM '62] [Brown–York '92] [Hawking–Horowitz '95] [Horowitz–Mayers '98] [Balasubramanian–Kraus '98]⋯

Volume integral? $\int d^d x\, T_{00}$? $\int d^d x\, T^{00}$? $\int d^d x\, T^0_0$? $+\sqrt{g}$? Precise form?

Plan

✓ 1. Introduction

✓ 2. Flow equation and bulk construction

✓ 3. Holographic geometry at finite T

4. Conserved charge in GR

5. Summary

Conserved charges in general relativity

[Aoki–Onogi–SY '20]

Question

What is a precise definition of energy in GR?

$\int d^d x\, T_{00}$? $\int d^d x\, T^{00}$? $\int d^d x\, T_0^0$? $+\sqrt{g}$?

Proposal

[Aoki-Onogi-SY '20]

A precise definition of energy available in a general D-dim curved spacetime

$$E = \int d^{D-1}x \sqrt{|g|}\,(-T^0_0)$$

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$$E = \int d^{D-1}x \sqrt{|g|}\, (-T^0_0)$$

$$\Uparrow \qquad \xi^\mu = -\delta^\mu_0$$

A [conserved] charge associated with a [Killing] vector

$$Q = \int d^{D-1}x \sqrt{|g|}\, T^0_\mu\, \xi^\mu$$

$$\nabla_\mu \xi_\nu(x) + \nabla_\nu \xi_\mu(x) = 0$$

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Q1. Was this known?

Formally, yes. ➡ V. Fock, *The Theory of Space, Time and Gravitation* (Pergamon Press, New York 1959)

The quantity $I = \int T^{\mu 0} \varphi_\mu \sqrt{-g}\, dx_1 dx_2 dx_3$ will be constant, $\cdots$, if the vector φ_μ satisfies the equation $\nabla_\nu \varphi_\mu + \nabla_\mu \varphi_\nu = 0$.

Proposal

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Q2. Then what's new?

Our claim is this definition can be applied to **generic spacetime including BH**!

(This definition has been forgotten and not been well studied for some reason.)

Evidence

[Aoki-Onogi-SY '20]

Let us compute mass of the Schwarzschild ((A)dS)BH.

Einstein equation $$R_{\mu\nu} - \frac{1}{2}g_{\mu\nu}R + \Lambda g_{\mu\nu} = 0.$$

Sch BH solution $$ds^2 = -f(r)(dx^0)^2 + \frac{1}{f(r)}dr^2 + r^2\tilde{g}_{ij}dx^i dx^j$$

where $$f(r) = \frac{-2\Lambda r^2}{(d-2)(d-1)} + k - \frac{2G_N M}{r^{d-3}}.$$ $${}^{(d-2)}R_{ij} = (d-3)k\tilde{g}_{ij}$$

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∃ a Killing vector $\xi^\mu = -\delta^\mu_0$ ➡ The energy is conserved. ➡ Energy = Mass

$$E = \int d^{d-1}\vec{x}\, \sqrt{|g|}(-T^0{}_0),$$

where the matter energy momentum tensor is

$$T_{\mu\nu} = \frac{1}{8\pi G_N}\left(R_{\mu\nu} - \frac{1}{2} g_{\mu\nu} R + \Lambda g_{\mu\nu}\right)$$

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Does this vanish since there is no matter in EOM and BH is a vacuum solution?

➡ Our answer is NO! $T_{\mu\nu}$ vanishes except singularity!! Cf. electron in electromagnetism.

Evidence

[Aoki-Onogi-SY '20]

The quickest way to see this
is to compute energy momentum tensors including contribution of singularity

$$R^0{}_0 = -\frac{1}{2r^{d-2}}\partial_r\left(r^{d-2}\partial_r f(r)\right) = R^r{}_r,$$

$$R^i{}_j = \delta^i_j\left[\frac{(d-3)k}{r^2} - \frac{1}{r^{d-2}}\partial_r\left(r^{d-3}f(r)\right)\right]$$

$$\Rightarrow \quad T^0_0 = \frac{d-2}{16\,\pi\,G_N\,r^{d-2}}\,\partial_r\,(r^{d-3}\,\delta f(r))$$

where $\delta f(r) = \frac{-2\,G_N\,M}{r^{d-3}}$

Evidence

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➡ $$T^0_0 = \frac{d-2}{16\,\pi\,G_N\,r^{d-2}}\,\partial_r\,(r^{d-3}\,\delta f(r))$$

where $\delta f(r) = \frac{-2\,G_N\,M}{r^{d-3}}$

➡ $$E = -\int d^{d-1}\vec{x}\,\sqrt{|\tilde{g}|}\,\frac{d-2}{16\pi G_N}\partial_r\left(r^{d-3}\delta f\right) = \rho V_{d-2}$$

where $V_{d-2} = \int d^{d-2}x\sqrt{|\tilde{g}|}$ $\quad \rho = (d-2)M/(8\pi)$

This result matches the known result computed by other method.

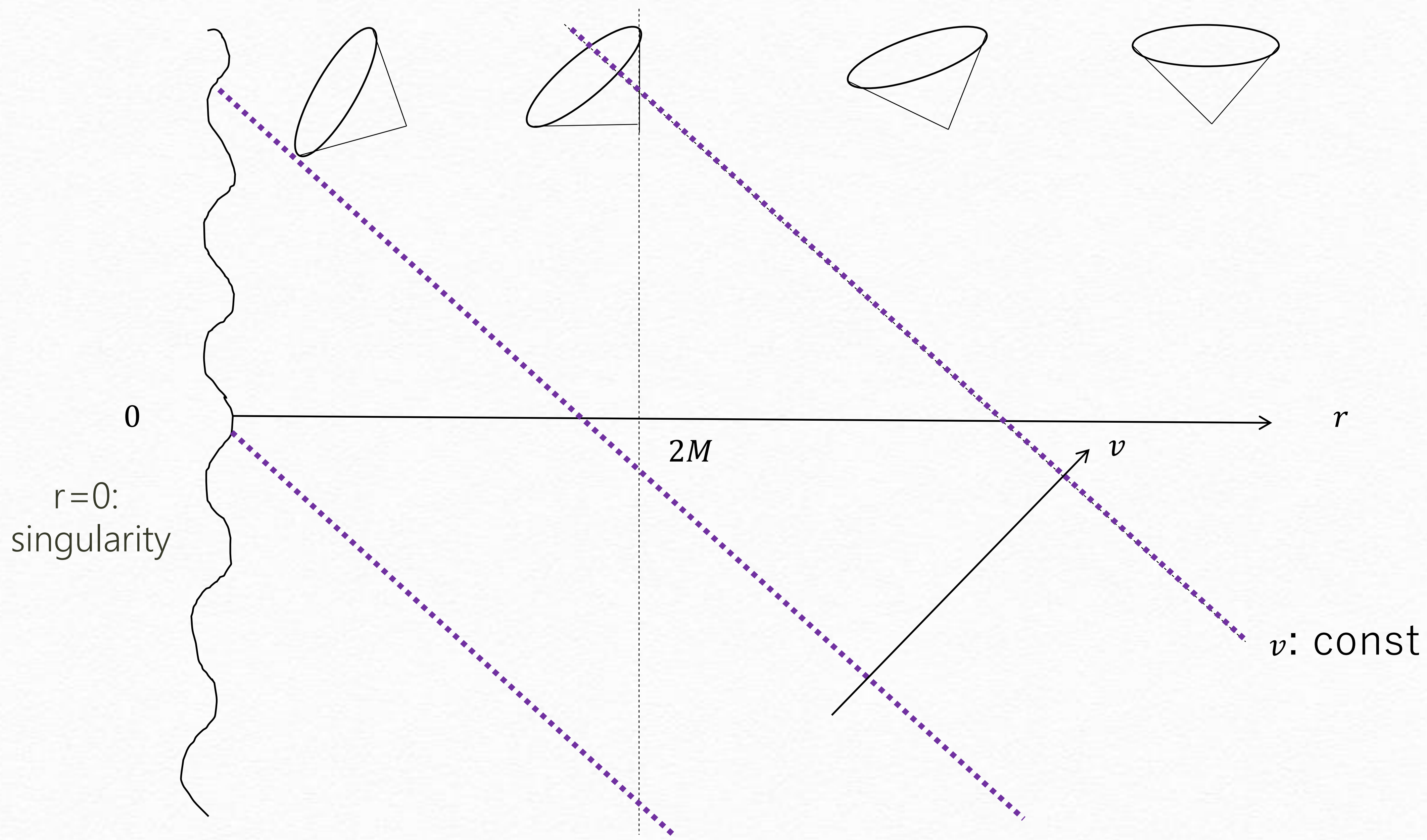
Volume-integration path

Eddington-Finkelstein coordinate (No horizon)

$$ds^2 = -f(r)dv^2 + 2\,dvdr + r^2\,\tilde{g}_{ij}\,dx^i dx^j$$

$t^* = v - r$

Finkelstein diagram



Evidence

[Aoki-Onogi-SY '20]

The charged (Reissner-Nordstrom) ((A)dS)BH.

EOM

$$G_{MN} + g_{MN}\Lambda = \kappa^2(-\frac{1}{4}g_{MN}F_{SR}F^{SR} + g^{RS}F_{MR}F_{NS})$$

$$\frac{1}{\sqrt{|g|}}\partial_M(\sqrt{|g|}F^{MN}) = 0$$

RN BH solution

$$ds^2 = -f(r)(dx^0)^2 + \frac{1}{f(r)}dr^2 + r^2\tilde{g}_{ij}dx^i dx^j$$

$$A_M = (-\frac{q}{r^{D-3}} + \frac{q}{r_+^{D-3}})\delta_M^0$$

where

$$f = \frac{r^2}{L^2} + k - \frac{m}{r^{D-3}} + \frac{Q^2}{r^{2(D-3)}}, \quad \frac{1}{L^2} = \frac{-\Lambda}{(D-2)(D-1)/2}, \quad Q^2 = \frac{D-3}{D-2}\kappa^2 q^2$$

Evidence

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The charged (Reissner-Nordstrom) ((A)dS)BH.

EOM $G_{MN} + g_{MN}\Lambda = \kappa^2(-\frac{1}{4}g_{MN}F_{SR}F^{SR} + g^{RS}F_{MR}F_{NS})$ $\frac{1}{\sqrt{|g|}}\partial_M(\sqrt{|g|}F^{MN}) = 0$

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where $f = \frac{r^2}{L^2} + k - \frac{m}{r^{D-3}} + \frac{Q^2}{r^{2(D-3)}}, \quad \frac{1}{L^2} = \frac{-\Lambda}{(D-2)(D-1)/2}, \quad Q^2 = \frac{D-3}{D-2}\kappa^2 q^2$

∃ a Killing vector $\xi^\mu = -\delta_0^\mu$ ➡ The energy is conserved. ➡ Energy = Mass

$$E = \int d^{d-1}\vec{x}\,\sqrt{|g|}(-T^0{}_0),$$

where the matter energy momentum tensor is $T_{\mu\nu} = \frac{1}{8\pi G_N}(R_{\mu\nu} - \frac{1}{2}g_{\mu\nu}R + \Lambda g_{\mu\nu})$

➡ This diverges as self-energy of an electron does in electromagnetism.
➡ We subtract the divergent contribution of the charged part.

➡ $$E = -\int d^{d-1}\vec{x}\,\sqrt{|\tilde{g}|}\frac{d-2}{16\pi G_N}\partial_r\left(r^{d-3}\delta f\right) = \rho V_{d-2}$$

This result matches the known result computed by other method.

Evidence

[Aoki-Onogi-SY '20]

The BTZ BH.

Einstein equation
$$R_{\mu\nu} - \frac{1}{2} g_{\mu\nu} R + \Lambda g_{\mu\nu} = 0.$$

BTZ BH solution
$$ds^2 = -f(r)dt^2 + \frac{1}{f(r)} dr^2 + r^2 (d\phi - \omega(r) dt)^2$$
$$\omega(r) = \frac{G_N J}{2r^2}$$

where
$$f(r) = \frac{r^2}{L^2} - 2G_N M \theta(r) + \frac{G_N^2 J^2}{4r^2}$$

Evidence

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The BTZ BH.

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∃ a Killing vector $\xi^\mu = -\delta_0^\mu$ ➡ The energy (=Mass) is conserved. $$E = \int d^{d-1}\vec{x}\,\sqrt{|g|}(-T^0{}_0).$$

∃ a Killing vector $\xi^\mu = \delta_\phi^\mu$ ➡ The angular momentum is conserved. $$P_\phi = \int d^2x \sqrt{|g|} T^0{}_\phi.$$

Energy momentum tensors: $$T_0^0 = \frac{1}{16\,\pi\, G_N\, r}\,\partial_r\,(-2\,G_N M\,\theta(r))$$

$$T_\phi^0 = \frac{-1}{16\,\pi\, G_N\, r}\,\partial_r\,(r^3\,\omega'(r))$$

➡ $$E = \frac{M}{4} \qquad P_\phi = \frac{J}{8}$$

This result matches the known result computed by other method.

New result?

[Aoki-Onogi-SY '20]

➡ Correction to known mass formula for a compact star

Ansatz: Stationary spherically symmetric $ds^2 = -f(r)(dx^0)^2 + h(r)dr^2 + r^2\tilde{g}_{ij}dx^i dx^j$

Matter stress tensor -> perfect fluid $T^0{}_0 = -\rho(r), \quad T^r{}_r = P(r), \quad T^i_j = \delta^i_j P(r)$

⬇

Toleman-Oppenheimer-Volkoff equation

Inside the star: $$-\frac{dP(r)}{dr} = \frac{G_d M(r)}{r^{d-2}}\left(P(r)+\rho(r)\right)h(r)\left\{d-3+\frac{r^{d-1}}{(d-2)M(r)}\left(8\pi P(r) - \frac{2\Lambda}{(d-1)G_d}\right)\right\}$$

where $$\frac{1}{h(r)} = k - \frac{2G_d M(r)}{r^{d-3}} - \frac{2\Lambda r^2}{(d-2)(d-1)} \qquad M(r) = \frac{8\pi}{d-2}\int_0^r ds s^{d-2}\rho(s), \quad M(0)=0$$

At the surface: $P(r)|_{r=R} = 0$

Outside the star: $r > R, \quad \rho(r) = P(r) = 0 \qquad f(r) = \frac{1}{h(r)} = k - \frac{2G_d M(R)}{r^{d-3}} - \frac{2\Lambda r^2}{(d-2)(d-1)}$

Mass for a compact star

[Aoki-Onogi-SY '20]

$$
\begin{aligned}
E &= -\int d^{d-2}x \int_0^\infty dr \sqrt{|g|} T^0{}_0 \\
&= V_{d-1} \int_0^R \sqrt{f(r)h(r)} r^{d-2} \rho(r) \\
&= \frac{(d-2)V_{d-2}}{8\pi} \int_0^R dr \sqrt{f(r)h(r)} \frac{dM(r)}{dr} \\
&= \frac{(d-2)V_{d-2}}{8\pi} \left[\boxed{M(R)} \boxed{- \int_0^R dr \frac{M(r)}{2} \frac{d}{dr} \log|f(r)h(r)|} \right]
\end{aligned}
$$

ADM mass   Corrections due to the internal structure of star

The correction term should be taken into account to evaluate the correct mass for a compact star.

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[Aoki-Onogi-SY '20]

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<u>Intuitive argument from electromagnetism</u>

Q: Which is a correct definition for an electric charge on a curved spacetime?

① $Q_{surface} = \int_{\Sigma_2} d^2\sigma_i \, F^{0i}$

② $Q_{volume} = \int_{V_3} d^3x \sqrt{|g|} \, j^0 = \int_{V_3} d^3x \sqrt{|g|} \, \nabla_\mu F^{0\mu}$

The answer should be compatible with the charge quantization!

Plan

✓ 1. Introduction

✓ 2. Flow equation and bulk construction

✓ 3. Holographic geometry at finite T

✓ 4. Conserved charge in GR

5. Summary

Summary

・We investigated holographic geometry for free scalars at finite temperature by the flow equation approach.

・The resulting holographic geometry has the following properties:

(i) It is an asymptotic AdSBH geometry with unknown matter contribution.

(ii) It has no coordinate singularity and milder curvature singularity.

・We presented a precise definition of a conserved charge associated with a Killing vector available on a general curved spacetime.

・Mass of well-known BHs was reproduced from the definition with non-vanishing energy momentum tensor at singularity.

・The definition leads to correction to known mass formula of a compact star. In particular, the mass cannot be written only by a surface integral.

Future works

- Holographic geometry for interacting theory or YM theory?

- How to encode dynamics beyond geometry? For excited states?

working in progress [Aoki-Balog-SY]

- How to reconstruct a local bulk operator? Causality from flow equation?

cf. [HKLL]

- Another application of the presented definition?

working in progress [Aoki-Onogi-SY]

$\vdots$

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⋮

Thank you!