Gravitational instantons and anomalous chiral symmetry breaking

Yu Hamada (Kyoto Univ.)

Based on [arXiv:2009.08728]

w/
Jan M. Pawlowski (Heidelberg U.)
Masatoshi Yamada (Heidelberg U.)

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Higgs discovery

https://www.theguardian.com/science/blog/2012/jul/04/higgs-boson-discovered-live-coverage-cern

Problems in SM

- Dark matter
- Cosmological constant
- masses of neutrinos
- Baryon asymmetry in universe
- **Quantum gravity**

Problems in SM

- Dark matter
- Cosmological constant
- masses of neutrinos
- Baryon asymmetry in universe
- <u>**Quantum gravity**</u>

$\rightarrow$ (Einstein) gravity is not renormalizable

$\rightarrow$ need quantum gravity $\rightarrow$ string theory!

Problems in SM

- Dark matter
- Cosmological constant
- masses of neutrinos
- Baryon asymmetry in universe
- **Quantum gravity**

→ (Einstein) gravity is not renormalizable

→ need quantum gravity → string theory!

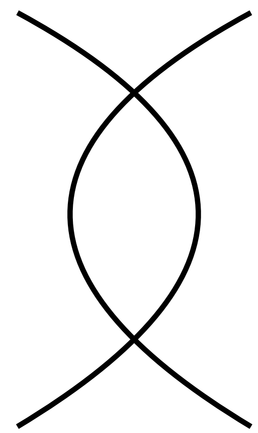
Other possibilities?

What's renormalizability?

- some textbooks say:

If **all UV divergences in Feynman diagrams are absorbed by a finite number of parameters** in the theory, then the theory is renormalizable.

(ex.) $\mathscr{L} = \frac{1}{2}(\partial_\mu \phi)^2 - \frac{\lambda}{4!}\phi^4$



$\sim \delta\lambda_{1-\text{loop}}\ \phi^4$

(counter term)

- non-renormalizability means:

need **infinite number of parameters** to absorb them

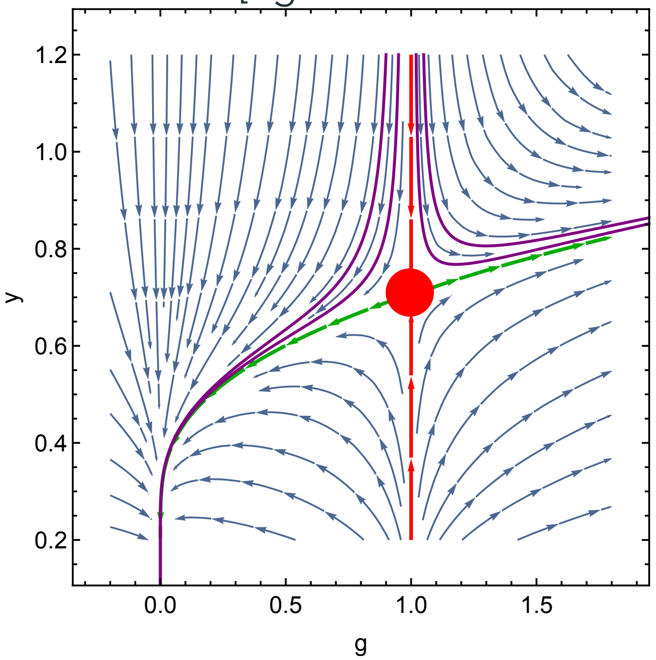
→ The theory has **no predictability.**

But, this is just a "perturbative renormalizability".

Asymptotic safety

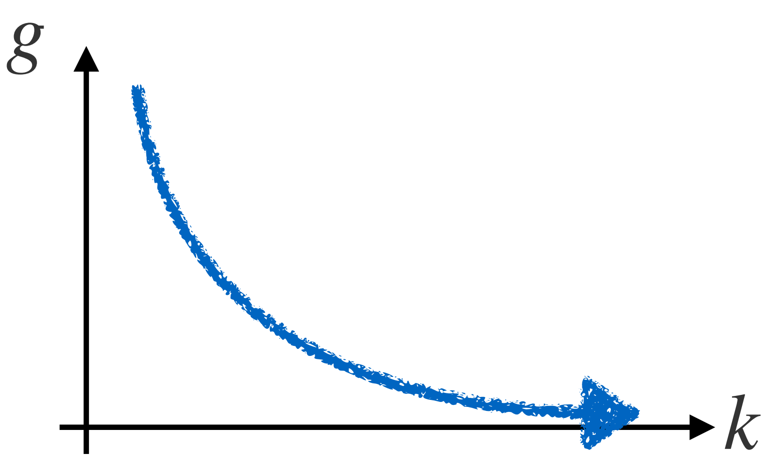
[S. Weinberg, '80]

[fig from 1810.07612



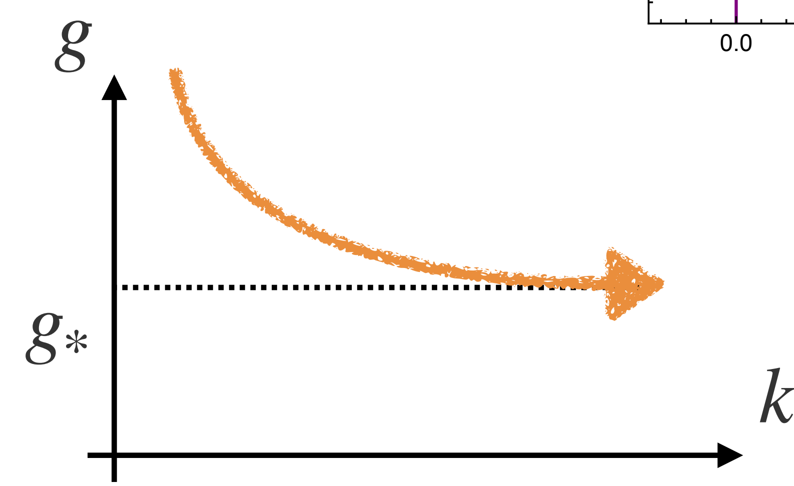
- A theory that has a non-trivial RG fixed pt at UV is **non-perturbatively renormalizable**.

  → **Asymptotic safety**



asymptotic freedom

(e.g. QCD)



asymptotic safety

(e.g. non-linear σ models in 3d)

- **Recently, a possibility has been pointed out that gravity is asymptotically safe and is a consistent QFT.**

→Asymptotic safety scenario of quantum gravity

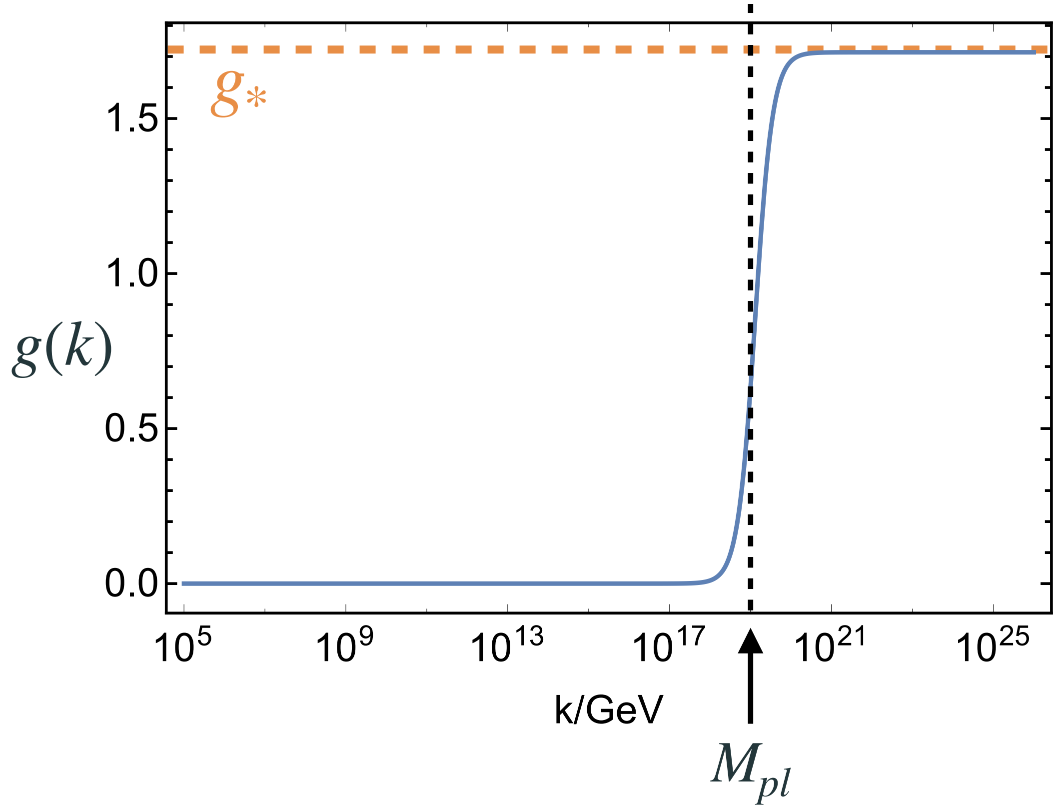
Evidence of asymptotically safe gravity

- eg.) Einstein-Hilbert truncation: [Reuter '98] [Souma '99]

$$\Gamma_k = \frac{1}{16\pi G(k)} \int d^4x \sqrt{g}\,(R - 2\Lambda(k))$$

$$g(k) \equiv G k^2$$

[arXiv:1709.03696]



- Solve RG equation (Wetterich eq.):

$$\partial_t \Gamma_k = \frac{1}{2}\mathrm{Tr}\left[\left(\Gamma_k^{(2)} + R_k\right)^{-1} \partial_t R_k\right]$$

$t \equiv \log k$ $\quad R_k$: cutoff function

- **No UV divergence**

- Fixed pt. still exists even when other operators such as $R^n, R_{\mu\nu}R^{\mu\nu}, \ldots$ are included.

→ **renormalizable QFT for gravity?**

Open problems

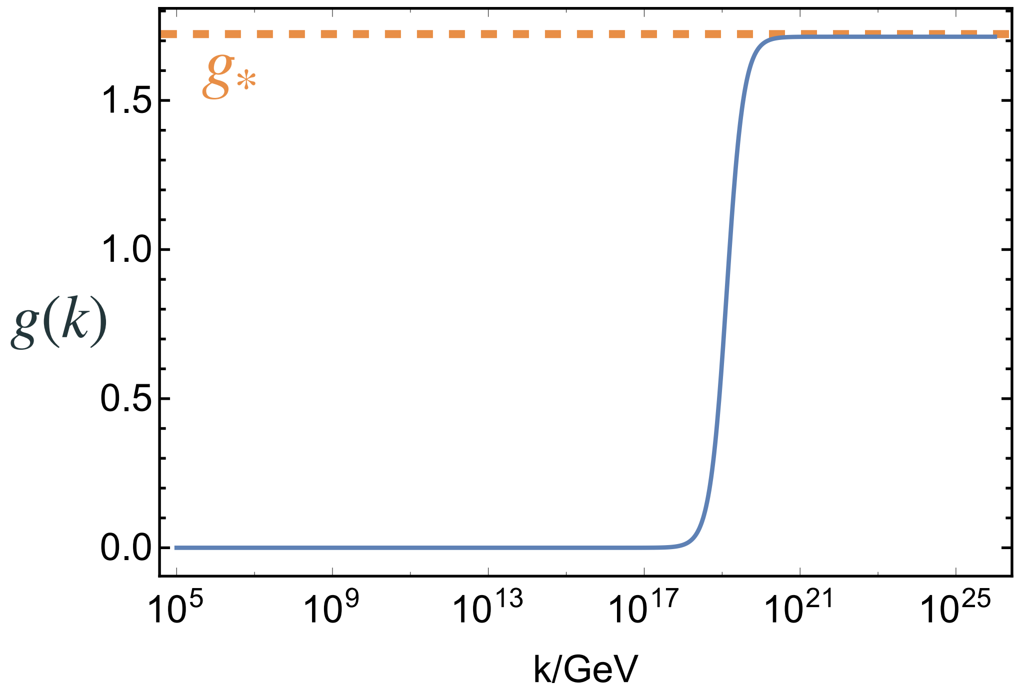
- These analysis are done in truncated parameter space
  $\rightarrow$ must consider an RG flow in infinite-dimensional parameter space
- Higher derivative terms necessarily appear → **unitarity?**
- Based on Euclidean space (Wick rotation is not justified in gravity)
- How to investigate dynamics of quantum spacetime？
  (Black hole entropy, singularity, emergence of spacetime？)
- Weak gravity conjecture?

Phenomenological implication?

[arXiv:1709.03696]

- Gravity is strong coupling for $k \gtrsim M_{pl}$
- could cause chiral symmetry breaking of fermions like QCD?

$$\Rightarrow \langle \bar{\psi}\psi \rangle \neq 0$$



→ **If so, all fermions acquire dynamical masses $\sim M_{pl}$**

→ inconsistent with light fermions in our world

→ **phenomenologically exclude asymptotic safety scenario**

[Eichhorn-Gies, '11] [Eichhorn, '12] [Meibohm-Pawlowski, '16] [Eichhorn-Lippoldt, '16] [Eichhorn-Held, '17]

This talk:

Gravitational instantons can trigger chiral sym. breaking!

Outline

- Introduction
- Gravitational instanton
- Chiral sym. breaking induced by grav. instanton
- Summary

Gravitational instanton

Gravitational instanton

- Gravity is a gauge theory of local Lorentz sym: $SO(4) \simeq SU(2) \times SU(2)$

$$\mathscr{L} \supset \bar{\psi}\gamma^\mu \left(\partial_\mu - \frac{i}{2}\sigma_{ab}\omega_\mu^{ab} \right) \psi \qquad \gamma^\mu \equiv \gamma^a e_a^\mu$$

$$\sigma^{ab} \equiv \frac{i}{4}\left[\gamma^a, \gamma^b\right]$$

Spin connection

~ $SO(4)$ gauge field

- instanton-like configuration with winding # for one $SU(2)$ sector

  →**gravitational instanton**

- self-dual curvature: $\tilde{R}_{\mu\nu} = \pm R_{\mu\nu}$ → Ricci flat: $R_{\mu\nu} = 0$

  → solution to vacuum Einstein eq.

analogue of Yang-Mills instanton

Example of grav. instanton

[Hawking '76]
[Eguchi-Hanson '78]

- Eguchi-Hanson metric
- Taub-NUT metric

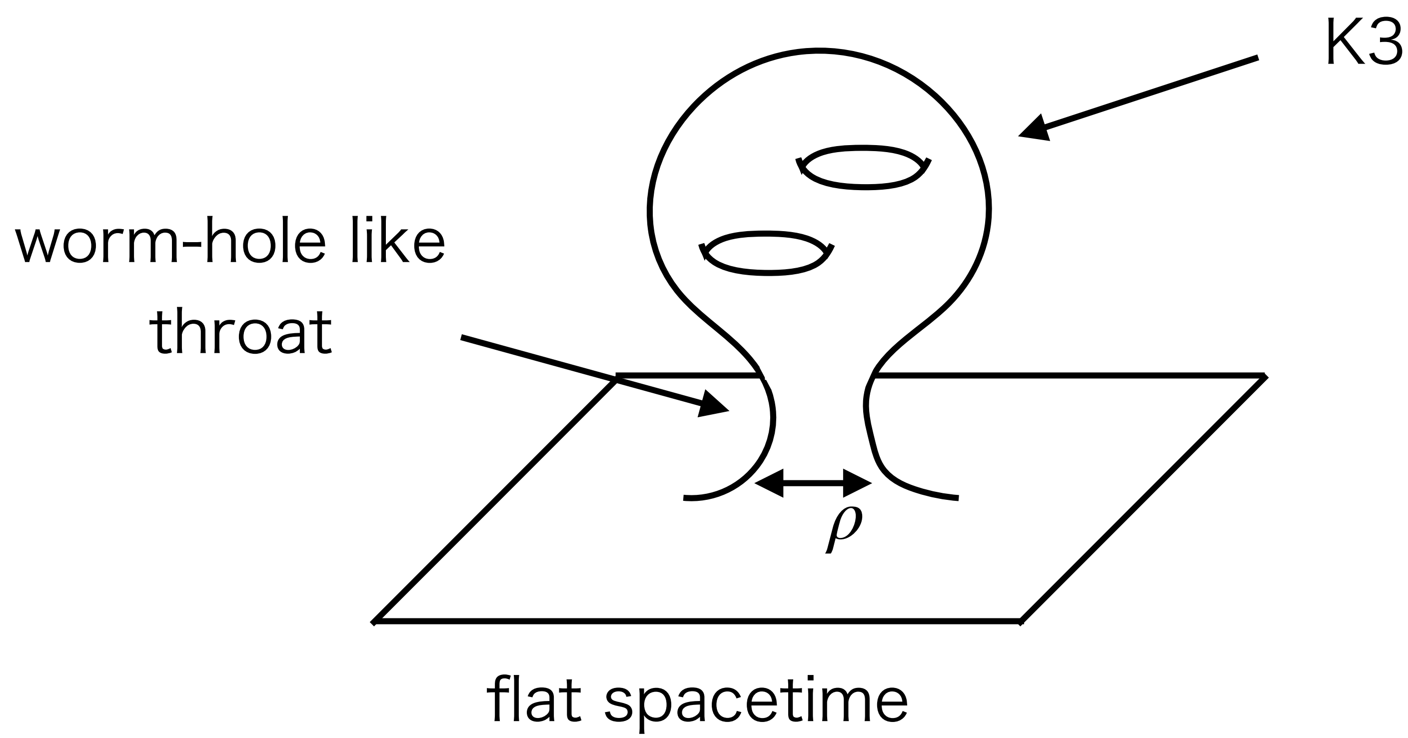
→ not considered in this talk

- **K3-surface (Calabi-Yau manifold)**

→ compact and closed manifold

→ **In path integral, it appears as a fluctuation around flat space:**

[Hebecker-Henkenjohann 1906.07728]



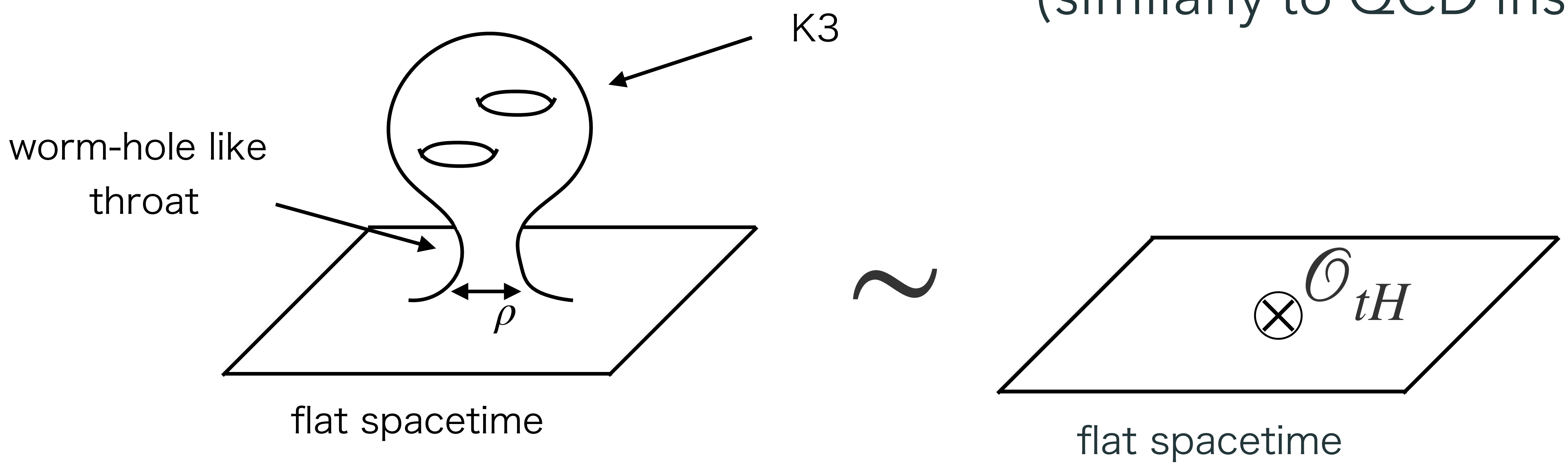
't Hooft vertex

['t Hooft '86]

- axial anomaly of fermions : $\langle \partial^\mu J^A_\mu \rangle \sim R\tilde{R}$

→ Grav. instanton induces an effective vertex called 't Hooft vertex.

(similarly to QCD instanton)



$$\mathcal{O}_{tH} = \lambda_{tH} \det_{st} \bar{\psi}_s \frac{1-\gamma_5}{2} \psi_t + (h.c.)$$

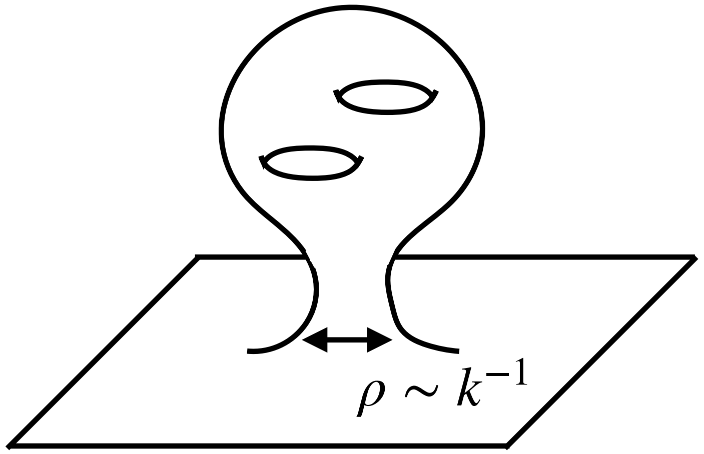
$\cancel{U(1)_A}$ operator

This is not induced by Taub-NUT, Eguchi-Hanson metrics.

't Hooft vertex and asymptotically safe gravity

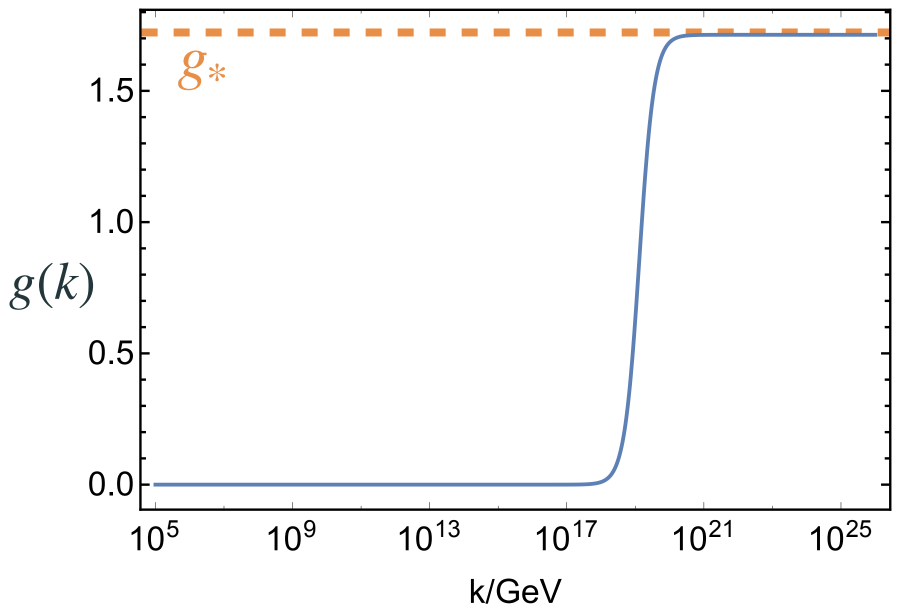
- dim. analysis based on the dilute gas approx.

$$\Rightarrow \lambda_{tH}(k) \sim k^{-2} e^{-S_{inst.}}$$



instanton action: $S_{inst.} \simeq S_{wormhole} \simeq \dfrac{1}{g_N(k)}$

[arXiv:1709.03696]



UV ($k \gg M_{pl}$) : strong (scale invariant)

IR ($k \ll M_{pl}$) : exponentially suppress

What does this affect ? $\rightarrow$ causes chiral sym. breaking: $\langle \bar{\psi}\psi \rangle \neq 0$

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Chiral sym. breaking induced by grav. instanton

Nambu-Jona-Lasinio model

- 2-flavor NJL model + gluon and gravity

$$\boxed{\Gamma_k = \int d^4x\sqrt{g}\left[\bar{\psi} i \nabla\!\!\!/ \psi + \underline{\lambda_q \mathscr{L}_q} + \underline{\lambda_{tH}\mathscr{L}_{tH}}\right] + \frac{1}{16\pi G}\int d^4x\sqrt{g}\, R + \frac{1}{4g_s^2}\int d^4x\sqrt{g}\, (F^a_{\mu\nu})^2}$$

$\sigma - \pi$ channel : $\mathscr{L}_q = \left(\bar{\psi}\psi\right)^2 - \left(\bar{\psi}\gamma_5\sigma^a\psi\right)^2$

't Hooft vertex : $\mathscr{L}_{tH} = \frac{1}{2}\det_{st}\, \bar{\psi}_s \frac{1-\gamma_5}{2}\psi_t + (h.c.)$

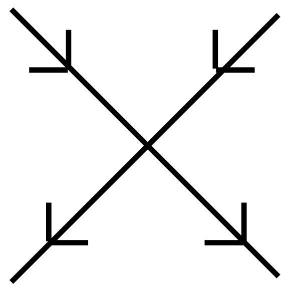
- global $SU(2)_L \times SU(2)_R$ symmetry

- RG analysis for running coupling: $\lambda_{tH}, \lambda_q, g_N, g_s$

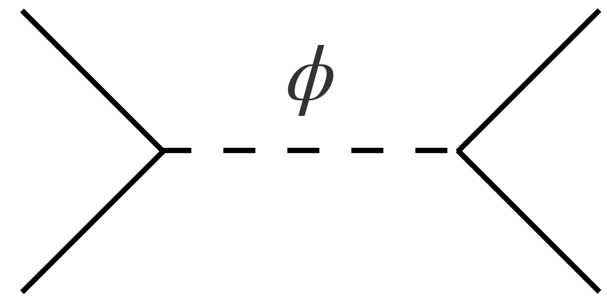
Chiral sym breaking = divergence of λ_q

- In NJL model, Chiral sym. breaking = $\boxed{\lambda_q \to \infty \text{ at IR}}$
- Bosonization :

$\lambda_q \left(\bar{\psi}\psi\right)^2$

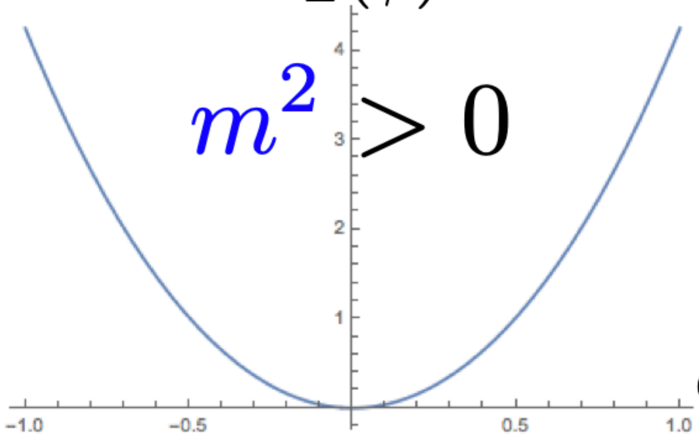


$\phi \sim \bar{\psi}\psi$



$$V(\phi) = m^2\phi^2 + \cdots \qquad m^2 = \frac{1}{\lambda_q}$$

$V_L(\phi)$

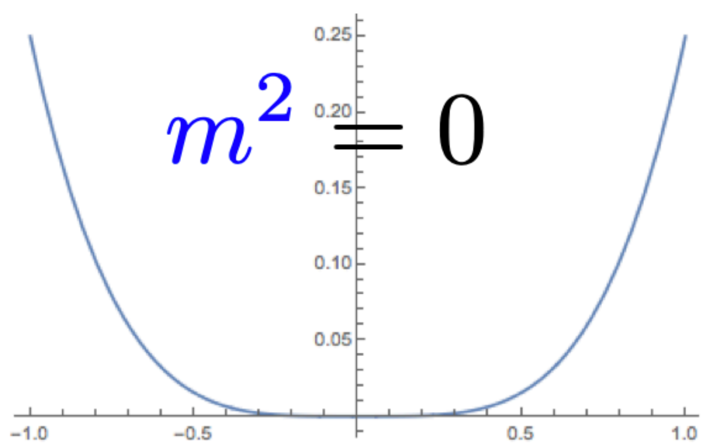


$m^2 > 0$

ϕ

$\langle\phi\rangle = 0$

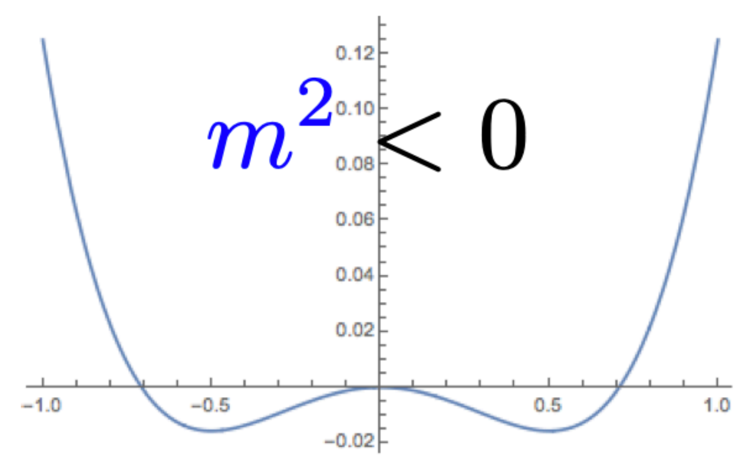
χ -symmetric



$m^2 = 0$

$\langle\phi\rangle = 0$

2nd order phase transition



$m^2 < 0$

$\langle\phi\rangle \neq 0$

χ -breaking

[Yamada-san's slide]

RG flow eq. for four-fermion couplings

- Functional renormalization group (Wetterich equation):

$$\partial_t \Gamma_k = \frac{1}{2}\mathrm{Tr}\left[\left(\Gamma_k^{(2)} + R_k\right)^{-1} \partial_t R_k\right]$$

$t \equiv \log k$ $\quad$ R_k : cutoff function

- RGE in the absence of grav. instantons:

[Eichhorn-Gies '11]

[Braun-Leonhardt-Pospiech '18]

$$\boxed{\begin{aligned}\partial_t \bar{\lambda}_q &= 2\bar{\lambda}_q - \frac{7}{2\pi^2}\bar{\lambda}_q^2 - \frac{8}{\pi^2}(\bar{\lambda}_q + \bar{\lambda}_{\mathrm{tH}})\bar{\lambda}_{\mathrm{tH}} + \cdots \\ \partial_t \bar{\lambda}_{\mathrm{top}} &= 2\bar{\lambda}_{\mathrm{tH}} + \frac{13}{3\pi^2}\bar{\lambda}_{\mathrm{tH}}^2 + \frac{1}{6\pi^2}\left(4\bar{\lambda}_{\mathrm{tH}} + \bar{\lambda}_q\right)\bar{\lambda}_q + \cdots\end{aligned}}$$

dim. less couplings: $\bar{\lambda}_q \equiv k^2 \lambda_q\,,\ \bar{\lambda}_{tH} \equiv k^2 \lambda_{tH}$

We have omitted contributions from simple graviton and gluon exchanges.

RG flow of four-fermion couplings

- It is difficult to derive RG eq. in the presence of grav. instantons.

→ **write down naive RG eq. using free parameters**

$$\partial_t \bar{\lambda}_q = 2\bar{\lambda}_q - \frac{7}{2\pi^2}\bar{\lambda}_q^2 - \frac{8}{\pi^2}(\bar{\lambda}_q + \bar{\lambda}_{\mathrm{tH}})\bar{\lambda}_{\mathrm{tH}}$$

$$\partial_t \bar{\lambda}_{\mathrm{tH}} = 2\bar{\lambda}_{\mathrm{tH}} + \frac{13}{3\pi^2}\bar{\lambda}_{\mathrm{tH}}^2 + \frac{1}{6\pi^2}\left(4\bar{\lambda}_{\mathrm{tH}} + \bar{\lambda}_q\right)\bar{\lambda}_q$$

$$+\gamma^{(1)} \underbrace{\frac{\beta_{g_N}}{g_N^2} \exp\left(-\frac{1}{g_N}\right)}_{\text{additive}} + \gamma^{(2)} \underbrace{\bar{\lambda}_{tH} \exp\left(-\frac{1}{g_N}\right)}_{\text{multiplicative}}$$

∂_t

single inst. $\sim \exp\left(-\frac{1}{g_N(k)}\right)$

$\gamma^{(i)}$: free parameters

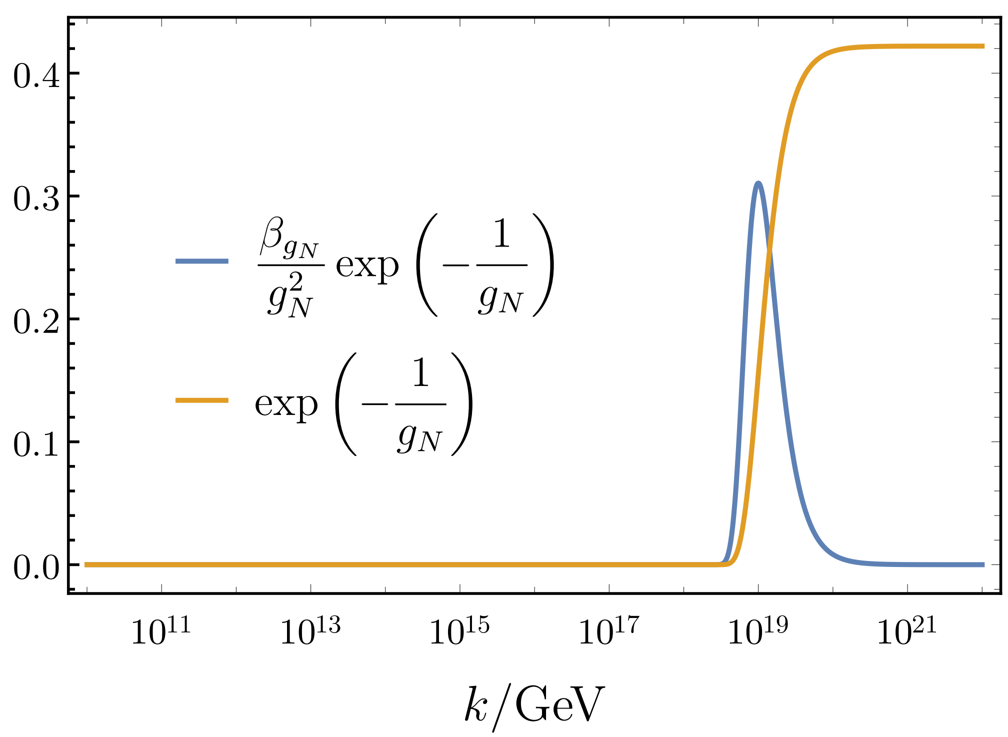
Fixed pt. annihilation

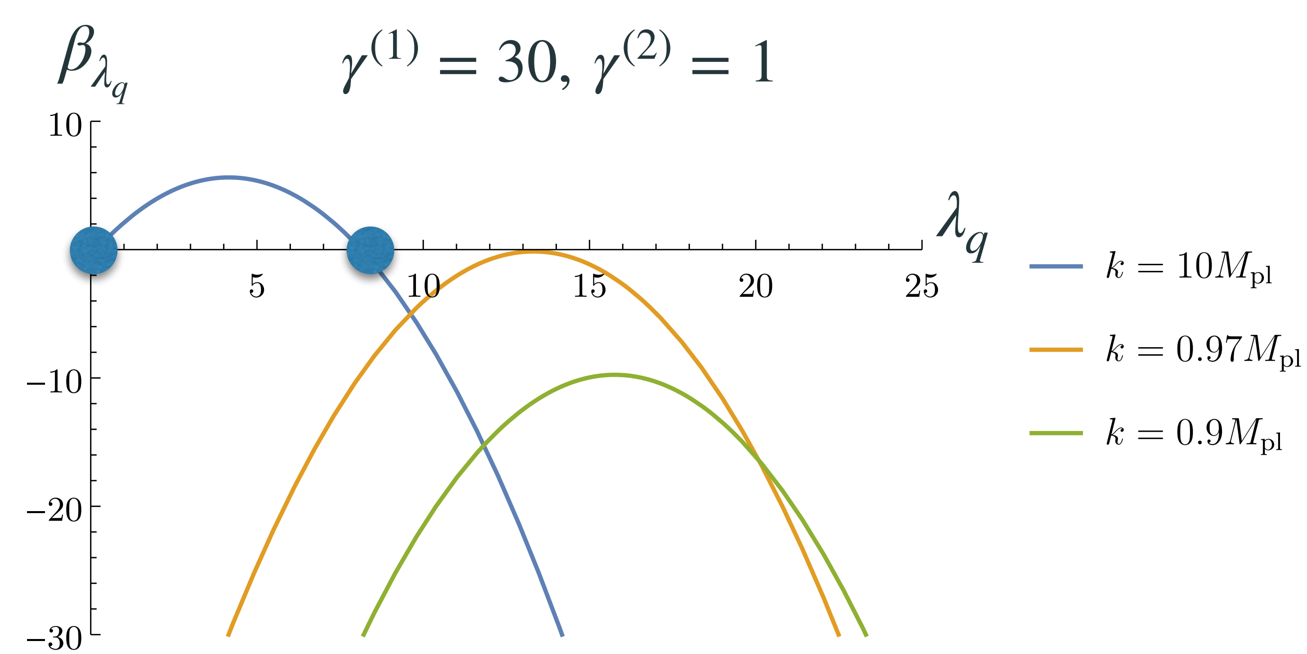
$$\partial_t \bar{\lambda}_q = 2\bar{\lambda}_q - \frac{7}{2\pi^2}\bar{\lambda}_q^2 - \frac{8}{\pi^2}(\bar{\lambda}_q + \bar{\lambda}_{\rm tH})\bar{\lambda}_{\rm tH}$$

$$\partial_t \bar{\lambda}_{\rm tH} = 2\bar{\lambda}_{\rm tH} + \frac{13}{3\pi^2}\bar{\lambda}_{\rm tH}^2 + \frac{1}{6\pi^2}\left(4\bar{\lambda}_{\rm tH} + \bar{\lambda}_q\right)\bar{\lambda}_q$$

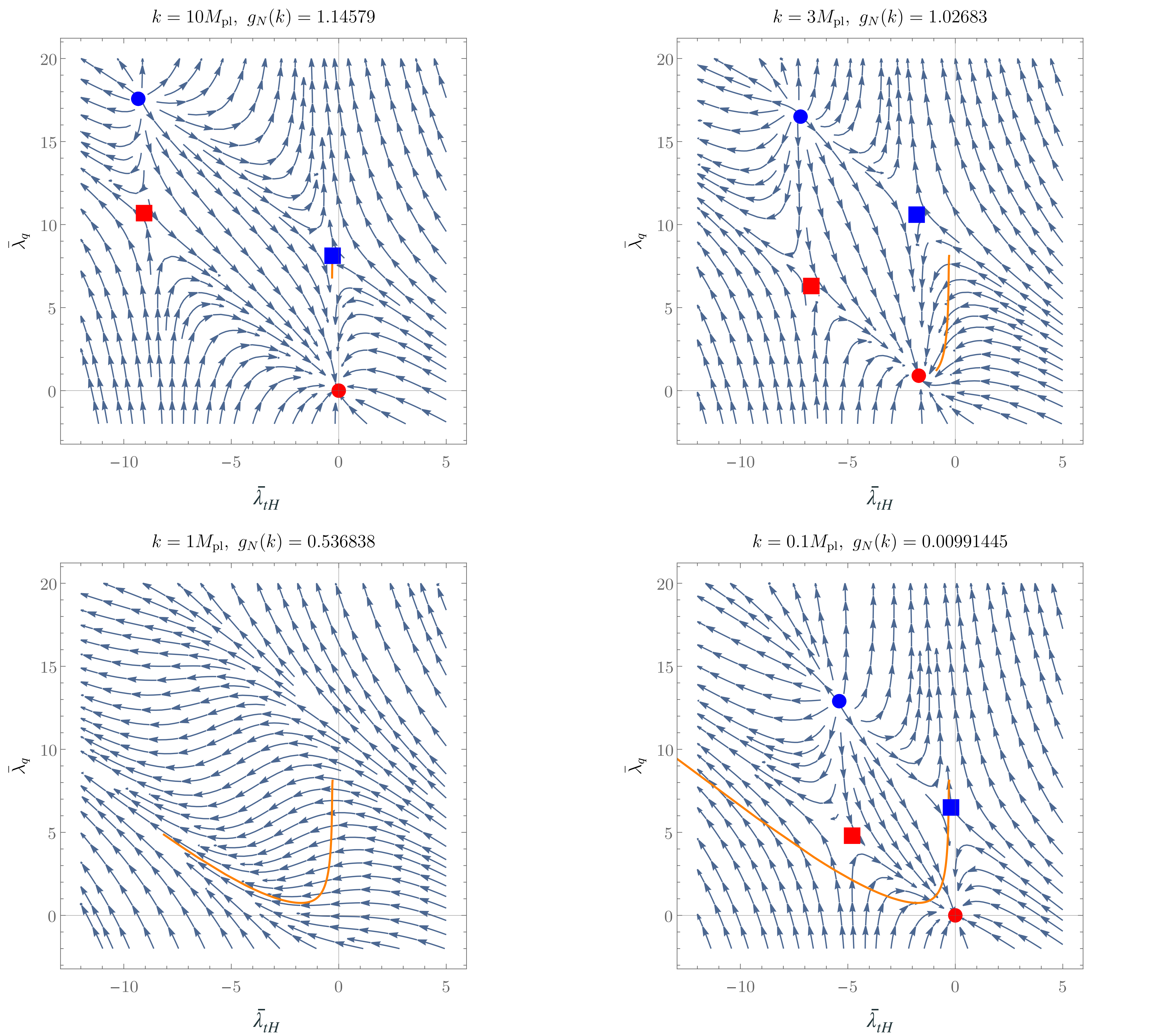
$$+\gamma^{(1)} \underbrace{\frac{\beta_{g_N}}{g_N^2} \exp\left(-\frac{1}{g_N}\right)}_{\text{additive}} + \gamma^{(2)} \underbrace{\bar{\lambda}_{tH} \exp\left(-\frac{1}{g_N}\right)}_{\text{multiplicative}}$$

- The former one is large around $k \sim M_{pl}$ → **IR fixed pt. disappear!**





Result of the flow $\gamma^{(1)} = 30,\ \gamma^{(2)} = 1$



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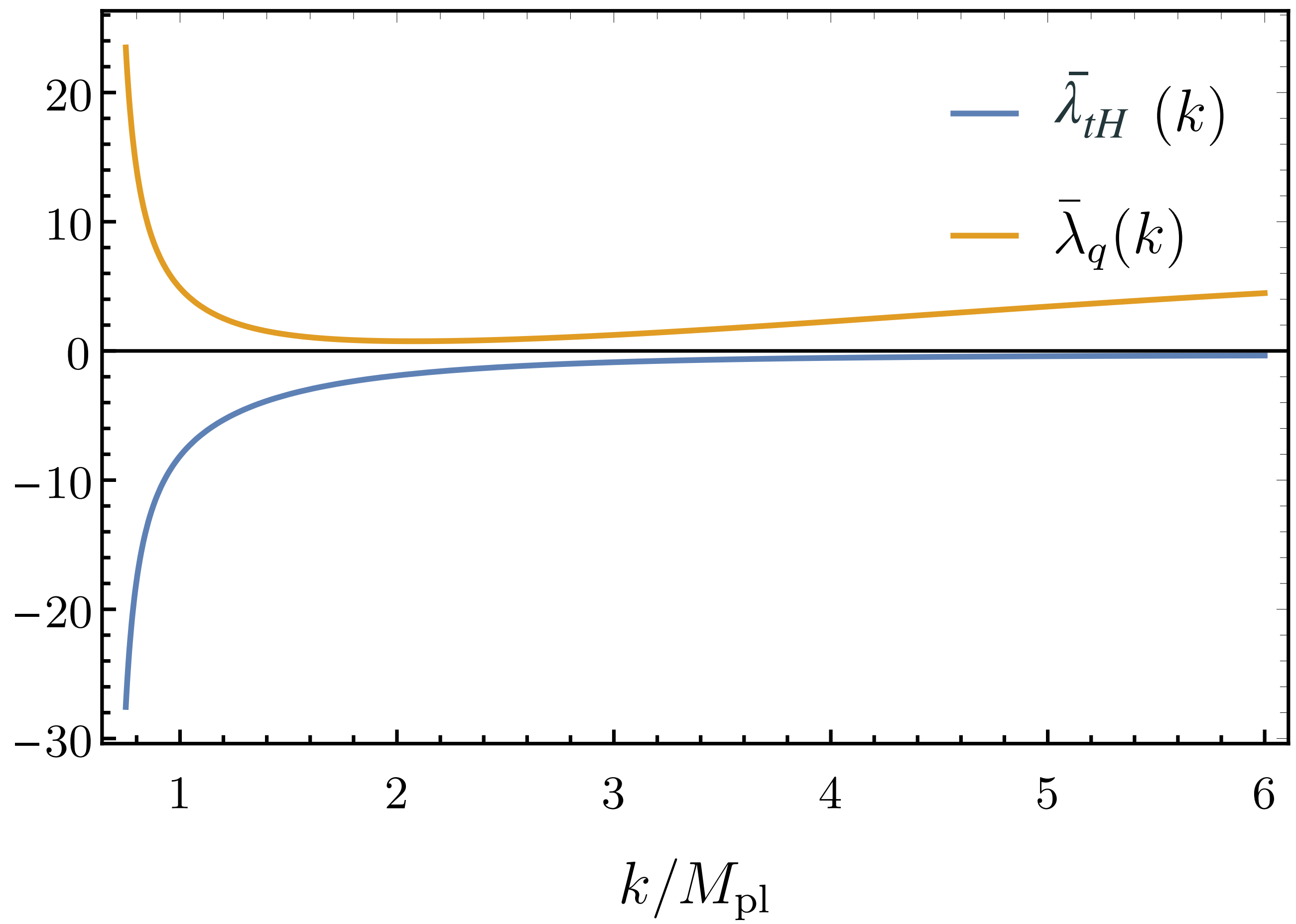
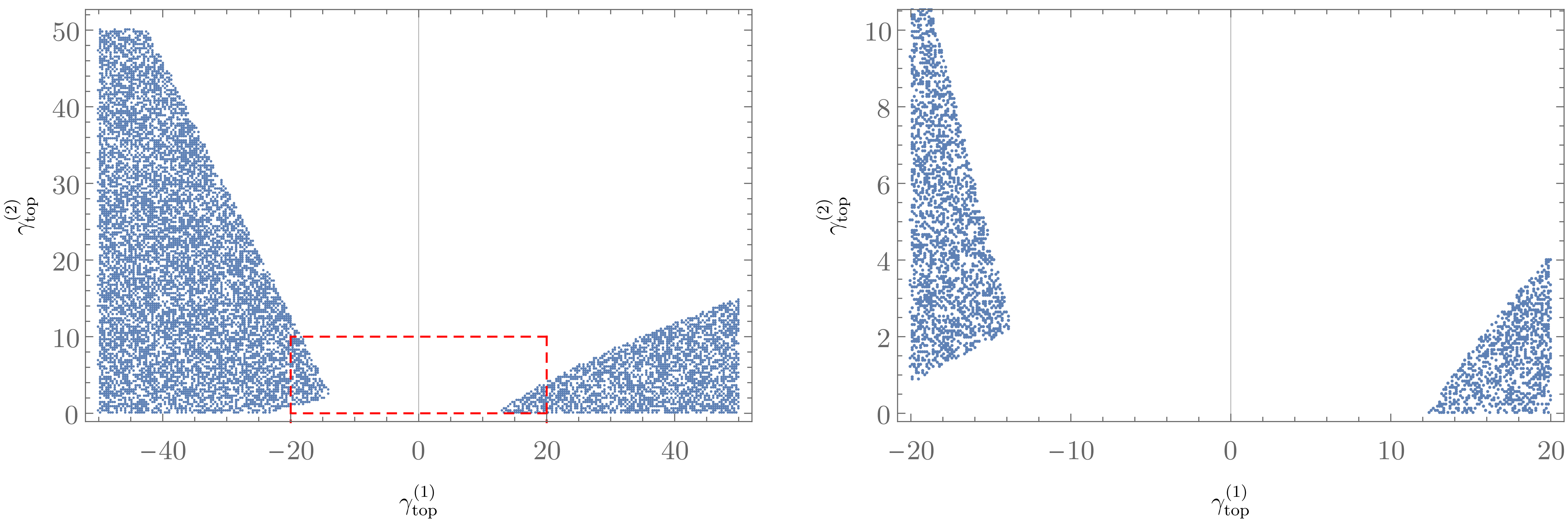


- $\bar{\lambda}_q$ diverges at $k \sim \mathcal{O}(0.1)\, M_{pl}$ → **χ-sym breaking !**

Parameter space

- How large free parameters $\gamma^{(i)}$ cause the chiral sym. breaking?



- **Sym. breaking is universal for $|\gamma^{(1)}| \gtrsim 12$.**

- This region leads to heavy fermions, and hence **phenomenologically excluded.**

Summary

- Gravity is strong coupling in the asymptotic safety scenario.

- **Gravitational instantons induce the 't Hooft vertex for fermions.**

- FRG analysis in NJL-like model

→ **Chiral sym. breaking $\langle\bar{\psi}\psi\rangle \neq 0$ occurs for some parameter region.**

- **Such parameter space is phenomenologically excluded.**

Future works:

- Parameters $\gamma^{(i)}$ are calculable in principle

  →**We can constrain UV theories.**

- Probe topological structure of spacetime using matter.

Back up

Asymptotically safe SM?

[arXiv: 1810.07615]

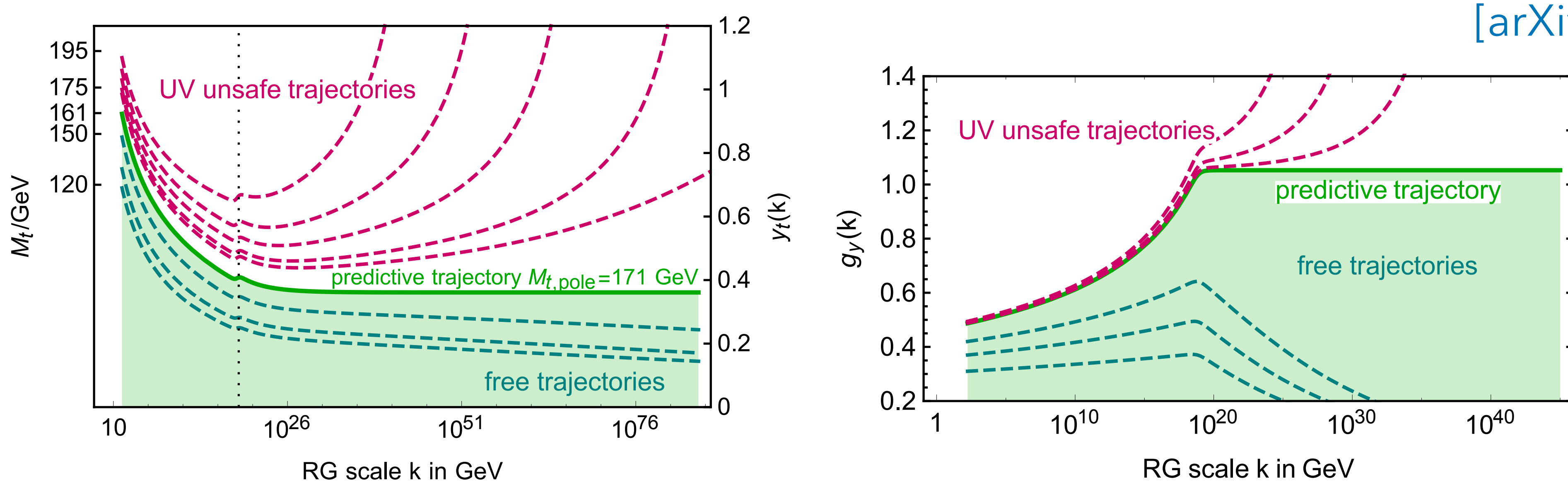


Figure 12: From [355] and [328].

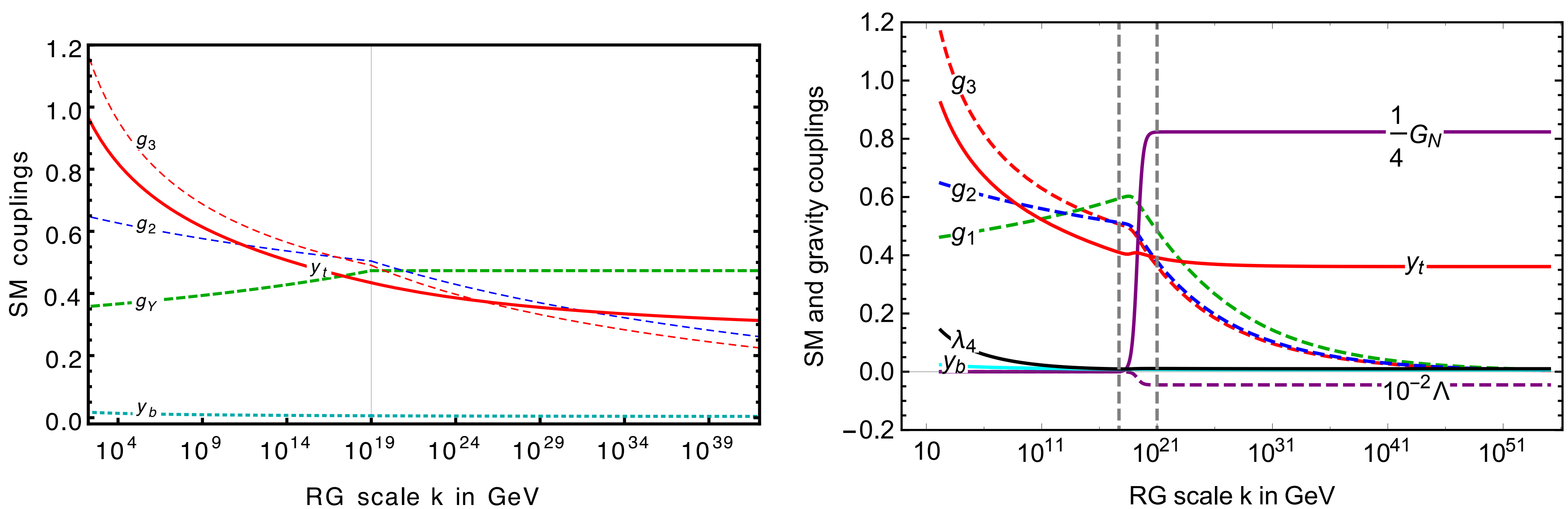


Figure 13: Both panels: RG flows in an approximation as in Eq. (56), see [354] and [355] for details. Left panel: Flow of gauge couplings and top and bottom Yukawa with quantum-gravity parameterized by $f_g = 9.8 \cdot 10^{-3}$ and $f_y = 1.13 \cdot 10^{-4}$ above the Planck scale and $f_g = 0 = f_y$ below the Planck scale as in [354]. Right panel: Standard-Model RG flow including running gravitational couplings as in [215] and is taken from [355].

Renormalization group (Wilsonian)

- Starting from a theory with UV cutoff Λ, let's construct an effective theory with energy scale k

→ integrating out only d.o.f. with momentum higher than k

$$e^{-S_k[\phi]} \equiv \int \mathscr{D}\phi_{|p|>|k|} \; e^{-S_\Lambda[\phi]}$$

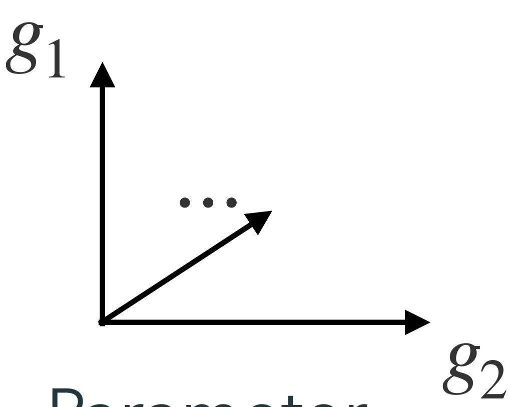
- Physics at energy scale k is described by the effective action S_k :

Running coupling constant

$$S_k[\phi] = \int d^4x \left[\frac{1}{2}(\partial_\mu \phi)^2 - \frac{g_1(k)}{4!}\phi^4 - \frac{g_2(k)}{6!}\phi^6 + \cdots \right]$$

Renormalization group flow

- k-dependence of running parameters are expressed by a flow in the parameter space.



Parameter space

$$\partial_t g_i(k) = -\beta_i(g(k)) \qquad t \equiv -\log k$$

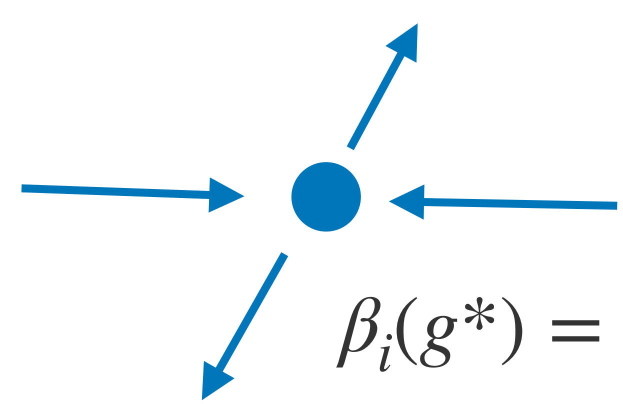
IR: $k = \mu$ $(t = e^{\Lambda/\mu})$

UV: $k = \Lambda$ $(t = 0)$

- The flow does not move at a fixed point : $g_i = g_i^*$

flowing-out direction: **relevant**

flowing-in direction: **irrelevant**

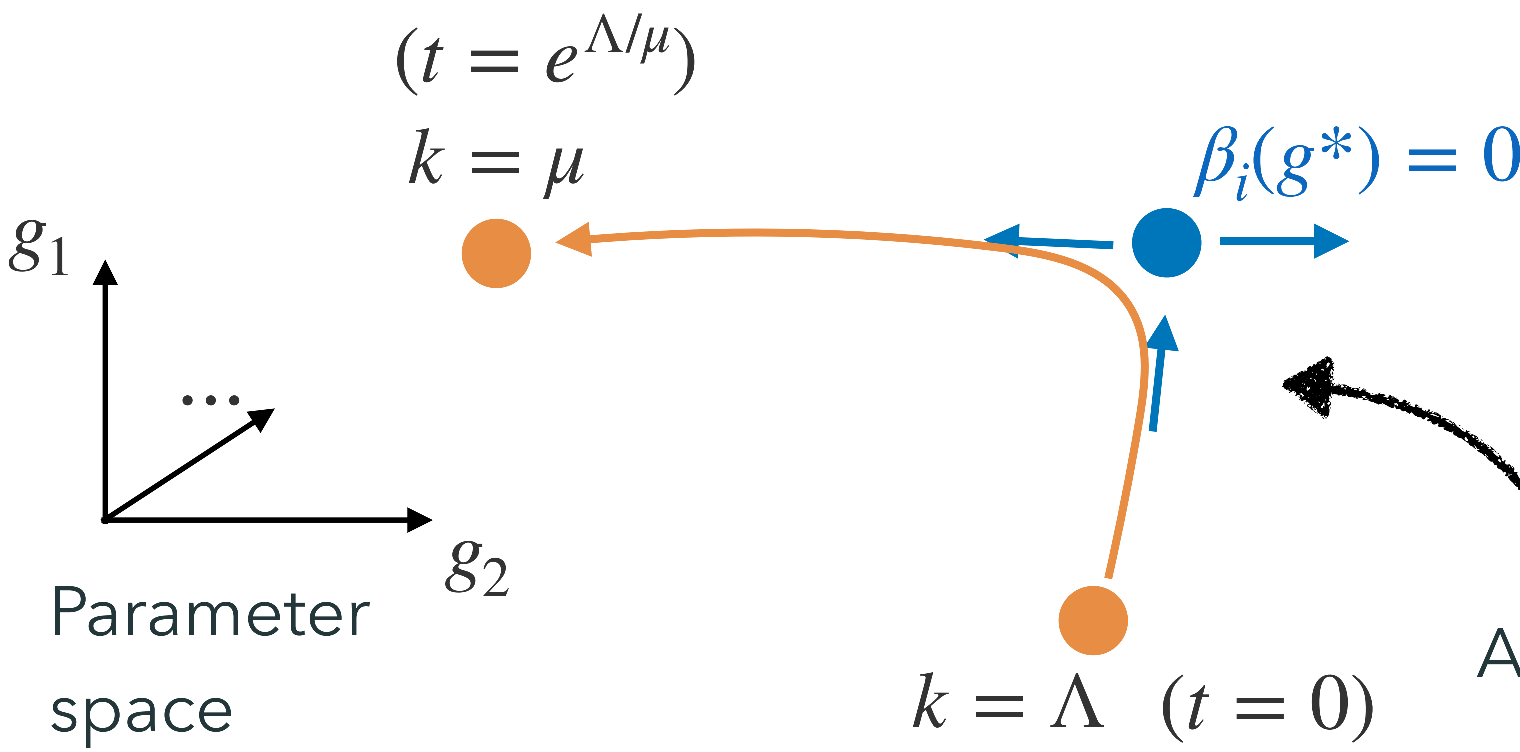


$$\beta_i(g^*) = 0$$

Renormalizability

- Is it possible to **take the continuum limit $\Lambda \to \infty$ ($e^{\Lambda/\mu} \to \infty$) keeping all parameters finite?**

- To do so, tune the parameters at $k = \Lambda$ s.t. the flow passes nearby a fixed pt.



At $g_i \simeq g_i^*$, the flow is very slow.

Renormalizability

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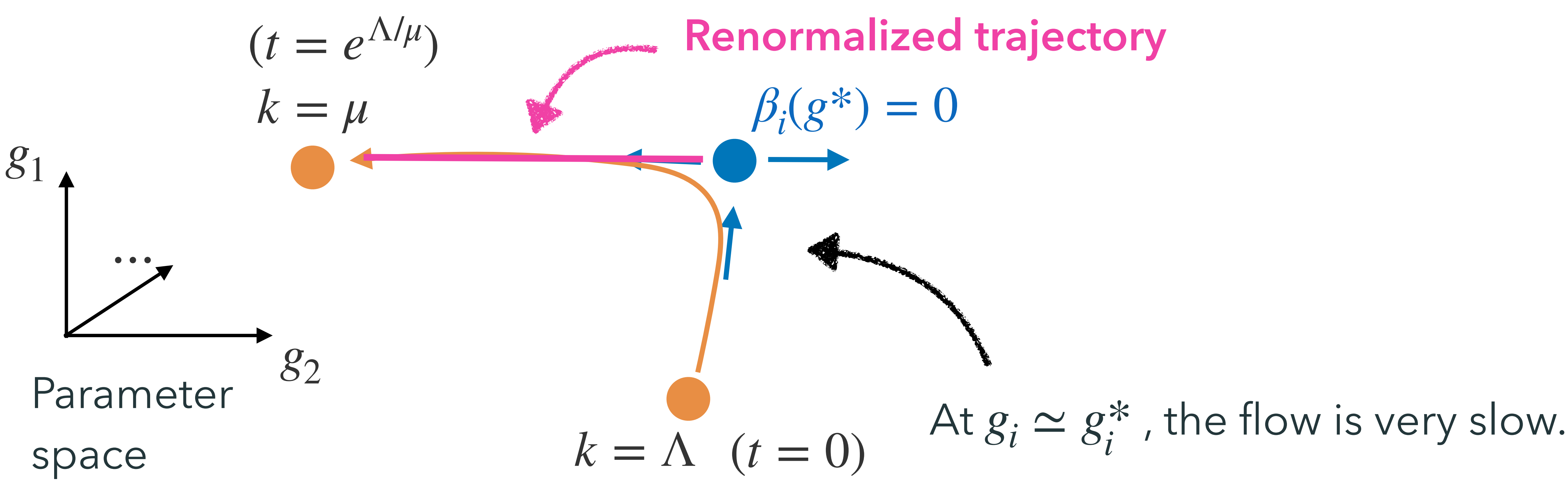

Renormalizability

- Is it possible to **take the continuum limit $\Lambda \to \infty$ ($e^{\Lambda/\mu} \to \infty$) keeping all parameters finite?**

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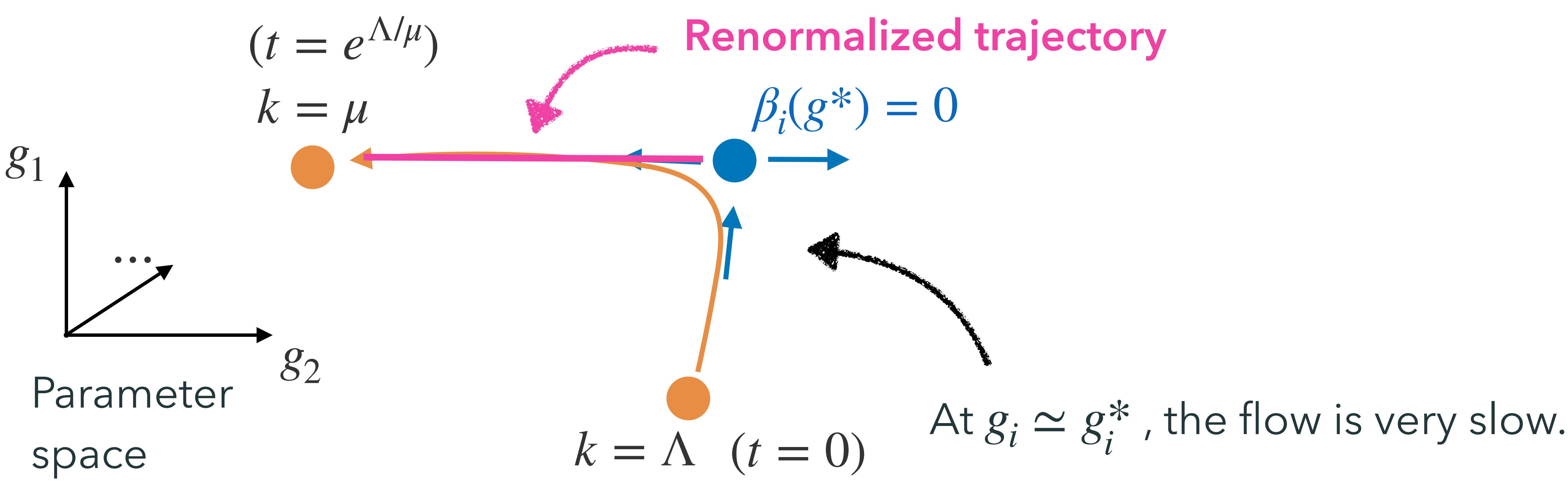
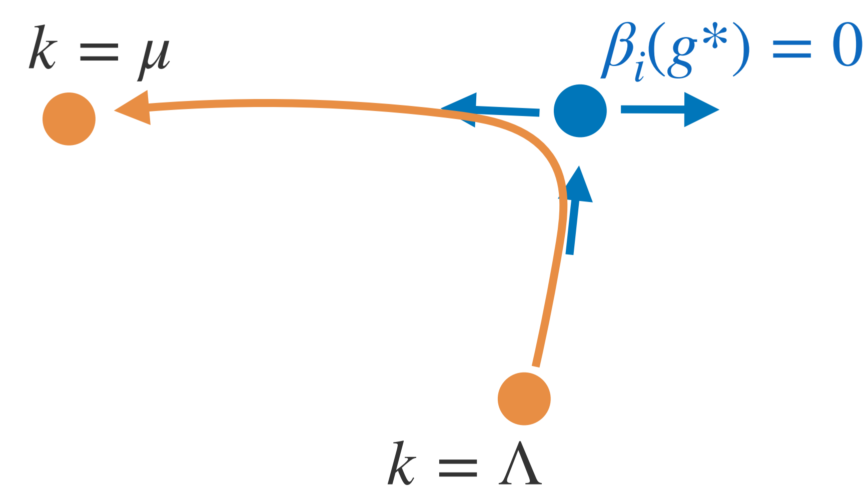


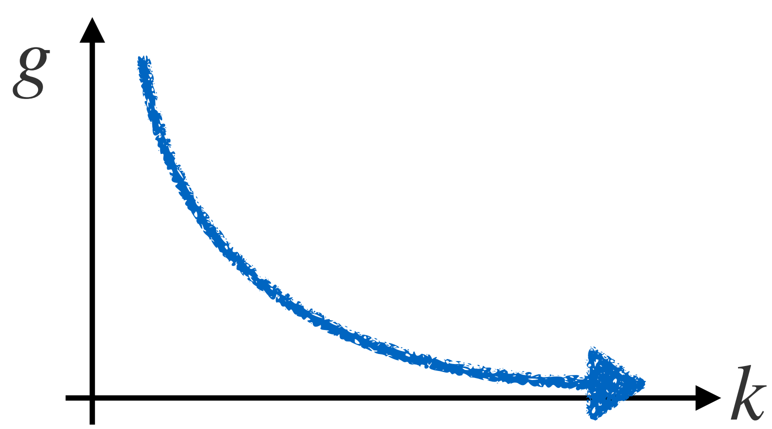
- Then, the parameters at $k = \mu$ are finite and insensitive to UV physics → **This is renormalizability!**

Asymptotic safety

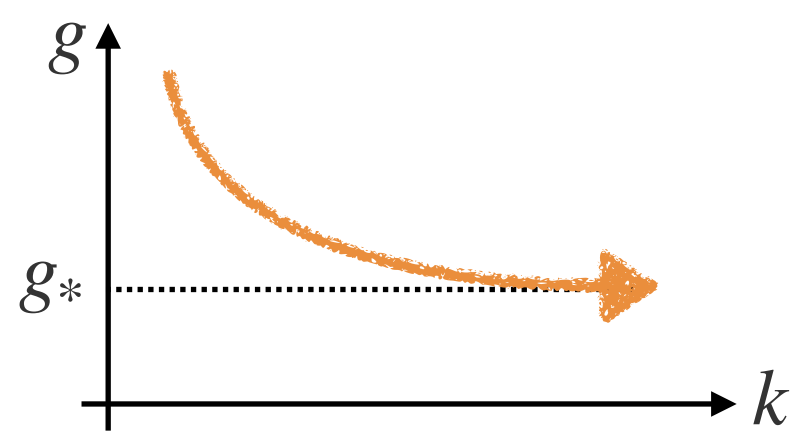
[S. Weinberg, '80]

- A theory which has a non-trivial RG fixed pt. at UV is called **asymptotically safe** and is **non-perturbatively renormalizable**.





asymptotic freedom
(e.g. QCD)



asymptotic safety
(e.g. non-linear σ)

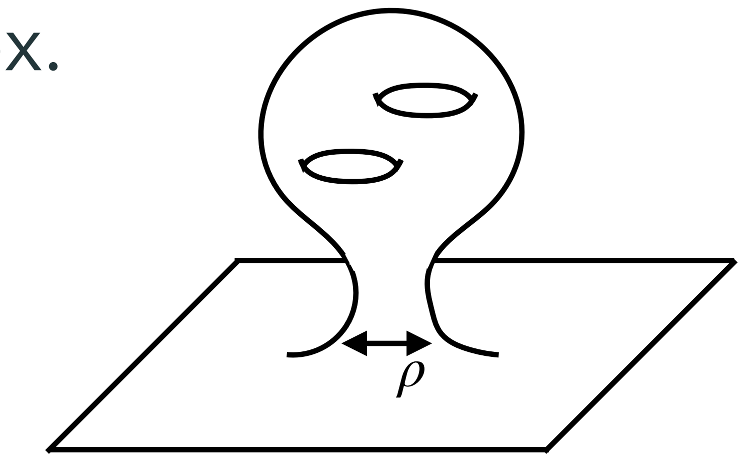
- **Recently, a possibility has been pointed out that gravity is asymptotically safe and is a consistent QFT.**

→Asymptotic safety scenario of quantum gravity

't Hooft vertex and asymptotically safe gravity

- dim. analysis based on the dilute gas approx.

$$\lambda_{tH} \sim \rho^2 \exp\left(-\frac{\rho^2}{G}\right)$$

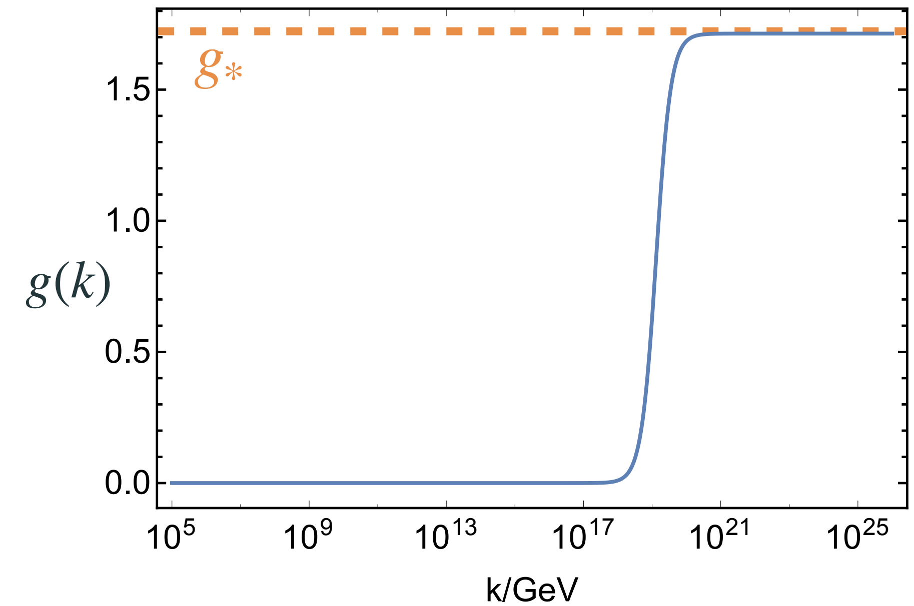


- assume an energy scale k

$$\rho \to k^{-1} \qquad G \to G(k) = g_N(k)/k^2$$

$$\Rightarrow \lambda_{tH}(k) \sim k^{-2} \exp\left(-\frac{1}{g_N(k)}\right)$$

[arXiv:1709.03696]



UV ($k \gg M_{pl}$) : instantons are active (scale invariant)

IR ($k \ll M_{pl}$) : exponentially suppress

What does this affect ? $\to$ causes chiral sym. breaking: $\langle \bar{\psi}\psi \rangle \neq 0$

Bosonization

$$Z = \int \mathscr{D}\psi \mathscr{D}\bar{\psi} e^{-S[\psi,\bar{\psi}]} \qquad S[\psi,\bar{\psi}] = i\bar{\psi}\partial\psi + \frac{1}{2}\lambda_\sigma \left[(\bar{\psi}\psi)^2 - (\bar{\psi}\gamma_5\sigma^a\psi)^2\right]$$

multiply: $1 = N\int \mathscr{D}\phi \ e^{-\int d^4x\left[\frac{1}{2}m_\phi^2 \overrightarrow{\phi}^2\right]}$

Shift: $\phi \to \phi + \frac{i}{\sqrt{2}m_\sigma^2}(\bar{\psi}\psi)$ with $m_\sigma^2 = 1/\lambda_\sigma$

$$\vec{\tau} = (i, \gamma_5)$$

$$\to S[\psi,\bar{\psi},\phi] = \int d^4x \left[i\bar{\psi}\partial\psi + \frac{1}{2}m_\sigma^2 \overrightarrow{\phi}^2 + \frac{i}{\sqrt{2}}\bar{\psi}(\vec{\tau}\cdot\overrightarrow{\phi})\psi\right]$$

EOM: $\overrightarrow{\phi} = \frac{1}{\sqrt{2}m_\sigma^2}\vec{\tau}\ (\bar{\psi}\psi)$ でもとに戻る

$\therefore \langle|\phi|\rangle \neq 0 \Rightarrow \langle\bar{\psi}\psi\rangle \neq 0$

$\overrightarrow{\phi}$ を回す方向がpion: NG mode